INTERTEMPORAL DISTRIBUTIONS OF A RICE PILE MODEL

PABLO M. GLEISER*
Departament de Fisica Fonamental (5ta. Planta), Facultat de Fisica
Universitat de Barcelona, Diagonal 647, 08028 Barcelona, España
gleiser@fnn.ub.es

SERGIO A. CANNAS† and FRANCISCO A. TAMARIT‡
Facultad de Matemática, Astronomía y Física (FaMAF)
Universidad Nacional de Córdoba, Ciudad Universitaria
Córdoba, 5000, Argentina
†*cannas@famaf.unc.edu.ar*
‡*tamarit@famaf.unc.edu.ar*

In this work we study the distribution of time intervals between avalanches in a rice pile model. This model has shown that the crossover from power law to stretched exponential behaviors observed experimentally in the granular dynamics of rice piles can be well described as a long-range effect resulting from a change in the transport properties of individual grains. In this work we show that the change in the transport properties is also reflected in the behavior of the distribution of time intervals between avalanches.

Keywords: Self-organized criticality; granular media; avalanches.

1. Introduction

In 1996 Frette *et al.*[1,2] performed a series of experiments where rice grains were slowly added in a narrow gap between two glass plates. They found that the avalanche size distribution for grains with a large aspect ratio presents a power law behavior, while a stretched exponential behavior is observed for rounder grains. The rice pile experiments showed that self-organized criticality (SOC)[3] is not a universal phenomenon and depends on the microscopic structure of the grains. The dynamics for elongated grains is dominated by local mechanisms and displays SOC, while for rounder grains the effects of inertia leads to a nonlocal process, and the system does not display SOC.

*Permanent address: Facultad de Matemática, Astronomía y Física (FaMAF), Universidad Nacional de Córdoba, Ciudad Córdoba, 5000, Argentina.

In a recent work we have presented a new microscopic model for granular flow[4] where both stretched exponential and power law avalanche size distributions are observed. In this model a single parameter ρ controls the average rolling distance of individual grains, which is expected to depend on its aspect ratio. The ability of an individual grain to roll a distance r is described by a long-range rolling probability of the form

$$P(r) = \begin{cases} \dfrac{A}{r^\rho} & \text{if } 1 \leq r \leq L\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{1}$$

where A is a normalization constant, L is the system size, $r = 1, 2, \ldots, L$ and $0 \leq \rho \leq \infty$. The $\rho \to \infty$ limit corresponds to a deterministic nearest neighbors movement ($P(r) = \delta_{r,1}$), thus describing the case where the grains do not roll, and recovering the Oslo model.[5] In the opposite limit, $\rho = 0$, the mean rolling distance is $\langle r \rangle \approx (1/2)L$, and the grains propagate typically halfway through the system, independently of the system size L. This behavior is consistent with the one observed in the experiments with round rice.[1] Within this scenario two distinct regimes appear regarding the qualitative behavior of the avalanche distribution: a short-range *sliding* regime[4] characterized by a power law avalanche size distribution for large avalanches, and a long-range *rolling* regime[4] characterized by a stretched exponential avalanche size distribution.

Given two consecutive time steps the energy difference between the profiles was used in the rice pile experiments[1] to measure the energy dissipated after an avalanche. However, we have observed, in the numerical simulations of our model, that the change in energy after an avalanche may be either positive or negative. After a big avalanche the potential energy clearly decreases. However, if a single grain is dropped into the system and rolls a distance r without affecting any other grains, the potential energy of the new profile will be higher than the profile before adding the grain. Thus it is important to distinguish between these two possible kind of events. One kind increases the potential energy of the system after the addition of a grain, while the other decreases the potential energy. In the following section we will define the model and characterize these different kind of events.

2. The Model

Our model[4] is defined as follows. We consider a one-dimensional lattice of size $L\,(1 \leq i \leq L)$, each site i having associated an integer variable $h(i)$ representing the local height of the pile. The local slope is then given by $\sigma(i) = h(i) - h(i+1)$. The grains enter into the system from the left ($i = 1$) and may drop off at the rightmost site $i = L + 1$, imposing $h(L + 1) = 0$ for all times. Every time the local slope $\sigma(i)$ of a site i exceeds a local critical value $\sigma(i) > \sigma_c(i)$, the topmost grain at site i rolls r sites to the right with probability $P(r)$ given by Eq. (1). Then, the heights of sites i and $i+r$ are recalculated as $h(i) \to h(i) - 1$ and $h(i+r) \to h(i+r) + 1$ and the corresponding local slopes are modified accordingly. Each time a grain leaves a column i we assign it a new critical slope, which may take the values $\sigma_c(i) = 1$ or $\sigma_c(i) = 2$ with equal probability. This process is repeated until all the local slopes satisfy $\sigma(i) \leq \sigma_c(i)$. An avalanche starts when $\sigma(1) > \sigma_c(1)$ and when it stops (i.e., $\sigma(i) \leq \sigma_c(i)\ \forall\, i$) new grains are added until a new avalanche is initiated.

3. Results

Once the system reaches the stationary state an avalanche may be defined as the total energy dissipated between two consecutive profiles. As we mentioned in the introduction the energy difference between these profiles may be either positive or negative. To characterize these different kinds of events we have studied the distribution of time intervals Δt between avalanches of the same kind. That is, the distribution of time intervals between avalanches where the potential energy of the system either increases or decreases.

In Fig. 1 we present the distribution of time intervals $P_g(\Delta t)$ between avalanches when the system gains potential energy after a grain is added. Two different values of the rolling parameter are presented, $\rho = 1.6$, which corresponds to the rolling regime,[4] and $\rho = 10.0$, which corresponds to the sliding regime.[4] The system size for both curves is $L = 800$. The distribution of time intervals presents an exponential decay $P_g(\Delta t) \sim \exp(-\Delta t/t_g)$ both in the sliding and the rolling regime. For $\rho = 1.6$,

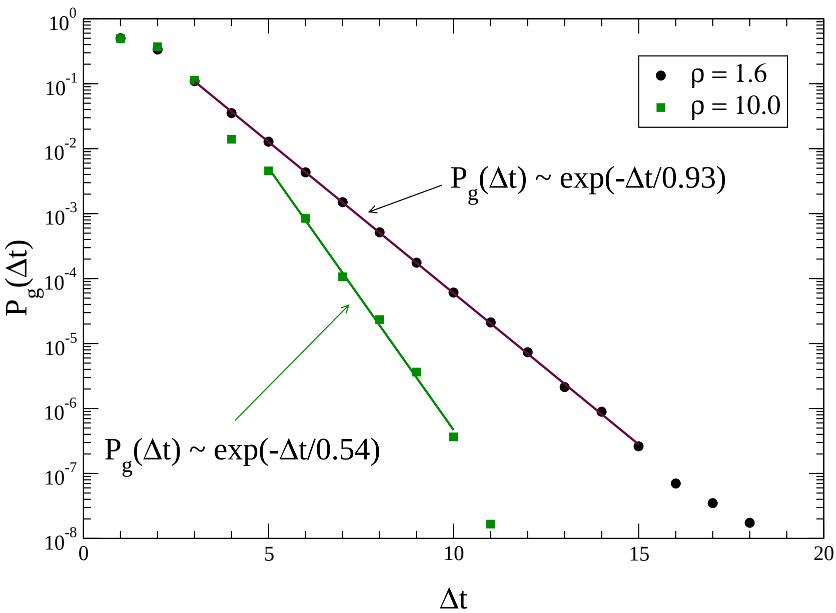


Fig. 1. Distribution of time intervals $P_g(\Delta t)$ for two different values of the rolling parameter, $\rho = 1.6$ and $\rho = 10.0$ and system size $L = 800$. Both distributions present an exponential decay $\exp(\Delta t/t_g)$. The straight lines indicate the best fits.

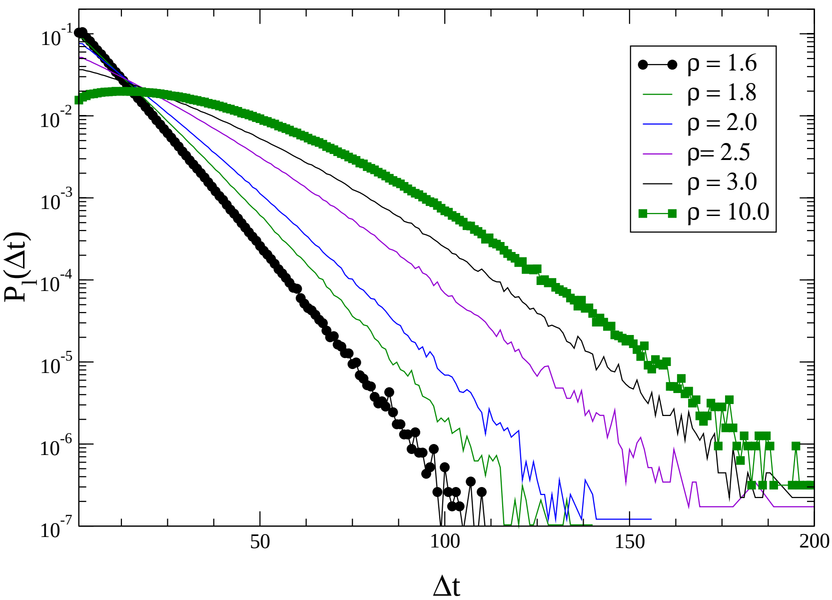


Fig. 2. Normal–log plot of the distribution $P_l(\Delta t)$ for six different values of ρ and system size $L = 1600$. A clear exponential decay is observed for $\rho = 1.6$. As ρ increases P_l deviates from this behavior.

the characteristic time $t_g = 0.93$. As ρ grows t_g decreases almost linearly until it reaches the saturation value $t_g = 0.54$, as in the $\rho = 10.0$ fit, also presented in Fig. 1.

A stronger dependency on ρ is observed in the distribution of time intervals $P_l(\Delta t)$ between avalanches when the system looses potential energy after the addition of a grain. In Fig. 2 we present the behavior of $P_l(\Delta t)$ for six different values of the rolling parameter: $\rho = 1.6$, 1.8, 2.0, 2.5, 3.0, and 10.0 for a system with size $L = 1600$. When $\rho = 1.6$ a clear exponential decay is observed. However as ρ grows the distribution P_l deviates from this behavior. We will focus our interest in the behavior of the system in the rolling regime, that is for round grains ($\rho = 1.6$), and also in the sliding regime, which corresponds to elongated grains ($\rho = 10.0$).

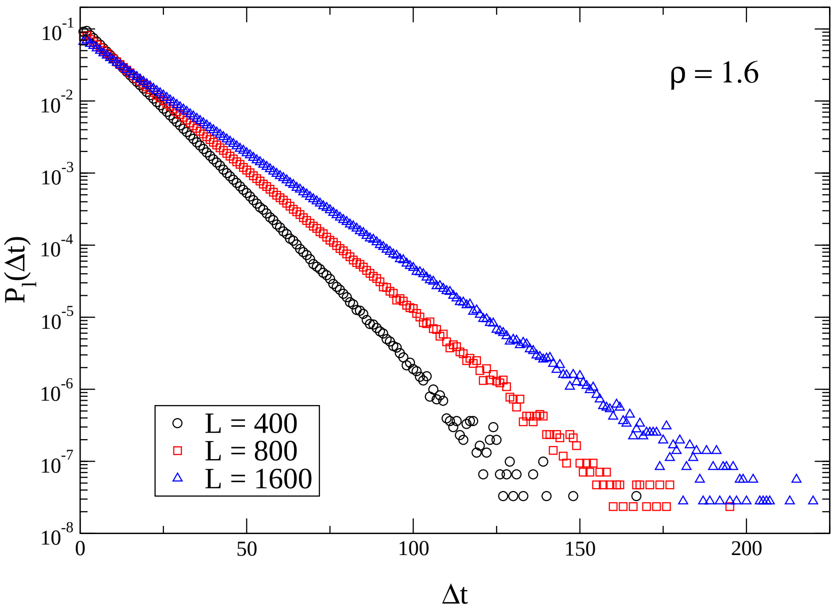
As we said in the rolling regime ($\rho = 1.6$) a clear exponential decay $P_l(\Delta) \sim \exp(-\Delta t/t_l)$ is observed. In Fig. 3(a) we present the behavior of $P_l(\Delta t)$ when $\rho = 1.6$ for three different system sizes, $L = 400$, 800 and 1600. The data collapse presented in Fig. 3(b) shows that when $\rho = 1.6$, the distribution $P_l(\Delta t)$ obeys the following finite size scaling behavior

$$P_l(\Delta t) \sim L^{-\beta_{lr}} f\left(\frac{\Delta t}{L^{\nu_{lr}}}\right), \tag{2}$$

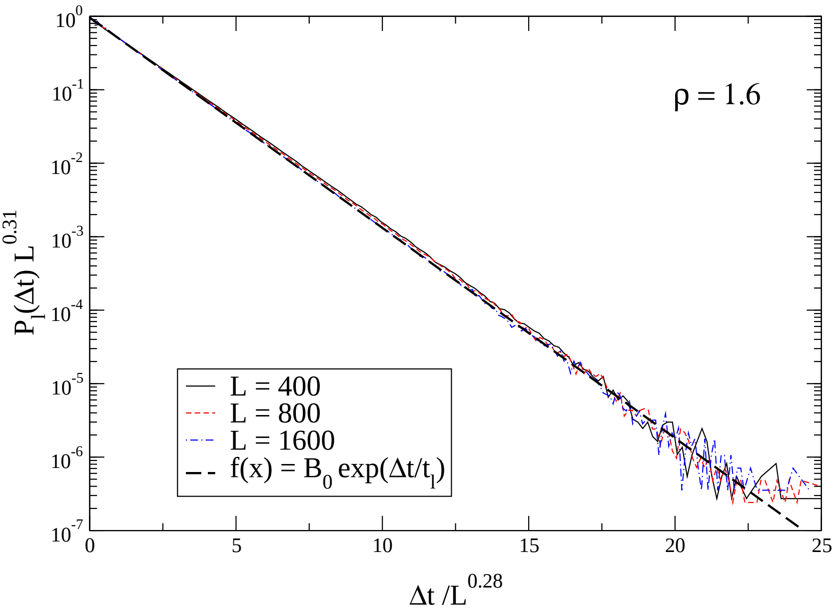
where $\beta_{lr} = 0.31$ and $\nu_{lr} = 0.28$ with

$$f(x) = B_0 \exp\left(-\frac{x}{t_l}\right), \tag{3}$$

where $B_0 = 0.96$ and $t_l = 1.52$.

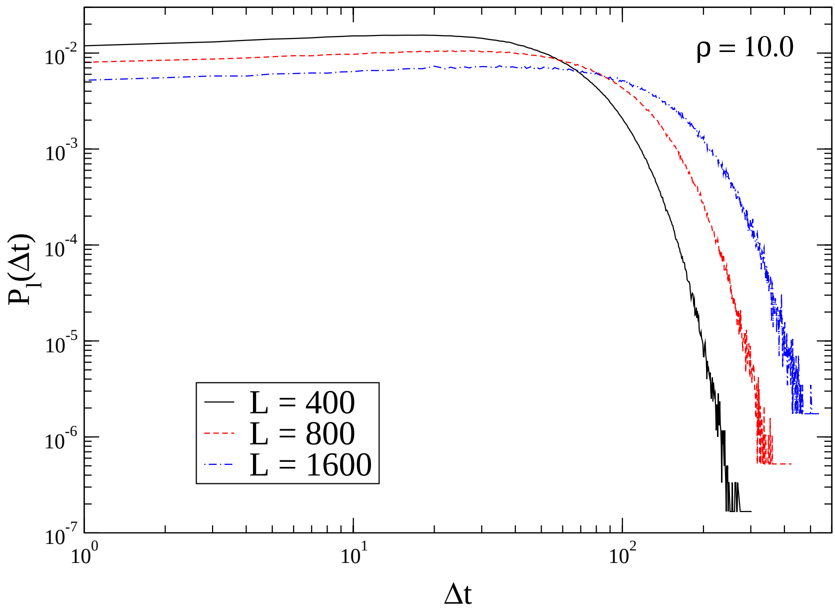


(a)

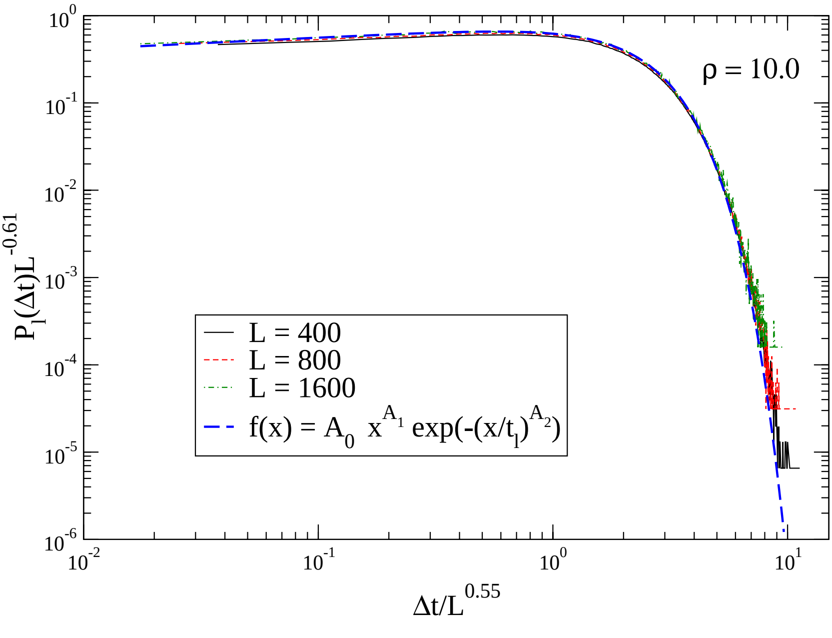


(b)

Fig. 3. (a) Distribution of time intervals between avalanches $P_l(\Delta t)$ when $\rho = 1.6$ for three different system sizes $L = 400$, 800 and 1600. In (b) a data collapse of the same data is presented. The dashed lines indicates the best fit given by the exponential function (3).



(a)



(b)

Fig. 4. (a) Log–log plot of the distribution $P_l(\Delta t)$ when $\rho = 10.0$ for three different system sizes $L = 400$, 800 and 1600. In (b) a data collapse of the same data is presented. The dashed line indicates the best fit given by function (5).

To analyze the behavior of the distribution P_l in the sliding regime we will consider the rolling parameter value $\rho = 10.0$. In Fig. 4(a) we present a log–log plot of the distribution $P_l(\Delta t)$ for three different system sizes $L = 400$, 800 and 1600 when $\rho = 10.0$. The data collapse presented in Fig. 4(b) shows that when $\rho = 10.0$, the distribution $P_l(\Delta t)$ obeys the following finite size scaling behavior

$$P_l(\Delta t) \sim L^{\beta_{ls}} f\left(\frac{\Delta t}{L^{\nu_{ls}}}\right), \quad (4)$$

where $\beta_{ls} = 0.61$ and $\nu_{ls} = 0.55$ with

$$f(x) = A_0 x^{A_1} \exp\left(-\left(\frac{x}{t_l}\right)^{A_2}\right), \quad (5)$$

with $A_0 = 0.76$, $A_1 = 0.13$, $A_2 = 1.87$ and $t_l = 2.38$. Note that the power law exponent A_1 is very small, nevertheless it plays an important role in the fit for small Δt.

4. Conclusion

In this work we have studied the intertemporal structure between avalanches in a rice pile model.[4] We observed that if the addition of a single grain, which increases the potential energy of the pile, is considered as an avalanche, the distribution $P_g(\Delta t)$ of time intervals Δt between these kind of avalanches presents an exponential decay $P_g(\Delta t) = \exp(\Delta t/t_g)$ for all ρ. The characteristic time t_g presents a linear dependence with ρ. We have also studied the distribution $P_l(\Delta t)$ of time intervals Δt between avalanches when the system looses potential energy after the addition of a single grain. We observed that in the rolling regime the distribution $P_l(\Delta t)$ presents a clear exponential decay, while in the sliding regime $P_l(\Delta t)$ can be well described by a stretched exponential, weighted by a power law, that corrects the distribution for small Δt. It would be interesting to test experimentally if these different behaviors can be observed in real rice piles and display a behavior accordingly to the results presented in this work.

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