Robust dynamical decoupling

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Abstract

Quantum computers, which process information encoded in quantum mechanical systems, hold the potential to solve some of the hardest computational problems. A substantial obstacle for the further development of quantum computers is the fact that the life time of quantum information is usually too short to allow practical computation. A promising method to increase the life time, known as dynamical decoupling, consists of applying a periodic series of inversion pulses to the quantum bits. In the present review, we give an overview of this technique and compare different pulse sequences proposed earlier. We show that pulse imperfections, which are always present in experimental implementations, limit the performance of dynamical decoupling. The loss of coherence due to the accumulation of pulse errors can even exceed the perturbation from the environment. This effect can be largely eliminated by a judicious design of pulses and sequences. The corresponding sequences are largely immune to pulse imperfections and provide an increase of the coherence time of the system by several orders of magnitude.

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arXiv:1110.6334v1 [quant-ph] 28 Oct 2011

I. INTRODUCTION

During the last decade, it was shown that quantum mechanical systems have the potential for processing information more efficiently than classical systems [1–4]. However, it remains difficult to realize this potential because quantum systems are extremely sensitive to perturbations. These perturbations come from external degrees of freedom or from the finite precision with which the systems can be realized and controlled by external fields. This loss of quantum information to the environment is called decoherence [5]. Different results show that a state is more sensitive to decoherence as the number of qubits increases [6–13]. This is also manifested by the impossibility to time reverse a quantum evolution when an initially localized excitation spreads over a system [14–17]. As the information is distributed over a increasing number of qubits, the evolution becomes more sensitive to the perturbation [17, 18]. In a similar vein, this sensitivity with the systems size limits the distance over which one can transfer information or analogously limits the number of qubits that one can control reliably [19, 20]. This is manifested as a localization effect for the quantum information [19–27]. In order to overcome these limitations for allowing quantum information processing with large number of qubits, methods for reducing the decoherence effects have to be developed.

One can tackle the problem by correcting the errors generated by the perturbations, but this is only possible if the perturbation is small enough to keep the quantumness of the system [28–30]. Therefore, one needs first to reduce the perturbation effects. The pioneering strategies for reducing decoherence were introduced in the Nuclear Magnetic Resonance (NMR) community, in particular by Erwin Hahn who showed that inverting a spin-1/2 system (a qubit) corresponds to an effective change of the sign of the perturbation Hamiltonian and therefore generates a time reversal of the corresponding evolution [31]. This leads to the formation of an echo that later was formalized as a Loschmidt echo [18]. These manipulations were extended to the so-called decoupling methods [32–36], which disconnect effectively the environment.

In the context of this review, we describe the environment as a spin-bath, without loss of generality. Considering spins 1/2 as qubits, two different types of decoupling methods can be distinguished. In the first case, the qubit system is well distinguished from the environment. Its energy level splitting differs significantly from that of the bath. As a result, the coupling

between them is much smaller than the difference of their energy level splittings. The interaction can then always be approximated by an Ising-type (zz) interaction, which causes dephasing of the system qubit but no qubit flips. The decoupling methods required for these cases are called heteronuclear decoupling within the NMR community and they can involve manipulation only on the spin-system [31–33, 35, 36] or only at the environment [37].

In the second case, the system and the environment have similar energy level splittings. This is the case of a homonuclear system where the general form of the coupling must be retained and it can induce flips of the system qubit as well as dephasing. In this case, decoupling will generally affect the complete system plus "bath" [38–43].

In this review we focus on decoupling the system from the environment by applying control pulses only to the system. During the last years this technique has gathered a lot of interest because it requires relatively modest resources: it requires no overhead of information encoding, measurements or feedback. The method is known as dynamical decoupling (DD). Since its initial introduction [44], a lot of effort has been invested to find sequences with improved error suppression [45–54]. The optimal design of DD sequences depends of the different sources of errors that have to be eliminated. The first to be considered is the nature of the SE interaction, i.e., a pure dephasing when only the 1/2-spin operator S_z is present, a pure spin-flip interaction (S_x and/or S_y are present) or a general interaction with S_x, S_y and S_z. Sequences like Carr-Purcell (CP) and Carr-Purcell-Meiboom-Gill (CPMG), which use rotations around a single axis, are useful when only two operators of S_x, S_y and S_z are present, but in order to fight against a general interaction, pulses along different spatial direction have to be applied. The shortest sequence that fulfills this condition is the XY-4 sequence [35, 44]. An actual implementation must take into account, in addition to the above issues, the effect of pulse imperfections [45, 54–58]. Fighting the effect of pulse errors was in fact the original motivation for the development of the XY-4 sequence [35]. Another experimental consideration is the amount of power deposited in the system, which often must be kept small to avoid heating effect or damage to the sample.

DD technique is becoming an important tool for quantum information processing [49, 54–56, 59–67] as well as in spectroscopy [68–71] and imaging [72–75]. In most cases, the goal is to preserve a given input state, but it may also be combined with gate operations [76–80]. In many cases, experimental results show that one of the main limitations for improving the decoupling efficiency are the non-ideal properties of the decoupling pulses [54–56, 67].

Decoherence effects due to the environment can in principle be reduced by shortening the cycle time τ_c. However, in reality the pulse lengths are finite and the DD cycle time is limited by the available hardware. It is therefore important to find the best finite time sequences for decoupling [45, 54, 55, 67, 81–84]. Furthermore, the pulses do not only have finite lengths, they also do not implement perfect rotations. Thus, increasing the number of pulses can result in a large overall error that destroys the qubit coherence instead of preserving it. As a result, the performance of the decoupling sequence may have an optimum at a finite cycle time [54, 55, 67]. In this review, we summarize the different DD strategies for fighting the effect of pulse imperfections and we show that they must be considered in the designing of useful DD sequences.

The paper is structured as follows: In section II we give some basics of dynamical decoupling, in section III we introduce the effects of pulse imperfections and in section IV we describe the strategies to fight against the imperfections effects. In the last section we draw some conclusions and perspectives.

II. BASICS OF DYNAMICAL DECOUPLING

A. The system

We consider a single qubit $\hat{S}$ as the system that is coupled to the bath. In a resonantly rotating frame of reference [85], the free evolution Hamiltonian is

$$\widehat{\mathcal{H}}_f = \widehat{\mathcal{H}}_{SE} + \widehat{\mathcal{H}}_E, \tag{1}$$

where $\widehat{\mathcal{H}}_E$ is the environment Hamiltonian and

$$\widehat{\mathcal{H}}_{SE} = \sum_{\beta} \left(b_z^{\beta} \hat{E}_z^{\beta} \hat{S}_z + b_y^{\beta} \hat{E}_y^{\beta} \hat{S}_y + b_x^{\beta} \hat{E}_x^{\beta} \hat{S}_x \right) \tag{2}$$

is a general interaction between the system and the environment. $\hat{E}_u^{\beta}$ are operators of the environment and b_u^{β} the SE coupling strength. The index β runs over all modes of the environment. Dephasing is due to an interaction that affects the z component of the spin-system operator, and spin-flips are done trough the x and/or y operators. A heteronuclear spin-spin interaction involves a pure dephasing interaction. This type of interaction that is naturally encountered when the system can be distinguished from the environment can be

found in a wide range of solid-state spin systems, as for example nuclear spin systems in NMR [32, 33, 55, 65], electron spins in diamonds [56], electron spins in quantum dots [86], donors in silicon [87], etc. In other cases when the system and environment have similar energies, the SE interaction can include terms along the x, y and z axis.

B. DD sequences with a single rotation axis

DD is achieved by iteratively applying to the system a series of stroboscopic control pulses in cycles of period τ_c [44]. Over that period, the time-averaged SE interaction can be described by an averaged or effective Hamiltonian [88]. The goal of DD is the elimination of the effective SE interaction. This can be seen by looking at Hahn's pioneering spin-echo experiment [31] (See Fig. 1b). It is based on the application of a π-pulse to the spin system at a time τ after the spins were left to evolve in the magnetic field. This pulse effectively changes the sign of the system-environment (SE) interaction, in this case the Zeeman interaction with the magnetic field. Letting the system to evolve for a refocusing period or time reversed evolution during the same duration τ generates the echo. If the magnetic field is static, the dynamics is completely reversed and the initial state of the spin recovered. This is observed as the echo. However if the magnetic field fluctuates, its effect cannot be reversed completely. Thus, the echo amplitude decays as a function of the refocusing time [31, 32]. This decay contains information about the time-dependence of the environment.

To reduce the decay rate of the echo due to a time-dependent environment, Carr and Purcell introduced a variant of the Hahn-echo sequence, where the single π-pulse is replaced by a series of pulses separated by intervals of duration τ [32]. This CP sequence reduces the changes induced by the environment if the pulse intervals are shorter than the correlation time of the environment. However, as the number of pulses increases, pulse errors tend to accumulate. Their combined effect can destroy the state of the system, rather than preserving it against the effect of the environment. This was noticed by Meiboom and Gill who proposed a modification of the CP sequence for compensating pulse errors, the CPMG sequence [33].

CP and CPMG sequences are useful only when two of the spin operators S_x, S_y and S_z
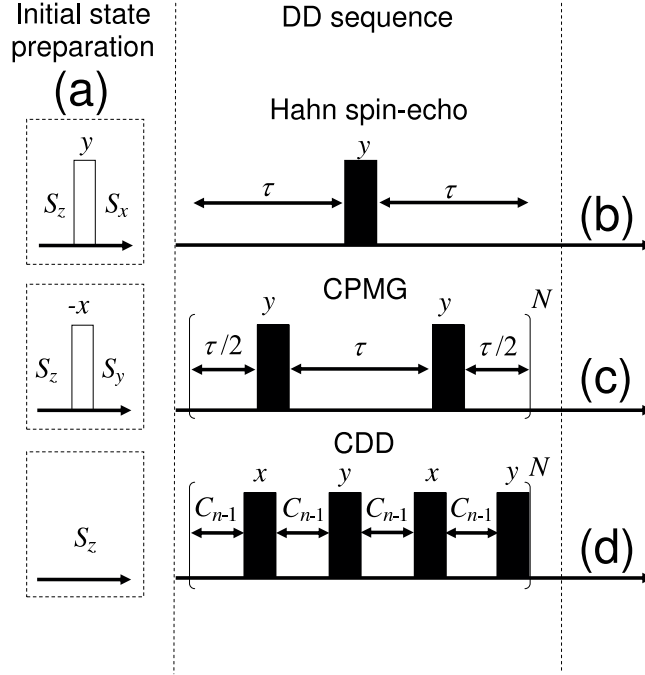
Figure 1: Dynamical decoupling pulse sequences. The empty and solid rectangles represent 90°
and 180° pulses, respectively and N represents the number of iterations of the cycle. (a) Initial
state preparation. (b) Hahn spin-echo sequence. (c) CPMG sequence. (d) CDD sequence of order
n, $\mathrm{CDD}_n = \mathrm{C}_n$ and $\mathrm{C}_0 = \tau$.

are affected by the environment. They can be written as

$$f_{\tau/2}\hat{Y}f_\tau\hat{Y}f_{\tau/2}, \tag{3}$$

where f_τ is a free evolution operator and $\hat{Y}$ is a π pulse around the Y axis (and the analogous
for $\hat{X}$). The difference between the CP and CPMG sequence is the orientation of the
rotation axis with respect to the initial condition. For applications in quantum information
processing, this distinction is not relevant, since all operations have to be independent of
the initial condition. CP or CPMG is the shortest sequence of pulses for decoupling a SE
interaction that affects only two of the spin components S_z, S_y and S_z [44].

Usually the average Hamiltonian generated by DD sequences can be described by a series
expansion, such as the Magnus expansion[89]. All the higher-order terms in this expansion
describe imperfections, which reduce the fidelity of the sequence and should be eliminated.
Improving the DD performance is therefore related to reducing the contribution of higher
order terms. This is closely related to efforts for developing better decoupling sequences for
NMR [34]. For quantum information processing (QIP), this lead to the design of sequences

that make DD more effective, such as concatenated dynamical decoupling [46, 81]. An important innovation was due to G. Uhrig [47], who proposed a sequence with non-equidistant pulse spacings, while all the standard sequences like CPMG are based on equidistant pulses. The UDD sequence is defined by

$$\text{UDD}_N = f_{\tau_{N+1}} \hat{Y} f_{\tau_N} \hat{Y} ... \hat{Y} f_{\tau_2} \hat{Y} f_{\tau_1}, \tag{4}$$

where the delays $\tau_i = t_i - t_{i-1}$ are determined by the positions

$$t_i = \tau_c \sin^2 \left[\frac{\pi i}{2 (N + 1)} \right] \tag{5}$$

of the pulses with $t_{N+1} = \tau_c$ and $t_0 = 0$. The lowest nontrivial order is equal to the CPMG sequence, $\text{UDD}_2 = \text{CPMG}$.

The effect of the non equidistant pulses can be discussed in the context of filter theory: DD can be considered as an environmental noise filter, where the distribution of pulses generates different filter shapes as a function of the frequencies. The overlap of this filter function with the spectral distribution of the environmental noise determines the decoherence rate [90]. Analogously, the filter shapes can be connected to diffraction patterns induced by interferences in the time domain [65]. The Uhrig DD sequence was shown theoretically to be the optimal sequence for reducing low frequency noise [47, 90, 91]. This prediction was confirmed experimentally [49, 61, 68]. However, it appears that non-equidistant sequences perform better only for particular noise spectral densities that increase for higher frequencies and have a strong cut off. In usual case, where the spectral density decrease smoothly with the frequency, as usually happens with NMR spin baths, equidistant sequences were predicted [90–92] and demonstrated [49, 55, 56, 62, 63, 65]to be the best option [65].

This filter function description can be traced back to previous NMR approaches [93] and to work on universal dynamical control [94]. Choosing the times for the pulses leads to a variety of sequences that can be optimized according to the spectral density of the bath [48, 49, 52, 92, 94–96].

If the SE interaction is a pure dephasing one, it is sufficient to apply pulses in one direction. However because every experimental setup has finite precision, pulse errors create an effective general SE interaction [55, 67, 81]. In this case, it was shown that sequences that are designed for general SE interaction perform better than 1D sequences [54–56].

C. DD sequences with multiple rotation axis

If the system-environment interaction includes all three components of the system spin operator, decoupling can only be achieved if the sequence includes rotations around at least two different axes. The first decoupling sequence of this type is the XY-4 sequence, which alternates rotations around the x- and y-axes (see Fig. 1 d, $n = 1$). This sequence was initially used to eliminate the effect of pulse errors in the CP and CPMG sequences [35]. It is also the shortest sequence for DD for general SE interactions [44]. In quantum information processing, where we consider the initial state to be unknown, the CP and CPMG sequences, which correspond to a train of identical π-pulses, are identical [55]. However, their effect on the quantum state depends on the (generally unknown) initial condition: If the initial condition is oriented along the rotation axis of the pulses, flip angle errors of the first pulse are refocused by the second pulse. However, for components perpendicular to the rotation axis, the pulse errors of all pulses add and cause rapid decay of the coherence, even in the absence of system-environment interactions [35, 55]. This motivated the development of the XY-4 sequence. In addition, pulse imperfections convert an ising-type system environment interaction into an effective general SE interaction [55, 67, 81], which is not eliminated by the CP/CPMG sequence, but is partially eliminated by the XY-4 sequence. In the QIP community, the XY-4 sequence is usually referred to as periodic dynamical decoupling (PDD).

The XY-4 sequence is also the building block for concatenated DD (CDD) sequences that improve the decoupling efficiency [46, 81]. The CDD scheme recursively concatenates lower order sequences to increase the decoupling power. The CDD evolution operator for its original version for a recursion order of n is given by

$$\mathrm{CDD}_n = C_n = \hat{Y}C_{n-1}\hat{X}C_{n-1}\hat{Y}C_{n-1}\hat{X}C_{n-1}, \tag{6}$$

where $C_0 = f_\tau$ and $\mathrm{CDD}_1 = $XY-4. Figure 1 shows a general scheme for this process. Each level of concatenation reduces the norm of the first non-vanishing order term of the Magnus expansion of the previous level, provided that the norm was small enough to begin with [46, 81]. This reduction comes at the expense of an extension of the cycle time by a factor of four. If the delays between the pulses are allowed to be non-equidistant like in UDD, it becomes possible to create hybrid sequences, such as CUDD [50] and QDD [53, 97, 98].

III. EFFECTS OF IMPERFECTIONS

Since the precision of any real operation is finite, the control fields used for decoupling introduce errors. Depending on the sequence, these errors can accumulate. If the number of pulses is large and the sequence is not properly designed, the accumulated pulse errors can lead to severe loss of coherence than the effect of the environment. Designing effective decoupling sequences that suppress environmental effects without degrading the system, even if the control fields have errors, requires a careful analysis of the relevant errors and appropriate strategies for combining rotations in such a way that the errors cancel rather than accumulate.

One example of a non-ideal control pulse is its finite duration, which implies a minimum achievable cycle time. The effects introduced by finite pulse lengths have been considered in different theoretical works [45, 81, 82]. These works predict that high order CDD or UDD sequences in general lose their advantages when the delays between pulses or pulse length are strongly constrained. While the limitation on the cycle reduces the maximal achievable DD performance, pulse errors can be even more destructive. In most cases, the dominant cause of errors is a deviation between the ideal and the actual amplitude the of control fields. The result of this amplitude error is that the rotation angle experimentally implemented deviates from π, typically by a few percent. Another important error occurs when the control field is not applied in resonance with the transition of the qubit. This off-resonant effect produces a rotation in which the flip angle and the rotation axis deviate from the ideal ones.

An example of the destructive effects of pulse imperfection is illustrated in the left panel of figure 2. Here, we measured the magnetization decay of the ^{13}C nuclear spins during two different DD sequences. The sample used for this experiment was polycrystalline adamantane [54, 55, 65, 67]. The dephasing of the nuclear spins originates from the interaction with an environment consisting of ^{1}H nuclear spins and can be considered as a pure dephasing process. The first sequence considered in the figure is CPMG. In this case we can observe that the decay of the magnetization is imperceptibly slow when the system is initially oriented parallel to the rotation axis of the pulse (longitudinal state). As we discuss below, this is an indication that the pulse errors have no effect on this initial state. In contrast, for a transverse initial state, the errors of the individual pulses accumulate and lead to a rapid decay, as shown in Fig. 2. A similar behavior is found for the UDD sequence [55, 65].
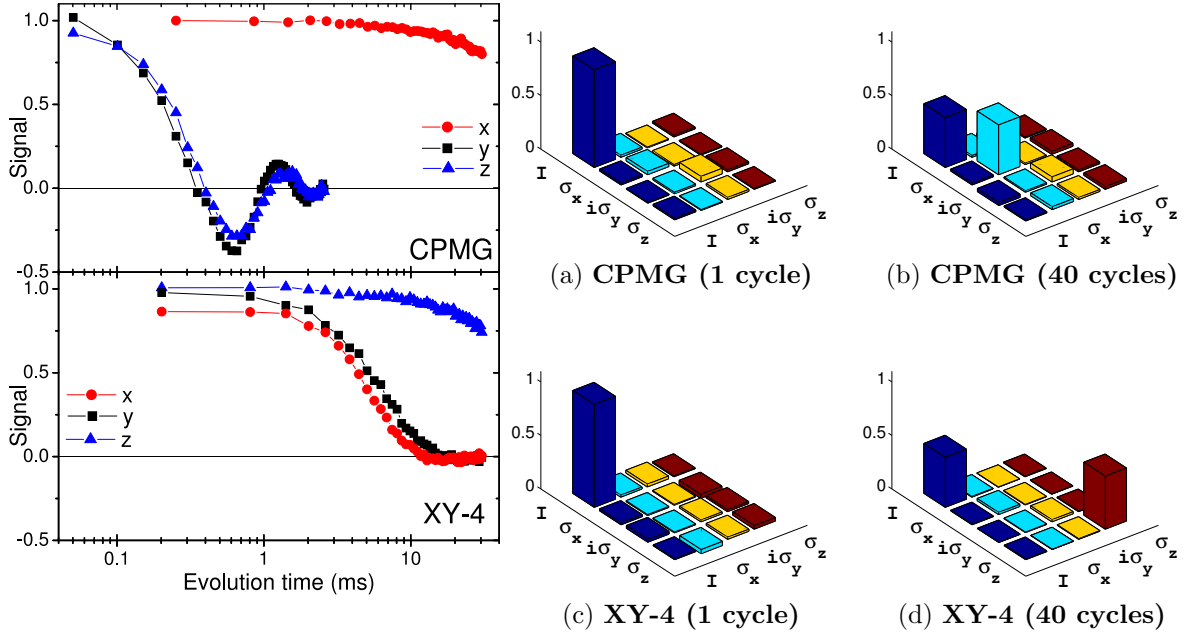
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Figure 2: Comparison between two basic DD sequences: CPMG, which is not robust against errors, and the self-correcting sequence XY-4. The left panel shows the normalized magnetization as a function of time. The delay $\tau = 40\mu$s between the pulses is constant and identical for both sequences. The panels on the left shows the real part of the process matrices χ for CPMG and XY-4. The imaginary part, which is very small, is not shown.

The second DD sequence considered in Fig. 2 is the XY-4 sequence, which consists of pulses applied along the x and y axes. The alternating phases of the pulses results in a partial cancellation of pulse errors, independent of the initial condition [35, 36]. As a result, the performance of this sequence is much more symmetric with respect to the initial state in the xy-plane and the average decay times are significantly longer [54, 55].

In the context of quantum information processing, it is important that the performance of gate operations be independent of the initial conditions (which typically are unknown). A common choice for quantifying the performance of a general quantum operation is then the fidelity F [99]:

$$F = \frac{|Tr(AB^{\dagger})|}{\sqrt{Tr(AA^{\dagger})Tr(BB^{\dagger})}}. \tag{7}$$

Here, A is the target propagator for the process and B the actual propagator. For the

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present situation, where the goal is a quantum memory, the target propagator is the identity operation I.

We can not assume that the actual propagators are unitary. We therefore write the process as

$$\rho_f = \sum_{nm} \chi_{mn} E_m \rho_i E_n^\dagger \tag{8}$$

where ρ_i and ρ_f are the density matrices at the beginning and end of the process. The operators E_m must form a basis. For the present case, we choose them as $E_m = (I, \sigma_x, i\sigma_y, \sigma_z)$. The ideal and actual processes can therefore be quantified by the matrix elements χ_{mn}. For the target evolution, the χ-matrix is

$$\chi_I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

The matrix elements for the actual process are determined experimentally by quantum process tomography [3, 100]. We use them to calculate the process fidelity from eq. (7). In figure 2, we compare the real part of the experimental χ matrices for the two sequences. After one cycle, the process matrices for both sequences represent a quantum operation that is close to the identity operation. The fidelity between the experimental matrices and the ideal matrix (9) is 0.988 and 0.989 for the CPMG and XY-4 cycles, respectively. Measured over 40 cycles, the process matrices of the two sequences differ significantly from the identity operation but also from each order. For the XY-4 sequence, the non-vanishing elements are χ_{11} and χ_{44}, while for CPMG the non-vanishing elements are χ_{11} and χ_{22}. The two matrices represent therefore qualitatively different processes. The XY-4 sequence destroys all transversal magnetization, in this case the resulting density matrix is

$$\rho_f = \chi_{11}\rho_i + \chi_{44}\sigma_z\rho_i\sigma_z. \tag{10}$$

This corresponds to a projection of the density operator onto the z-axis, i.e. to a complete dephasing of the transverse components in the xy-plane.

The CPMG sequence, conversely, projects the density operator onto the x-axis:

$$\rho_f = \chi_{11}\rho_i + \chi_{22}\sigma_x\rho_i\sigma_x. \tag{11}$$

The CPMG is a spin lock [101] sequence that allows retains magnetization in the x direction but destroys all components perpendicular to it. Since a real experimental implementation always generates a distribution of control field amplitudes, spins at different positions precess with different rates around the direction of the rf field. As a result, the perpendicular components become completely randomized after a sufficiently large overall flip angle as shown in the left panel of Fig. 2.

The performance of experimentally accessible DD sequences is limited by the pulse errors [54–56, 67]. In many situations, the dominating error contributions are flip angle and offset errors. In the next section we show different strategies to make decoupling sequences robust against such errors.

IV. ROBUST DECOUPLING SEQUENCES

We have to make decoupling insensitive to pulse imperfections. Different possibilities for generating high-fidelity sequences have been proposed in the context of quantum information processing [46, 54, 81]. Here, we discuss two possible approaches: first we show that it is possible to replace individual refocusing pulses by compensated pulses that implement very precise inversions, and then we discuss sequences that are inherently robust, i.e. insensitive to the imperfections of the individual pulses.

A. Robust Pulses

The simplest approach to make a sequence robust is by replacing every standard pulse by a robust composite pulse [102]. The composite pulses are sequences of consecutive pulses designed to be robust against various classes of imperfections generating therefore rotations that are close to the ideal rotation even in the presence errors. Particularly useful for quantum information applications are those composite pulses that produce compensated rotations for any initial condition, denominated in the NMR literature as class-A pulses [102].

Recent experiments have successfully used composite pulses to demonstrate the resulting increase of the performance of different DD sequences [54, 56]. These works have imple-

mented the composite pulse defined as:

$$(\pi)_{\pi/6+\phi} - (\pi)_\phi - (\pi)_{\pi/2+\phi} - (\pi)_\phi - (\pi)_{\pi/6+\phi}, \tag{12}$$

which is equivalent to a robust π rotation around the axis defined by ϕ followed by a $-\pi/3$ rotation around the z axis [103]. For cyclic sequences, which always consist of even numbers of π rotations, the effect of the additional z rotation vanishes if the flip angle errors are sufficiently small.

A comparison between standard sequences (not using robust pulses) against sequences with robust pulses has been reported in [54]. It was observed that robust pulses improve the performance at high duty cycles. However, for low duty cycles, standard sequences perform better. This is due to the shorter cycle time of the standard sequence if constant duty cycles are compared. Thus, if the objective is only to preserve a quantum state, the best performance is obtained at high duty cycles, using robust pulses. The best sequences that are suitable for parallel application of quantum gate operations are the self-correcting sequences discussed below.

The composite pulses are usually designed to correct flip angle errors and offset errors. For compensating the effects introduced by the finite length of pulses, some theoretical works have proposed that a finite pulse could be approximated as an instantaneous one by using an appropriate shaped pulse [104–106]. These works has provided analytical and numerical evidence that a careful shape design can strongly affects the performance of decoupling sequences.

B. Self-correcting sequences

An alternative to the use of composite pulses consists in making the decoupling sequences fault-tolerant without compensating the error of each pulse, but by designing them in such a way that the error introduced by one pulse is compensated by the other pulses of the cycle [54]. A straightforward strategy for designing improved sequences consists in concatenating one basic building block cycle into a longer and robust cycle.

The XY-4 cycle is often chosen as the building block for constructing self-correcting sequences. This cycle is the shortest DD sequence that cancels the zero-order average Hamiltonian for a general SE interaction and also has the advantage of being partially robust to

pulse imperfections.

In the spectroscopy and quantum computing communities, two versions of the XY-4 sequence are used [67]. The basic cycle originally introduced in NMR shows reflection symmetry with respect to the center of the cycle. In contrast to that, the sequence used in the quantum information community is time asymmetric. One consequence of this small difference is that in the symmetric version, the echoes are formed in the center of the windows between any two pulses, while in the case of asymmetric cycles, the echoes coincide with every second pulse, as shown in figure 3. The separation in time between the echoes is therefore twice as long in this case. If the environment is not static, the larger separation of the echoes leads to a faster decay of the echo amplitude [67].

If a robust pulse only contains π rotations, as in the case of (12), it is also possible to convert such a composite pulse into a decoupling cycle by inserting delays between the individual π rotations. This approach has been used in [54] to build an self-correcting sequence. The basic cycle is then

$$KDD_\phi = f_{\tau/2}(\pi)_{\pi/6+\phi}f_\tau(\pi)_\phi f_\tau(\pi)_{\pi/2+\phi}f_\tau(\pi)_\phi f_\tau(\pi)_{\pi/6+\phi}f_{\tau/2}. \tag{13}$$

The self-correcting sequence is created by combining 5-pulse blocks shifted in phase by $\pi/2$, such as $[KDD_\phi - KDD_{\phi+\pi/2}]^2$, where the lower index gives the overall phase of the block. The cyclic repetition of these 20 pulses is referred to as the KDD sequence [54].

In Fig. 4, we show how strongly errors in the flip angles of individual pulses affect the fidelity of the pulse sequence. Neglecting the effect of the environment, we calculate the fidelity F after the application of 20 pulses as a function of the flip angle error. The figure compares the fidelities for the CPMG, XY-4 and KDD cycles. For the CPMG sequence, the fidelity drops to <95% if the flip angle error exceeds $\approx$ 2 %. For the XY-4 sequence, this bandwidth increases to $\approx$ 10 % and for KDD to $\approx$ 30 %. KDD and XY-4 are obviously much less susceptible to pulse imperfections than CPMG. The low fidelities observed for CPMG is experimentally manifested by the fast decay of the transverse components, such as M_x in Fig. 2.
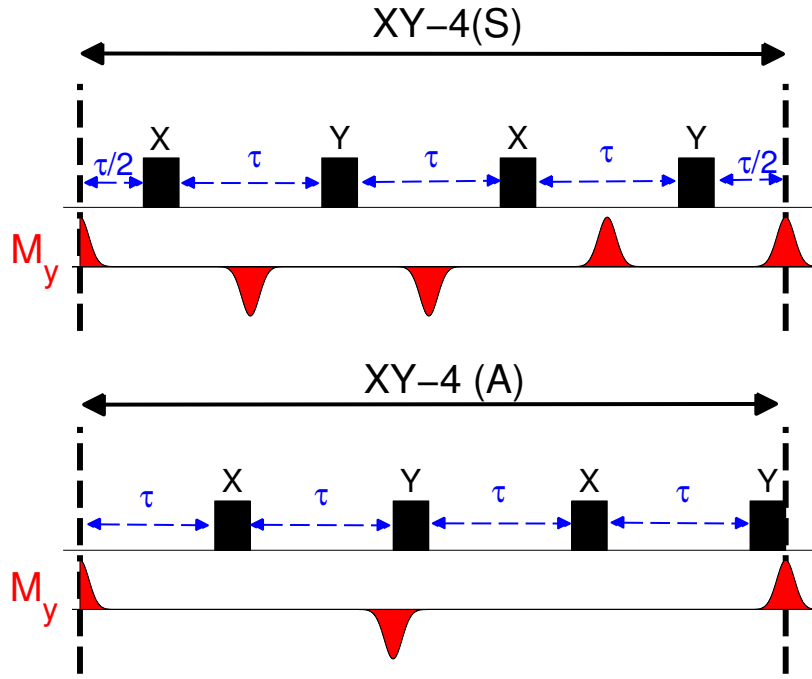
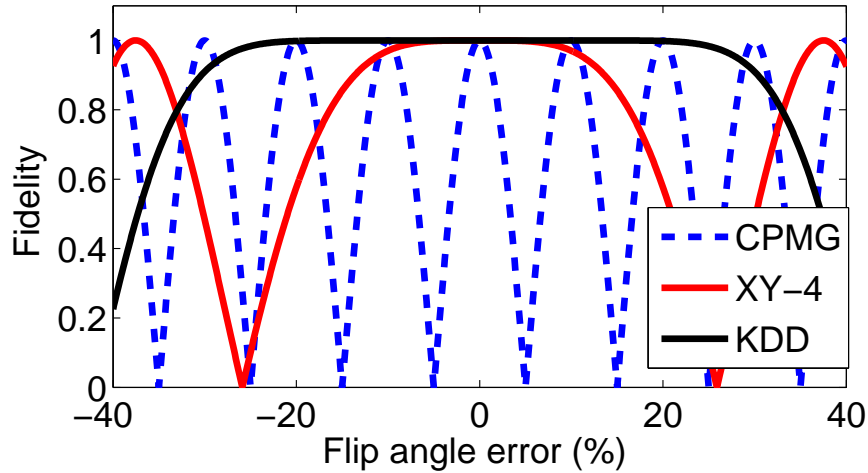Figure 3: Schematic representation of time symmetric XY-4(S) and asymmetric XY-4(A).

Figure 4: Simulation of fidelity as a function of the flip angle error for KDD, XY-4 and CPMG cycles.

C. Combining basic cycles

Every decoupling sequence contains unwanted terms in the average Hamiltonian. They can be reduced by combining different versions of the basic cycles in such a way that some of the error terms cancel. Two different versions of this procedure have been used: The basic cycles can be applied subsequently [36] or one cycle can be inserted into the delays of another cycle [46, 81]. The first approach was introduced in NMR, e.g. for designing high-

performance homonuclear decoupling sequences [38–43] or in high-resolution heteronuclear decoupling [31–33, 35–37]. Examples of DD sequences that can be constructed using this approach are XY-8 and XY-16 sequences [36]. Here the XY-8 is created combining a XY-4 cycle with its time-reversed image while XY-16 is created combining the XY-8 with its phase-shifted copy.

The second approach is the concatenation scheme proposed by Khodjasteh and Lidar [46, 81]. It generates the CDD sequence of order $n + 1$ by inserting CDD_n cycles into the delays of the XY-4 sequence (see figure 1). Ideally, each level of concatenation improves the decoupling performance and the tolerance to pulse imperfections; in practice, higher order sequences do not always perform better. It has been theoretically predicted [45, 81, 82] and later observed experimentally that the finite duration of the pulses and constrained delays between pulses result in the existence of optimal levels of concatenation [54, 55].

In figure 5, we demonstrate how the well-designed combination of basic cycles can lead to extended cycles with better error compensation. Here, we consider as the leading experimental imperfections deviations of the amplitude and frequency of the pulse. Neglecting the effect of the environment, we calculate the fidelity F after applying 1680 pulses to the system as a function of the two error parameters. Each panel contains the color-coded fidelity for one of six different decoupling sequences. In the top panels we can clearly see the improvement in the error tolerance due to the CDD scheme of concatenation. Panels 1, 4 and 5 show the same result for the sequential concatenation scheme, where only two cycles are combined at each step: concatenation of the XY-4 cycle with its time-inverted and phase-shifted copies forms the XY-8 and XY-16 sequences. The 16-pulse XY-16 cycle is significantly more robust than the 84-pulse CDD_3 cycle. The best performance is achieved by the KDD sequence, whose cycle consists of 20 pulses .

In Fig. 6, we compare the experimental performance of different self-correcting sequences. The performance of the CDD sequences always saturates or decreases with increasing duty cycle under the current experimental conditions [54]. However, instead of saturating, the relaxation time for the KDD sequence continues to increase, as in the case of sequences with robust pulses [67]. The KDD sequence combines the useful properties of robust sequences with those of sequences of robust pulses and can thus be used for both quantum computing and state preservation.
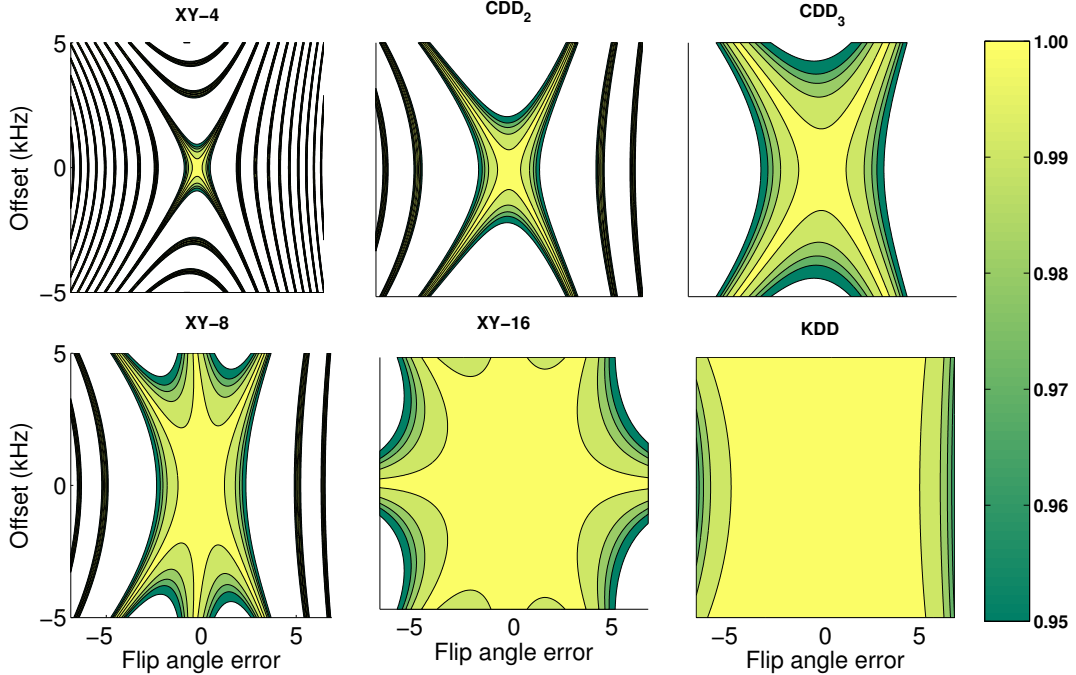
Figure 5: Error tolerance of different self-correcting sequences. The upper row shows the calculated fidelity F for CDD sequences, while the lower row shows the results for XY-8, XY-16 and KDD sequences. Each panel shows the fidelity after 1680 pulses as a function of flip-angle error and offset errors. The regions where the fidelity is lower than 0.95 are shown in white.

D. Time reversal symmetry

The symmetry of the basic building blocks has a key role in determining the performance of the concatenated higher order sequences. Two sequences constructed according to the same rules from a basic block have different propagators if the basic blocks are symmetric or not [54, 67]. If we concatenate four XY-4 cycles to the XY-16 sequence, e.g., we obtain new cycles, which are time-symmetric, independent of which version of the XY-4 sequence was used for the building blocks. Although all the odd order terms vanish in the average Hamiltonians of both versions, the even order terms of the sequences that are built from asymmetric blocks contain additional unwanted terms [67]. The different behavior of sequences consisting of symmetric vs. asymmetric blocks is illustrated in figure 7. If we start from the symmetric form of XY-4, the resulting XY-16 sequence shows much better performance than the sequence using the asymmetric XY-4 as the building block. Analogous results were obtained for the two versions of the XY-8 sequence [67].

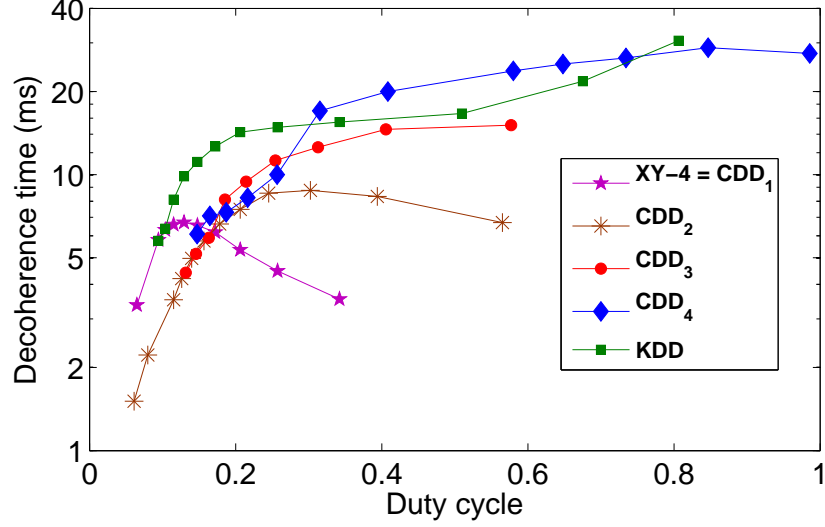Earlier experiments showed two different contributions to the overall fidelity loss [67]: A

Figure 6: Experimental decoherence times for different compensated DD sequences as a function of the duty cycle.
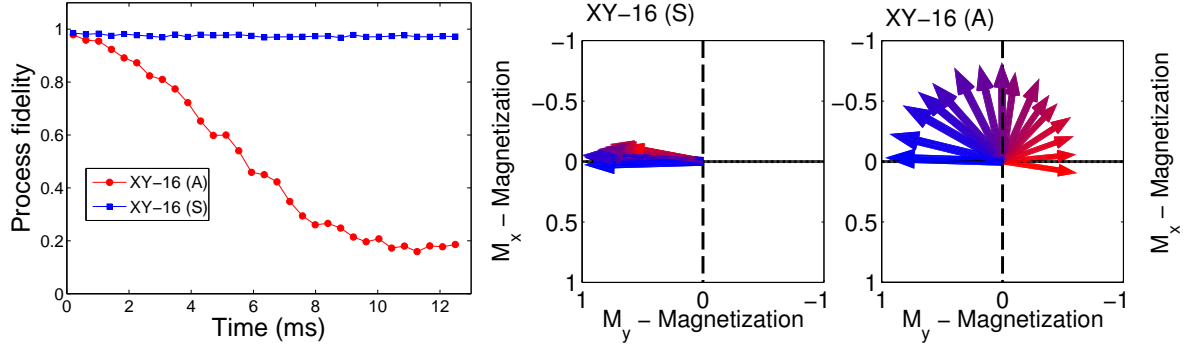

Figure 7: Comparison between the two forms of XY-16 sequence: XY-16 (S), built from symmetric form of the XY-4 cycle, and XY-16 (A), built from the asymmetric block. The delay $\tau = 10\mu s$ between the pulses is constant and identical for both sequences. The left panel shows the process fidelity as a function time. The right panel shows the Bloch vector in the xy plane at different times. The color code in the right panel denotes the time evolution, blue for the initial state and red for the final state.

precession around the z-axis, which can be attributed to the combined effect of flip-angle errors and an overall reduction of the amplitude, which results from the system-environment interaction. The combination of precession and reduction of amplitude is illustrated in right panel of Fig. 7. In this figure we show the xy-components of the magnetization at different

18

times during the XY-16 sequence. If the XY-16 sequence is built by the asymmetric form of XY-4 a distinct precession around the z-axis is observed. This causes a deviation from the desired evolution and reduces therefore the fidelity of the process. However, for the sequence consisting of symmetric blocks, the precession is negligible. These results suggest that pulse errors are better compensated by concatenating symmetric building blocks.
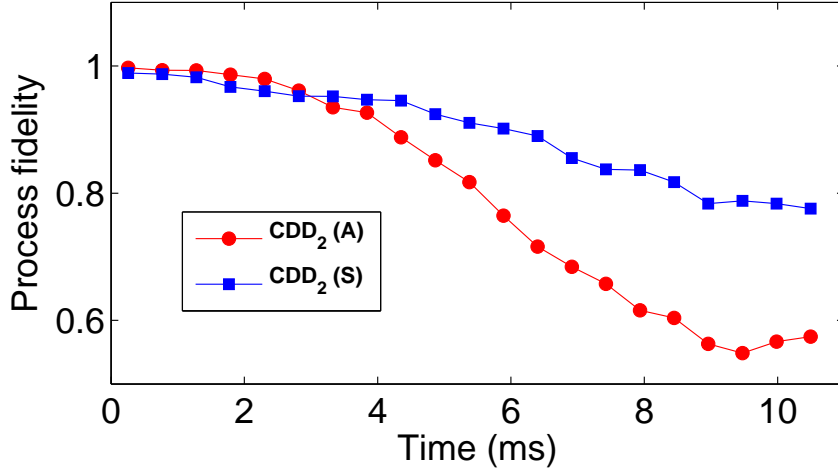

Figure 8: Experimental fidelity decay for CDD_2 built from symmetric and asymmetric blocks. The delay $\tau = 10\mu s$ between the pulses is constant and identical for both sequences.

The same concept can also be applied to CDD sequences. The conventional concatenation scheme of eq. (6) uses asymmetric building blocks and is not compatible with the symmetric version of XY-4. A new concatenation scheme was therefore proposed in [54, 67]. In this scheme, the symmetrized version of CDD is constructed as

$$CDD_{n+1} = [\sqrt{CDD_n} - X - CDD_n - Y - \sqrt{CDD_n}]^2. \tag{14}$$

In Fig. 8 we compare the process fidelities for the two versions of the CDD_2 sequence. As in the case of XY sequences, clearly, the symmetrized version, $CDD_2(S)$, shows a significantly improved performance, compared to the standard $CDD_2(A)$ version. In Ref. [67], it was experimentally observed that the performance of all DD sequences based on symmetric building blocks is better or equal to that of sequences using non-symmetric building blocks. This behavior is consistent with general arguments based on average Hamiltonian theory [107, 108].

V.   CONCLUSION AND PERSPECTIVES

Dynamical decoupling is becoming a standard technique for preserving the coherence of quantum mechanical systems, which does not need control over the environmental degrees of freedom. The technique aims to reduce decoherence rates by attenuating the system-environment interaction with a periodic sequence of π pulses applied to the qubits. The pioneering strategies for decoupling were introduced in the context of NMR spectroscopy [31]. Since then, many different decoupling sequences have been developed in the context of NMR [32, 33, 35, 36] or quantum information processing [44, 46, 47, 50, 53, 54, 81, 97, 98].

Generally, we can divide the DD sequences in two groups: (i) Sequences that involve pulses in a single spatial direction and (ii) sequences that contain pulses in different spatial directions. The type (i) sequences are strongly sensitive to pulse errors and are only capable of suppressing the effects of a purely dephasing environment or pure spin-flip interaction. Examples of such DD sequences are CPMG and UDD. The second group can suppress a general SE interaction and usually exhibits better tolerance to experimental imperfections. Example of such sequences are the XY-family (XY-4, XY-8 and XY-16), the CDD sequences and the KDD sequence.

Recent experiments have successfully implemented DD methods and demonstrated the resulting increase of the coherence times by several orders of magnitude [54–56, 67]. These works also showed that the main limitation to the reduction of the decay rates are the imperfections of the pulse. Two approaches have been used to correct this. The first approach replaces the inversion pulses by robust composite pulses [102], which generate rotations that are close to the target value even in the presence of pulse errors. In this case, the pulses are corrected individually. The second approach consists in designing the decoupling sequences in such a way that the error introduced by one pulse is compensated by the other pulses, without compensating the error of each pulse individually. The properties of basic decoupling cycles can be further improved by concatenating basic cycles into longer and more robust cycles. The concatenation can be made either by combining symmetry-related copies of a basic cycle subsequently [36] (resulting in the XY-8 and XY-16 sequences) or by inserting the basic cycle into the delays of another cycle [46, 81] (CDD sequences).

The time reversal symmetry of the basic building blocks is a useful criterion for minimizing error contributions. It has been demonstrated that the sequences built from symmetric build-

ing blocks often perform better and never worse than sequences built from non-symmetric blocks [54, 67]. This is a significant advantage, considering that the complexity of the sequences based on symmetric or asymmetric blocks are identical.

Earlier experiments [54] showed that the best sequences that are suitable for parallel application of quantum gate operations are the symmetric self-correcting sequences. However, as the delay between pulses decreases, sequences with robust pulses perform better. Thus, if the objective is only to preserve a quantum state, the best performance is achieved by using robust pulses to correct pulse errors. On the other hand, the KDD sequence introduced in [54] combines the useful properties of self-correcting sequence with those of robust pulses and can thus be used for both quantum memory and quantum computing.

During the last years, many advances have been achieved. However, for the application of the technique in real quantum devices, further studies will certainly be required. So far, most work has focused on single qubit systems. In the future, more experimental tests will be needed with multi-qubit systems. In the field of quantum computation, another important development may result from the combination of dynamical decoupling sequences with those techniques used to implement robust quantum gates [28, 30, 109, 110]. Since DD does not require auxiliary qubits or measurements, it can be used as an economical alternative to complement quantum error correction. Some theoretical works [76–79] proposed methods for combining the two methods but no experimental test was carried out to date. Future research on dynamical decoupling will also focus on applications outside of quantum information processing. Recent experiments have applied DD pulse sequences, for example, to probe the noise spectrum directly [69–71] and detect weak magnetic fields [72–75].

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