

Robustness of spin-chain state-transfer schemes

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This is a shortened and slightly edited version of a chapter [1] in the collection *Quantum State Transfer and Network Engineering*, edited by G.M. Nikolopoulos and I. Jex [2], where we review our own research about the robustness of spin-chain state-transfer schemes along with other approaches to the topic. Since our own research is documented elsewhere to a large extent we here restrict ourselves to a review of other approaches which might be useful to other researchers in the field.

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I. INTRODUCTION

A. Spin chains

The ground state of a one-dimensional ferromagnetic spin-1/2 chain is the all-up state $|\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \otimes \dots \otimes |\uparrow\rangle_N = |\uparrow\uparrow \dots \uparrow\rangle$. Here, the states $|\uparrow\rangle_i$ and $|\downarrow\rangle_i$ are the eigenstates of the Pauli spin operator σ_i^z acting at site i of an N -site chain, with eigenvalues $+1$ and -1 , respectively. A very simple excited state is then created by flipping a single spin; $\sigma_j^- |\uparrow\uparrow \dots \uparrow\rangle$, where $\sigma_j^- = \frac{1}{2}(\sigma_j^x - i\sigma_j^y)$. This state breaks translational invariance, which may be restored, however, by superposition:

$$|\psi_k\rangle := \frac{1}{\sqrt{N}} \sum_{j=1}^N e^{ikj} \sigma_j^- |\uparrow\uparrow \dots \uparrow\rangle. \quad (1)$$

Here $k = \frac{2\pi\nu}{N}$ ($\nu = 0, \dots, N-1$) is a dimensionless wave number, and we have temporarily assumed periodic boundary conditions. The state (1) is called a (single-) spin-wave state in magnetism [3]. Spin waves are often found to be low lying excited energy eigenstates of spin chain models, with the energy-momentum relation (dispersion relation) $\omega(k)$ depending on the detailed nature of the model Hamiltonian.

Information-carrying signals must be localized in space and time. The Fourier relation (1) may be inverted to represent a localized single spin-flip state $\sigma_j^- |\uparrow\uparrow \dots \uparrow\rangle$ as

a superposition of spin-wave states $|\psi_k\rangle$. This is the most sharply localized state available in a spin chain.

In quantum information science the single spin-1/2 system is called a quantum bit (qubit), its states $|\uparrow\rangle$ and $|\downarrow\rangle$ are mapped to the basis states $|0\rangle$ and $|1\rangle$ of the qubit, respectively, and the all-up ferromagnetic state is mapped to the computational basis state $|\mathbf{0}\rangle = |00\dots 0\rangle$ of the N -qubit chain. The single spin-flip state then is interpreted as one of the N computational basis states with a single non-zero qubit at site j : $\sigma_j^- |\uparrow\uparrow \dots \uparrow\rangle =: |\mathbf{j}\rangle$. As stated above that state may be expressed as a superposition of spin wave states (1) each of which propagates in space-time with amplitude $e^{i(kj-\omega t)}$, that is, with a phase velocity $v_\phi = \frac{\omega(k)}{k}$ which in general varies with k if the dispersion relation $\omega(k)$ is nonlinear. Dispersion then takes its toll by broadening and flattening the pulse in the course of time.

In the remainder of this introduction we will sketch some of the main lines of attack on the problem of quantum information transfer via quantum spin chains. All those proposals work well under ideal “design” conditions. Robustness of some of the schemes under less than ideal conditions is discussed subsequently.

B. Quantum information transfer in ideal quantum spin chains

To set the stage we define the following general nearest-neighbor coupled spin-1/2 chain Hamiltonian:

$$H = \frac{1}{2} \sum_{i=1}^N J_i [(1+\gamma)\sigma_i^x \sigma_{i+1}^x + (1-\gamma)\sigma_i^y \sigma_{i+1}^y + \Delta\sigma_i^z \sigma_{i+1}^z] + \sum_{i=1}^N h_i \sigma_i^z. \quad (2)$$

We identify site 1 with the fictitious site $N+1$, that is, $\sigma_{N+1}^\alpha \equiv \sigma_1^\alpha$ ($\alpha = x, y, z$). Then H describes a ring for $J_N \neq 0$ and an open chain of N spins for $J_N = 0$. If J_i (for $i \neq N$) and h_i do not depend on i we call the sys-

tem homogeneous, otherwise it is inhomogeneous. The symmetry of the spin-spin interaction is controlled by the anisotropy parameters Δ and γ . For $\gamma = 0$ and $\Delta = 1$ the model is the original Heisenberg model, also known

as XXX model, because all spin components experience the same coupling to their nearest neighbors. For $\gamma = 0$ and $\Delta \neq 1$ the model is called XXZ model, the important special case $\Delta = 0$ being known as XX model [142]. $\gamma = 0$ implies conservation of the total z spin component, a case of obvious importance in quantum information transfer, since the number of ones in the state of the N -qubit system as expressed in the computational basis is then a constant of the motion. If $\gamma \neq 0$ and $\Delta \neq 0, 1$ the model is referred to as the general XYZ chain. Finally, for $\gamma \neq 0$ and $\Delta = 0$ the model is known as the XY chain, a special case being $\gamma = \pm 1$ where we have an Ising chain in a transverse magnetic field (TI chain). The $\Delta = 0$ case of (2) may be mapped [4, 5] to a model of noninteracting spinless lattice fermions with nearest-neighbor hopping by means of a Jordan-Wigner transformation [6], with the number of fermions being conserved for the XX case, $\gamma = 0$. Hence the XX chain has been quite popular as a model for quantum information transfer due to its simple dynamics.

The earliest example [7] for quantum information transfer in a spin chain, however, employed a ferromagnetic Heisenberg chain in its ground state with homogeneous couplings to whose ends a “sender” and a “receiver” spin can be coupled by the experimenters, Alice and Bob, after Alice has prepared the sender spin in the single-qubit state she wishes to transmit. As the state of the combined system evolves, information is transferred to the other end of the chain, where Bob at some suitable time decouples the receiver spin from the rest of the system and measures or further processes it. The fidelity of this process is less than perfect, but higher than the maximal value of $2/3$ [8] for classical transmission of a quantum state, for $N \leq 80$. Osborne and Linden [9] pointed out the deleterious effect of dispersion on single-qubit quantum state transfer and proposed to encode the quantum information to be transmitted not in the state of a single spin, but in a spin-wave packet constructed so as to involve only the approximately linear part of the dispersion relation $\omega(k)$ of the spin-wave excitations in a homogeneous Heisenberg model ring. Burgarth et al. [10, 11] suggested to improve the Heisenberg chain transfer protocol by using two or more chains in parallel and performing measurements in order to increase the fidelity of transfer. That “multirail” protocol is treated in [12]. Besides the presence of dispersion effects the homogeneous Heisenberg chain has another disadvantage which limits its usefulness and which was pointed out by, among others, Subrahmanyam [13]: two flipped spins in the ground state of a ferromagnetic Heisenberg chain interact with each other, leading to distortions of a two spin-flip state. More technically speaking, this means that while single spin-wave states are energy eigenstates, two spin-wave states are not, making their treatment more complicated and ultimately leading to the intricacies of the Bethe Ansatz [14].

Interactions between elementary excitations can be avoided by going from the Heisenberg chain to the XX

chain by dropping the z part of the nearest-neighbor interaction, but dispersion cannot be avoided as long as homogeneous chains ($J_i \equiv J$ in (2)) are considered. This precludes *perfect state transfer* (to be discussed below) for all but very short chains [15]. However, recent work on *pretty good state transfer* [16] shows that arbitrarily patient observers will obtain fidelities arbitrarily close to 1 in homogeneous XX chains of length $N = P - 1, 2P - 1$ with P a prime, or $N = 2^M - 1$. The waiting time depends on the required deviation from unity and seems to grow exponentially in the chain length.

Haselgrove [17] discussed an interesting scheme involving time-dependent manipulation of the nearest-neighbor couplings between both sender and receiver qubits and the rest of the chain. Another attempt at improving the transfer fidelity of homogeneous quantum chains is the application of a sequence of two-qubit gates (switchable interactions) between the end of the spin-chain “wire” and the receiving qubit [18]; some results on the stability of this scheme against disorder were reported in [19]. These schemes involving time-dependent couplings or external fields are at the border of quantum optimal control theory, an extremely rapidly developing field of research which, however, is not within the scope of the present chapter.

Zenchuk [20] suggested to consider not perfect state transfer but “complete information transfer”: The state of the sender system S (a subsystem of the communication system under study) is encoded in the initial reduced density operator $\rho_S(0)$ of S . The quantum time evolution of a fairly arbitrary chain system then performs a linear mapping of $\rho_S(0)$ to $\rho_R(t)$, the reduced density operator of the receiver subsystem R . If the Hilbert space of R is at least as large as that of S the linear mapping $\rho_S(0) \mapsto \rho_R(t)$ can be inverted for almost all times t and the original information may be reconstructed.

The comprehensive review by Bose [21] covers the development of spin-chain quantum information transfer until the end of 2006. The special class of homogeneously coupled spin chains is discussed in [22].

Since quantum information transfer in strictly homogeneous chains suffers from dispersion effects as explained above, two main strategies have been developed, both based on the natural dynamics of inhomogeneous qubit chains. One approach uses “fully engineered” chains, where $\mathcal{O}(N)$ coupling constants or local fields have to be assigned specific values in order to achieve perfect state transfer (PST). The other approach employs “boundary-controlled” chains in which only $\mathcal{O}(1)$ coupling constants connecting sender and receiver qubits to the transmitting “wire” have to be adjusted in order to achieve optimized state transfer (OST). Both strategies will be explained below.

1. Perfect state transfer in fully engineered chains

Perfect state transfer in engineered chains is based on the observation that a quantum system generates periodic dynamics if its energy spectrum displays only commensurate energy differences. Examples are the harmonic oscillator with energies $E_n = \hbar\omega(n + 1/2)$ ($n \geq 0$) or the infinite square well with $E_n = n^2 E_1$ ($n \geq 1$). A popular exercise in elementary quantum mechanics shows that arbitrary wave functions develop periodically (up to a global phase) under a harmonic force: $\psi(x, t) = -\psi(x, t + \frac{2\pi}{\omega})$. More interesting in the present context, but less often discussed in quantum mechanics courses is the relation $\psi(x, t + \frac{\pi}{\omega}) = -i\psi(-x, t)$, meaning that in one half period the state of the oscillator develops into a perfect spatial mirror image of the original state. This mirroring property of the quantum oscillator rests on the commensurate energy spectrum and on the alternating parities of successive energy eigenstates. If these two properties can be carried over to a one-dimensional array of quantum mechanical objects that array can be used for perfect quantum state transfer.

A charged spin- J particle in a magnetic field in z direction shows a finite equidistant energy spectrum of $2J + 1$ levels, the energy eigenstates being the eigenstates of J_z . Transitions between these states are caused by the transverse spin components J_x and J_y . In a 1979 paper Cook and Shore [23] employed the analogy between a spin- J system and a $(2J + 1)$ -level atom to derive a model for stepwise laser excitation, obtaining periodic solutions which permitted complete population inversion. Nikolopoulos et al. [24] turned population inversion to spatial inversion by suggesting an array of quantum dots able to accommodate an electron each, with tunneling between neighboring quantum dots adjusted so as to mimic the dynamics of the spin- J system. The same line of thought was followed by Christandl et al. [15] who found PST in an inhomogeneous open N -spin XX chain with couplings $J_i = \sqrt{i(N-i)}$ and equidistant energy levels in the single spin-flip sector. Albanese et al. [25] generalized the concept to other state-mirroring systems using earlier results [26] on the construction of finite quantum systems with periodic dynamics. Shi et al. [27] discussed other spin chains involving external fields and displaying a commensurate energy spectrum.

Yung and Bose [28] and Karbach and Stolze [29] suggested a systematic approach to the construction of PST chains with a desired energy spectrum by solving a special type of Jacobian inverse eigenvalue problem [30, 31]. Given a set of N real numbers $E_1 < E_2 < \dots < E_N$, it is always possible to construct a unique *persymmetric Jacobi matrix*, that is, a real symmetric tridiagonal matrix with diagonal entries a_1, \dots, a_N and strictly positive super- / subdiagonal entries b_1, \dots, b_{N-1} , with the additional symmetry conditions $a_i = a_{N+1-i}$ and $b_i = b_{N-i}$ which has the prescribed numbers E_i as eigenvalues. Note that the number of given eigenvalues equals the total number of independent matrix elements a_i and b_i .

In an XX chain the b_i are related to the nearest-neighbor couplings J_i and the a_i are related to the local fields h_i in (2). The persymmetry of the matrix corresponds to the spatial symmetry of the spin chain and makes sure that successive eigenvalues correspond to eigenvectors of opposite parities. The eigenvalue spectrum can be chosen freely, as long as the energy differences are commensurate, to ensure PST. That freedom of choice may be used to provide the system with other desirable properties besides PST. In [29] it was shown, for example, that it is possible to perform PST in chains with nearly homogeneous couplings, deviating from $J_i = \text{const}$ only on the few percent scale. If the diagonal matrix elements a_i (the local fields h_i in the XX chain) vanish, the eigenvalues are symmetrically distributed about zero, that is, $E_i = \pm|E_i|$ and the number of unknowns in the inverse eigenvalue problem is greatly reduced. For that special case, a simple algorithm was recently proposed by Wang et al. [32]; for the more general case, many interrelated algorithms are known [30, 31].

A simple spin-1/2 XX chain with PST thus is given by

$$H = \frac{1}{2} \sum_{i=1}^{N-1} J_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y) \quad (J_i = J_{N-i}), \quad (3)$$

with the J_i chosen appropriately. Let us pause briefly to point out some important facts about this system. By the Jordan-Wigner transformation [6] (3) is equivalent to the Hamiltonian of noninteracting lattice fermions with hopping elements J_i . The number of fermions corresponds to the number of down spins and the ferromagnetic all-up state corresponds to the fermion vacuum. The Jacobi matrix discussed above in the context of the inverse eigenvalue problem is the Hamiltonian restricted to the subspace of a single fermion. All single-fermion states are transferred to their spatial mirror images at the same instant of time, and so are suitable many-fermion states, since the Jordan-Wigner fermions do not interact. This means that in contrast to many other quantum information transfer schemes, PST in the system (3) is not restricted to single-qubit states, as already noted in [25].

A generalization of the above PST systems was suggested by Kostak et al. [33] who discuss Hamiltonians which generate permutations between the sites of a network and can thus be employed for single-qubit perfect state transfer. A generalization of the PST concept in the mathematical sense is ‘‘almost PST’’ as defined recently [34] by replacing the periodic functions in the time evolution of the quantum system with almost periodic functions. A useful review on various aspects of PST was written by Kay [35].

2. Optimized state transfer in boundary-controlled chains

The second strategy employing spin chains, boundary-controlled chains, is restricted to the transfer of single-qubit states by construction. In this strategy the sender

qubit A and the receiver qubit B are only weakly coupled to the large system used to transfer the quantum information, the “quantum data bus”, for example, a homogeneous spin chain.

An early example for that concept was given in [36], for a ring of coupled harmonic oscillators. Within the single-excitation subspace the coupled oscillator chain is equivalent to a spin-1/2 XX chain. A very clear picture of what is going on in the weak-coupling scenario was given in the short paper by Wójcik et al. [37]. There a homogeneous N -site XX chain with $J_i \equiv 1$ is coupled to two end spins with coupling strength $\alpha < 1$. The eigenvalues and eigenvectors responsible for single-excitation transfer are determined analytically and it is found that for very small values of α only two or three (when N is odd) closely-spaced energy levels in the center of the spectrum are important. Oscillations between these states determine the quantum information transfer, and the transfer time increases as α decreases, while the deviation of the fidelity from unity scales as $\alpha^2 N$. The transfer time is $\mathcal{O}(\alpha^{-2})$ for even N and $\mathcal{O}(N^{\frac{1}{2}}\alpha^{-1})$ for odd N . Furthermore it is observed that after half the transfer time the two end spins become entangled for even N .

Another system proposed [38] to be used as a data bus is a spin ladder. Due to the excitation gap above the ladder’s ground state, perturbation theory can be used to eliminate the ladder to lowest order and to replace it by an effective coupling between qubits A and B . The ground-state entanglement between the end spins in some spin chains was suggested [39] to be used for teleportation or state transfer even at finite temperature T , as long as T is smaller than the smallest excitation gap which depends on the chain length. Under suitable conditions, end spins A and B weakly coupled to an intermediate chain can then be approximated by an effective two-spin model. Similar approximations are also considered in more recent spin bus scenarios [40]. The same general strategy, of separating the sender-receiver Hilbert space of the spins A and B from the rest of the system, was followed in [41]. Since the nature of the transfer medium connecting A and B is to a large extent irrelevant, that medium may even be a spin chain with random couplings, according to [42]. The separation of end spins from the remainder of the system may also be achieved by applying strong local fields to the end spins [43]. Thereby, two states of opposite parities and both strongly localized at the boundary spins are created, which are spectrally separated from the remainder of the Hilbert space, and which may be used for near-perfect state transfer [44]. This last example shows very clearly a drawback common to many of the boundary-controlled scenarios. The smaller the coupling between sender / receiver and data bus, the better is the quality of transfer. At the same time, unfortunately, the energy differences driving the dynamics become smaller and smaller and the transfer slows down, increasing the danger of the quantum information being destroyed by fluctuating interactions with the environment before it is transferred completely.

The boundary-controlled scenarios discussed up to now may be termed “weak-coupling” scenarios. We now briefly discuss the “optimal-coupling” scenario discovered in a numerical study by Zwick and Osenda [45] and at the same time developed analytically by Banchi et al. [46, 47]. As in [37], a homogeneous N -site XX chain with $J_i \equiv 1$ is coupled to two end spins with coupling strength $\alpha < 1$, however, α is optimized in a different way. For $\alpha = 1$, i.e. a completely homogeneous XX chain with $N + 2$ sites, the energy eigenvalues in the single-excitation subspace are proportional to $\cos k$, with $k = \frac{n\pi}{N+3}$, ($n = 1, \dots, N + 2$), while the corresponding eigenvector amplitudes at site i of the chain are $u_k(i) = \sqrt{\frac{2}{N+3}} \sin ki$. Note that in the vicinity of $k = \frac{\pi}{2}$ the energies are approximately linear in k , so that packets of spin waves from that region are approximately free from dispersion. For $\alpha \neq 1$ the situation changes, but it is still possible to treat the eigenvalue problem analytically. It turns out [47] that the initial state with the sender spin down (and all others up) corresponds to a wave packet with approximately Lorentzian probability distribution in k , with center and width depending on α . Since the energy spectrum also depends on α it is possible to jointly optimize the width of the wave packet and the linearity of the energy spectrum in order to achieve near-perfect state transfer. It turns out that the optimal value of α scales as $N^{-\frac{1}{6}}$. Further optimization is possible [48] if not only the first and last, but also the second and second to last bonds may be adjusted.

II. FULLY-ENGINEERED VS. BOUNDARY-CONTROLLED CHAINS

The topic of the present book chapter is the transfer of quantum information solely by the natural dynamics of a spin chain with fixed couplings, as explained in section IB. Although the perfect-transfer schemes of section IB 1 are able to transfer multi-qubit states, we shall restrict our attention to the most frequently discussed case, single-qubit state transfer. Since nothing in this world is perfect, including computer hardware and software, both classical and quantum, information transport in spin chains is vulnerable to two main sources of irregularity: external dynamic randomness, that is, fluctuating fields caused by the environment, and internal static randomness, caused by inaccurate implementation of the theoretical design of the chain. Here we shall exclusively deal with static randomness and with the robustness of both the perfect state transfer chains from section IB 1 and the optimized state transfer chains from section IB 2 against this kind of “manufacturing errors”.

(The work described in the remainder of this section of [1] is based on the material published in [49–51].)

III. OTHER THEORETICAL APPROACHES

In this section we review some studies dealing with the robustness of quantum information transfer schemes. We are restricting ourselves to the transfer of single-qubit states and we consider only static randomness inherent in the system, that is, fabrication errors. Furthermore, although interesting schemes involving different kinds of qubit networks [52–57] have been suggested, we will discuss only strictly one-dimensional systems. Despite these restrictions we are sure to have missed some important contributions in this rapidly developing field.

(In [1] we also discuss the contributions [24, 58–61].)

An influential early paper is the study [62] by De Chiara et al who considered the influence of disorder on the state transfer properties of the PST chain with linear energy spectrum [15]. Both the nearest-neighbor couplings J_i and the local z fields h_i ($h_i = 0$ in the ideal case) were assumed random. Borrowing from the language of single-particle transport in a one-dimensional tight-binding chain, one might call the disorder in h_i diagonal and the disorder in J_i off-diagonal. These two kinds of disorder are known to have fundamentally different effects on the localization and transport properties of nearest-neighbor coupled tight-binding chains [63, 64]. In fact, these two kinds of disorder also turn out to be very different for the spin chains in [62]: Coupling constant disorder is the more detrimental the longer the chains become, while magnetic field disorder apparently averages out for longer chains. Numerical evidence and perturbation calculations for weak disorder show that the fidelity [143] is a decreasing function of the two variables $N\varepsilon_J^2$ and ε_h^2/N , where ε_J measures the strength of the (relative) coupling disorder and ε_h does the same for the (absolute) field disorder. Note that the ideal coupling values $J_i = J\sqrt{i(N-i)}$ scale with the chain length so that a given relative disorder strength ε_J entails larger absolute changes in the couplings for longer chains. In contrast the random fields h_i do not scale with N , and ε_h^2/N is what the central limit theorem yields for the variance of the (zero) average field $\frac{1}{N}\sum_i h_i$.

Burrell and Osborne [65] investigated correlations in an infinite XX chain with random nearest-neighbor interactions and a random magnetic field and showed that all correlations are exponentially suppressed outside of an effective “light cone” whose radius grows at most logarithmically in time (and hence is no light cone at all). This means that information transfer out of a region of given size will take exponentially long times in the limit of an infinite system. This is Anderson localization at work: In one dimension all states go localized at arbitrarily small diagonal disorder [63], but things are different for off-diagonal disorder [64] since in that case there is always a delocalized state at the center of the band. Localization effects were also discussed by Keating et al [66], unfortunately without clear distinction between the two kinds of disorder. However, localization can be overcome for finite systems. In fact, it has been shown [67]

how to employ quantum error correction techniques to send a qubit with high fidelity using several imperfect spin chains in parallel, over distances large compared to the individual chain’s localization length. In temporally fluctuating fields things are different, see [68] for a study of localization properties in that case.

An early example for robustness considerations is the paper by Kay [69], which contains a section about manufacturing errors. This paper considers couplings beyond nearest neighbors in XX chains and also discusses the influence of, for example, timing errors when reading out the transmitted state.

Three different scenarios of quantum information transfer along XX spin chains are covered in [70]. The first scenario involves sequential SWAP operations effected by switching in turn every single spin coupling for an appropriate duration. The second scenario employs the natural dynamics of the PST system with linear spectrum without any external driving, while the third one achieves adiabatic state transfer by slowly switching all even and odd couplings appropriately. The transfer times for all these schemes scale as $\tau \sim N/J_{max}$, where J_{max} is the maximum coupling available. All three scenarios are studied in the presence of diagonal (magnetic field) and off-diagonal (exchange coupling) static randomness. It turns out that the sequential SWAP scheme is most susceptible to randomness, especially of the off-diagonal type. The linear-spectrum PST system is more robust than the sequential SWAP scheme, but the adiabatic state transfer scheme is most noise-tolerant, at least for the system sizes ($N \leq 51$) studied.

Ronke et al. [71] performed a comprehensive study of robustness of state transfer in short PST chains ($N \leq 15$) with linear spectrum [15]. The built-in perturbations considered were randomness in the nearest-neighbor couplings, site-dependent random magnetic fields, interactions between travelling excitations and unwanted next-nearest neighbor spin couplings. In addition, also handling errors, such as readout timing errors were studied. It was found that next-nearest neighbor spin couplings had the strongest detrimental effect on the quality of state transfer. The general behavior of the transfer fidelity was found to be consistent with an exponential decay with chain length and a Gaussian dependence on the disorder strength.

Bruderer et al. [72] suggested a smart hybrid approach unifying advantages of the fully-engineered and boundary-controlled state transfer schemes. Their idea amounts to optimizing the temporal structure (commensurate spectrum leading to perfect periodicity) and the spatial structure (boundary-localized states insensitive to perturbations from the interior of the chain) at the same time.

(In [1] a more detailed discussion follows. Also discussed is the work of [40] on short isotropic Heisenberg (XXX) antiferromagnetic chains, and of [73], where boundary-localized states were created by erecting high magnetic field barriers on sites close to the ends of a

homogeneous XX chain, and the implementation of the PST scheme suggested by Christandl et al. [15] in an array of 19 laterally coupled parallel waveguides [74].)

IV. EXPERIMENTAL IMPLEMENTATIONS

In this section we review some of the experimental implementations of state transfer using protocols based on spin chain channels and show their present limitations. Every proposal for physical qubits that allows to couple them permanently can be used to develop spin chains. Therefore, in solid-state systems, there are proposals to implement qubit chains using superconducting nanocircuits, such as charge qubits [75, 76], Josephson junctions [60, 77, 78] or flux qubits [79, 80]. The advances in semiconductor technology allow to couple quantum dots, so there are proposals using chains of charged quantum dots [24, 58, 81] or alternatively, excitons in quantum dots [82, 83]. Spin chains can be also simulated in optical lattices [84–90] or with nuclear spin systems in NMR [91–93]. Nitrogen vacancy centers in diamond [42, 94–97] constitute another promising solid-state system. However, only very few of these systems have actually developed into experimental implementations of quantum spin channels and in particular NMR was the pioneer setup for testing these protocols.

The main limitation to make these quantum channels a reality is decoherence, which not only affects the survival time of the quantum information [98], but also affects the distance over which it can be transmitted [62, 65–67, 99–103]. Perfect or high fidelity state transfer can be obtained by many of the theoretical methods described so far. However, if the ideal control Hamiltonian or system for the state transfer is affected by decoherence, the transfer fidelity can be remarkably reduced. Decoherence effects can come from either time dependent perturbations or even static ones. In order to show this, let us consider the simplest quantum channel of two qubits, where a SWAP operation transfers the state from one qubit to the other. The experimental implementation of a SWAP operation was first addressed within the field of liquid state NMR [104–106]. However, pioneering solid state NMR experiments performed by Müller, Kumar, Baumann, and Ernst [107] can now be identified as a SWAP operation. Even in this simple 2-qubit channel the swapping oscillation is damped by a decoherence rate that depends on the rate of interaction with the environment. Even worse, if the interaction rate with the environment becomes larger than the ideal swapping frequency between the two spins, the swap is frozen, manifesting an overdamped dynamics due to localization effects of the initial excitation [102, 108, 109].

Ideally quantum communication is expected to be performed by means of pure-state transfers. Consequently most of the theoretical approaches focused on pure-state communication processes, but experimental realization of pure states is a major challenge for present technologies.

Just recently a lot of progress is being made with superconducting devices, semiconductor technologies, optical lattices and Nitrogen vacancy centers where pure states can be generated and controlled, even at the single qubit level [110–115]. However, the first implementations of quantum computation were based on ensemble quantum computing using mixed states that mimic pure-state quantum evolutions, in particular with NMR setups [92, 93, 116]. State transfer in a solid state system has been observed in a ring of spins with dipolar (many-body) interactions [117, 118]. An initial polarization localized in a specific spin of the ring propagates around the ring and after a time related to the ring length, a constructive interference reappears in the form of an echo [117, 118]. This mesoscopic echo [119] reflects the quantum nature of the finite quantum spin-ring. However, the many-body nature of these spin-spin interactions made these systems to sensitive to perturbations which strongly reduce the amplitude of the echo [120]. In order to improve the state transfer fidelity an effective XX interaction Hamiltonian [144] has been experimentally implemented in a spin chain by Madi et al [91] by using global-pulse rotations of the spins in a liquid state sample. In Ref. [91], the evolution of the initial excitation was monitored in all the spins of the quantum channel. In that work two important features deserve special attention. On the one hand it is relevant to be able to generate, even if artificially, the simple one-body XX Hamiltonian for state transfer because it increases the transfer fidelity in comparison to more complicated many-body Hamiltonians. On the other hand, by comparing the experimentally observed evolution of polarization transfer to the ideal design, one can assess the decoherence effects reducing the transfer fidelity. These decoherence effects originate from the finite precision on control pulses and from interactions with external degrees of freedom changing the coupling strengths between the spins and/or their Zeeman energies.

Due to the non scalability of liquid state NMR, more recently, implementations of spin chains were attempted in solid-state NMR [121–127] which mimic the XX state transfer evolution. This approach is based on experimentally generating a double quantum Hamiltonian where the evolution of a locally prepared initial state can be mapped to the one generated by an XX interaction [128]. However, these systems do not allow individual addressing of the qubits, engineering of the coupling strengths or local manipulations for generating PST.

Up to now, PST protocols could only be implemented in systems of very few spins, mainly in trivial cases of 2 [104–106] and 3 [129, 130] spins interacting by a homogeneous XX interaction. Again, the XX interaction was not natural and in particular in these cases it was engineered by controlling the spins individually and generating the desired effective Hamiltonian. Similarly, but only requiring global control-pulses on a 6-spin system, effective chains of 2, 3, and 6 spins were generated for implementing PST protocols [131]. In this case the selec-

tive spin-spin coupling networks were created by exploiting selective quantum interferences in the time domain to filter out the undesired couplings while leaving intact the desired ones [131]. The process can be interpreted as a time-domain analog of Bragg gratings that filter the non-selected coupling strengths [132, 133]. Alternative state transfer protocols with spin chains for achieving arbitrarily high fidelities were constructed applying iterative state transfer [18] along chains of 3 and 4 spins by controlling only the boundary spins [134, 135]. Again, in all these cases decoherence by external degrees of freedom or finite precision control changes the coupling strengths or induces energy fluctuations of the spins affecting the transfer fidelity.

The decoherence effects also are the main limitation on the chain lengths. Decoherence effects increase as the number of qubits increases [101, 103, 136–139]. The sensitivity of the quantum states grows with the number of spins, causing imperfections, disorder or external influence on the couplings within the spin channel to in-

duce localization of the quantum information [62, 65–67, 99–101, 103, 140]. These localization effects were recently observed experimentally in three-dimensional spin-network topologies with about 7000 spins, where the localization effects were induced by finite precision control of the quantum gates driving the information transfer [101, 103]. Thus it is clear that the only way of building quantum computers or quantum simulators has to be based on developing robust methods of controlling the information and in particular the state transfer [141].

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- [144] Many experimental references employ the term XY or planar interaction for what we call XX interaction here.