Un modelo matemático para la dinámica de multitudes

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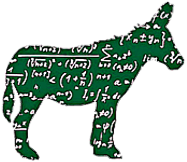
BIOMAT 2014
La Falda, Agosto de 2014



Some common features of living systems

- **Ability to develop a strategy**: living entities are able to develop specific strategies and have organization abilities (e.g. for crowds: trend toward the exit, avoiding clusters, avoiding walls and obstacles, perception of signals, etc).
- **Heterogeneity**: the ability to express the said strategy is not the same for all entities
- **Interactions**: living entities interact with other entities and with the surrounding environment in a non-local and non-linear way

The choice of the scale for crowd dynamics

* **Microscopic**: Pedestrians identified singularly by $\mathbf{x} = \mathbf{x}(t)$ and $\mathbf{v} = \mathbf{v}(t)$ $\rightarrow$ Large systems of ODE's [Helbing et al.: "Social force model", 1995]

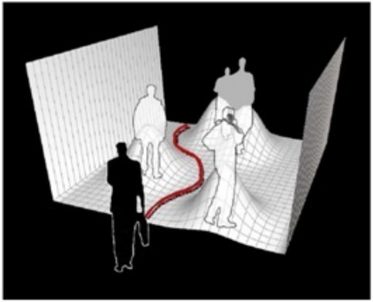
* **Macroscopic**: The crowd is assimilated to a continuum, its state being described by average quantities (density, linear momentum, and energy) regarded as time and space-dependent variables $\rightarrow$ Systems of PDE's [Hughes: a first order model, 2002]

$*$ The mean distance between pedestrians may be small or large, the ratio between the mean free path and the geometrical length scale (Knudsen number) spans a wide range of values in the same computational domain.

$*$ Using the macroscopic representation becomes a complex task because of the breakdown of continuum models in some regions of the physical domain.

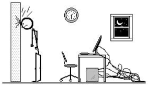
$*$ The study of living complex systems always needs a **multiscale approach**, where the dynamics at the large scale need to be properly related to the dynamics at the low scales.

* **Mesoscopic** (kinetic): The microscopic state of pedestrians is still identified by position and velocity but the system is represented statistically through a distribution function over such a microscopic state → Integro-differential equations



[J.P. Agnelli, F. Colasuonno, D. Knopoff, *A kinetic theory approach to the dynamics of crowd evacuation from bounded domains*, Math. Models Methods Appl. Sci., 25(1) (2015)]

- **What**: Evacuation of pedestrians from a room with one or more exits.
- **How**: Development of the kinetic approach by [N. Bellomo, A. Bellouquid, D. Knopoff (2013)] to include

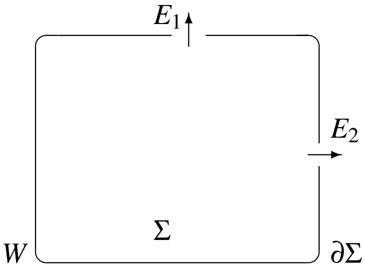


$\ast$ interactions with walls

$\ast$ flow through exit doors

Description of the system

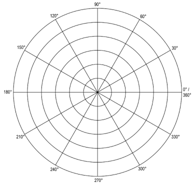
- Bounded domain $\Sigma \subset \mathbb{R}^2$, assumed convex (no obstacles)
- $E \subset \partial\Sigma$ outlet zone (exit), E could be the union of disjoint sets
- $W = \partial\Sigma \setminus E$ wall



- Pedestrians are viewed as **active particles**
- Microscopic state (continuous-discrete hybrid features):
  - * Position: $\mathbf{x} = (x, y)$
  - * Velocity : $\mathbf{v} = (v, \theta)$

The velocity direction θ takes values in the discrete set

$$I_\theta = \left\{ \theta_i = \frac{i-1}{n} 2\pi \, : \, i = 1, \dots, n \right\}$$



Generalized distribution function

We neglect the heterogeneity of pedestrians in changing the velocity modulus

- changes of the velocity direction θ: stochastic
- changes of the velocity modulus v: deterministic

From now on we consider the generalized distribution function

$$f(t,\mathbf{x},\theta) = \sum_{i=1}^{n} f_i(t,\mathbf{x})\delta(\theta-\theta_i), \quad f_i(t,\mathbf{x}) = f(t,\mathbf{x},\theta_i)$$

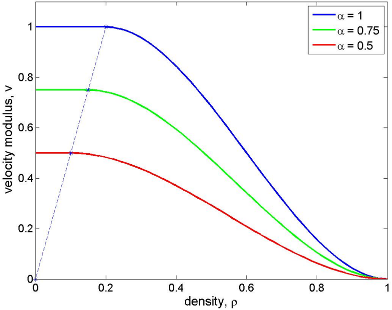
$f_i(t,\mathbf{x})d\mathbf{x}$ = number of pedestrians who, at time t are in the infinitesimal rectangle $[x, x+dx] \times [y, y+dy]$ and move with direction θ_i

- Local density: $\rho(t,\mathbf{x}) = \sum_{i=1}^{n} f_i(t,\mathbf{x})$

Velocity modulus and perceived density

Velocity modulus depends on

- (perceived) level of congestion
- quality of the environment, assessed by a parameter $\alpha \in [0, 1]$



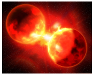
In the free flow zone ($\rho \leq \rho_c(\alpha) = \alpha/5$) pedestrians move with the maximal speed $v_m(\alpha) = \alpha$ allowed by the environment. In the slowdown zone ($\rho > \rho_c(\alpha)$) pedestrians have a velocity modulus which is heuristically modeled by the 3rd order polynomial joining the points $(\rho_c(\alpha), v_m(\alpha))$ and $(1, 0)$ and having horizontal tangent in such points.

The mathematical structure

$$\partial_t f_i(t,\mathbf{x}) + \mathrm{div}_{\mathbf{x}}(\mathbf{v}_i[\rho_i^p(t,\mathbf{x})] f_i(t,\mathbf{x})) = \mathcal{J}_i[f](t,\mathbf{x})$$

$i = 1, \ldots, n$

transport term:
net balance of particles in the elementary volume of the space of the microscopic states, moving with direction θ_i, due to transport



interaction term:
net balance of particles in the elementary volume of the space of the microscopic states, moving with direction θ_i, due to interactions

Interaction dynamics

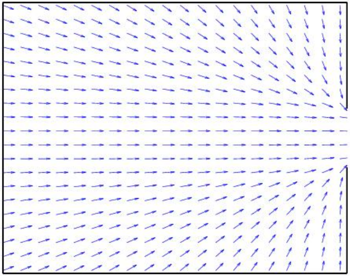
We take into account the following effects:

1. Geometrical effects
   - Exit
   - Walls
2. "Congestion" effects
   - Stream
   - Vacuum

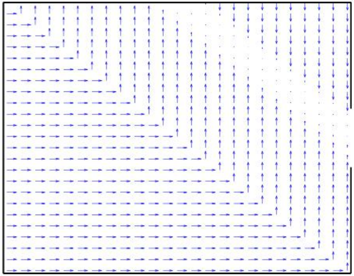
Optimal geometrical direction

$$\vec{\omega}_G(\mathbf{x},\theta_h) = (1-d_E(\mathbf{x}))\vec{\nu}_E(\mathbf{x}) + (1-d_W(\mathbf{x},\theta_h))\vec{\tau}_W(\mathbf{x},\theta_h)$$

whose direction θ_G is the optimal geometrical direction



$\vec{\nu}_E(\mathbf{x})$



$\vec{\tau}_W(\mathbf{x},\theta_8)$

Optimal geometrical direction

$$\vec{\omega}_G(\mathbf{x},\theta_h) = (1 - d_E(\mathbf{x}))\vec{\nu}_E(\mathbf{x}) + (1 - d_W(\mathbf{x},\theta_h))\vec{\tau}_W(\mathbf{x},\theta_h)$$

whose direction θ_G is the optimal geometrical direction

Geometrical transition probability or "table of games"

$$\mathcal{A}_h(i) = \beta_h(\alpha)\delta_{s,i} + (1 - \beta_h(\alpha))\delta_{h,i}, \quad i = 1, \ldots, n$$

where $s := \underset{j\in\{h-1,h+1\}}{\arg\min} \{d(\theta_G,\theta_j)\}$

$$d(\theta_*,\theta^*) = \begin{cases} |\theta_* - \theta^*| & \text{if} \quad |\theta_* - \theta^*| \leq \pi \\ 2\pi - |\theta_* - \theta^*| & \text{if} \quad |\theta_* - \theta^*| > \pi \end{cases}$$

$$\beta_h(\alpha) = \begin{cases} \alpha & \text{if} \quad d(\theta_h,\theta_G) \geq \Delta\theta \\ \alpha^{\frac{d(\theta_h,\theta_G)}{\Delta\theta}} & \text{if} \quad d(\theta_h,\theta_G) < \Delta\theta \end{cases}$$

Optimal interaction-based direction

$$\vec{\omega}_P(\mathbf{x}, \theta_h, \theta_k) = \varepsilon \vec{\sigma}_k + (1-\varepsilon)\vec{\gamma}(\mathbf{x}, \theta_h)$$

θ_P direction of $\vec{\omega}_P$

* ε close to 0 $\rightarrow$ normal conditions
* ε close to 1 $\rightarrow$ panic conditions

"Congestion" transition probability

$$\boxed{\mathcal{B}_{hk}(i)[\rho] = \beta_{hk}(\alpha)\rho\delta_{r,i} + (1-\beta_{hk}(\alpha)\rho)\delta_{h,i}, \quad i = 1, \ldots, n}$$

where $r := \underset{j \in \{h-1, h+1\}}{\arg\min} \{d(\theta_P, \theta_j)\}$

$$\beta_{hk}(\alpha) = \begin{cases} \alpha & \text{if} \quad d(\theta_h, \theta_P) \geq \Delta\theta \\ \alpha^{\frac{d(\theta_h, \theta_P)}{\Delta\theta}} & \text{if} \quad d(\theta_h, \theta_P) < \Delta\theta \end{cases}$$

Some words about panic

[Helbing D., Johansson A., *Pedestrian, Crowd and Evacuation Dynamics*, (2009).]

Panic: *Breakdown of ordered, cooperative behavior of individuals due to anxious reactions to a certain event.*

Typical features of panic conditions

- people develop blind actionism
- move considerably faster
- moving and passing of a bottleneck becomes incoordinated
- herding behavior, i.e. people do what other people do

Interaction terms

$$\mathcal{J}_i[f](t,\mathbf{x}) = \mathcal{J}_i^G[f](t,\mathbf{x}) + \mathcal{J}_i^P[f](t,\mathbf{x})$$

$$\mathcal{J}_i^G[f](t,\mathbf{x}) = \mu[\rho(t,\mathbf{x})]\left(\sum_{h=1}^{n} \mathcal{A}_h(i) f_h(t,\mathbf{x}) - f_i(t,\mathbf{x})\right)$$

$$\mathcal{J}_i^P[f](t,\mathbf{x}) = \eta[\rho(t,\mathbf{x})]\left(\sum_{h,k=1}^{n} \mathcal{B}_{hk}(i)[\rho] f_h(t,\mathbf{x}) f_k(t,\mathbf{x}) - f_i(t,\mathbf{x})\rho(t,\mathbf{x})\right)$$

$\mathcal{J}_i^G$: difference between the gain and the loss of particles moving with direction θ_i due to geometrical effects

$\mathcal{J}_i^P$: difference between the gain and the loss of particles moving with direction θ_i due to interactions among particles

Case-studies

The specific case-studies are selected to analyze the **influence on evacuation time** of

1. the **exit size**
2. the **initial distribution**
3. the **parameter ε**

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In our simulations:

- Σ square domain of side length 10 m
- quality of the environment $\alpha = 1$
- 8 different velocity directions in $I_\theta = \left\{\frac{i-1}{8}2\pi : i = 1, \ldots, 8\right\}$

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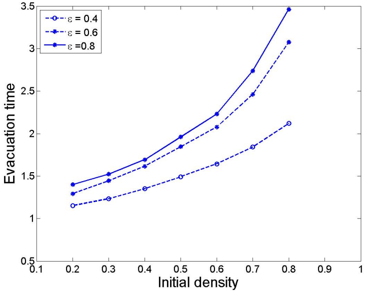
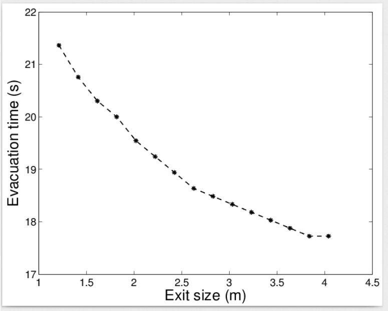
The influence of initial distribution and exit size





* density referred to $\rho_M = 7$ ped/m^2,
* velocity modulus referred to $v_M = 2$ m/s,
* time referred to the minimal evacuation time $T_{ev,0} = 13$ s.

The role of the parameter ε