numbers. The existence of the maximum provides limited opportunities to enhance ...at
...gular, differentially heated cavities, which would possibly be potentiated in cavities with
...ting horizontal walls, and in tall cavities with multiple divisions. From a theoretical point
...ue of S at which the maximum overall Nusselt number occurs at a given Ra is shown to
... characterize the transition from a shallow cavity regime to a slender cavity regime.

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ESTIMATION OF THE OCCURANCE OF THE PHASE-CHANGE PROCESS IN SPHERICAL COORDINATES

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ABSTRACT

We study heat conduction problems in spherical coordinates with mixed boundary conditions. We obtain sufficient and/or necessary conditions among the data in order to get a phase-change process. For the spherical coordinates case we consider a problem in a hollow sphere $r_1 < r < r_2$, where the boundary conditions are the heat flux ($q > 0$) on the surface $r = r_1$ and the temperature the surface ($b > 0$) on the surface $r = r_2$, and an initial condition in the hollow sphere is also considered. We analyse both, the case with or without source. We explicit the relation between the heat flux q, the fixed temperature b and the thermal conductivity k of the initial phase, in order to have a change of phase in the material. © 1999 Elsevier Science Ltd

Introduction

Heat transfer problems with phase-change such as melting and freezing have been studied in the last century due to their wide scientific and technological applications. The modeling of solidification systems is a problem of a great mathematical and industrial significance. Phase-change problems appear frequently in industrial process and other problems of technological interest [1,3,5,9,10]. For example, a review of a long bibliography on moving and free boundary problems

: equation, particulary concerning the Stefan problem, is presented in [17] with a large y.

y heat conduction problems in spherical coordinates with mixed boundary conditions. ufficient and/or necessary conditions among the data in order to estimate the occurance hange process. We consider a heat conduction problem in a hollow sphere $r_1 < r < r_2$, oundary conditions are the heat flux ($q > 0$) on the surface $r = r_1$ and the temperature he surface $r = r_2$, and an initial condition in the hollow sphere is also considered. We h, the case with or without source. We explicit the relation among the heat flux q, the rature b and the thermal conductivity k of the initial phase, in order to have a change of material. We suppose, without loss of generality, that the phase-change temperature he data monotonicity the corresponding phase-change interface begins at $r = r_1$.

y the following heat conduction problem: we consider a hollow sphere with radii r_1, r_2 the initial temperature $\theta_0 = \theta_0(r) \geq 0$, having a heat flux $q = q(t) > 0$ on the internal r_1) and a temperature condition $b = b(t) > 0$ on the external surface ($r = r_2$). Then duction problem for the initial (liquid) phase is given by the following mathematical :

$$k(\theta_{rr} + \tfrac{2}{r}\theta_r) = \rho c \theta_t, \qquad r_1 < r < r_2, \qquad t > 0; \tag{1.1}$$

$$\theta(r,0) = \theta_0(r), \qquad r_1 \leq r \leq r_2; \tag{1.2}$$

$$k\theta_r(r_1,t) = q(t), \qquad t > 0; \tag{1.3}$$

$$\theta(r_2,t) = b(t), \qquad t > 0. \tag{1.4}$$

me that the data satisfy the hypotheses that ensure the existence and uniqueness of the he Problem Ps.

r, the following two possibilities can occur:

eat conduction problem is defined for all $t > 0$;

exists a time $T_w < \infty$ (called, a waiting-time) such that another phase (i.e., the solid ears for $t > T_w$ and then we will have a solidification process, i.e. a two-phase Stefan this case, there exists a front (free-boundary) $r = s(t)$ which separates the liquid and ases whose initial position is given by $s(T_w) = r_1$. Moreover, for $t < T_w$ we have a heat problem for the initial liquid phase.

These two only possibilities depend on the data θ_0, q, k, b. We try to clarify this dependence by finding necessary or sufficient conditions on data in order to estimate the different possibilities.

In [6,12,13] the unidimensional one-phase Stefan problem with prescribed flux or convective boundary conditions at $x = 0$ is studied. This paper was motivated by [2,14,16], where some explicit results were obtained for the one-dimensional case. First we analyse the heat conduction problem in spherical coordinates (Ps), and then we study the corresponding problem with a source term.

Heat Conduction Problems in Spherical Coordinates

Without a Source Term

In order to study the possibilities a) and b) for the problem Ps we consider the steady-state heat conduction problem (called Problem P_∞), corresponding to (1.1)-(1.4), which is given by :

$$\theta''_\infty + \frac{2}{r}\theta'_\infty = 0, \quad r_1 \leq r \leq r_2;$$

$$\theta_\infty(r_2) = b > 0; \qquad k\theta'_\infty(r_1) = q > 0,$$

where b and q are positive constants.

The solution of Problem P_∞ is given by:

$$\theta_\infty(r) = b + \tfrac{q}{k} r_1^2 \left(\tfrac{1}{r_2} - \tfrac{1}{r}\right), \quad r_1 \leq r \leq r_2. \tag{2.1}$$

The temperature at the interior surface $r = r_1$ is given by $\theta_\infty(r_1) = b + \frac{q}{k} r_1^2 (\frac{1}{r_2} - \frac{1}{r_1})$ and we can conclude that if the heat flux q satisfies the following inequality,

$$q > Q_\infty = kb \tfrac{r_2}{r_1} \tfrac{1}{(r_2 - r_1)}, \tag{2.2}$$

then the temperature $\theta_\infty(r_1) < 0$, that means that we have a steady state phase-change process [14]. This condition is also a necessary condition to have a change of phase [15]. From this, it is natural to think that we can expect to have a change of phase for the evolution case (Problem Ps), if $q > Q(t)$ (with $Q(t) > Q_\infty$) for $t > t_Q$, for a suitable positive time t_Q.

The answer to this question is given below.

we will obtain sufficient condition on data of problem (Ps) in order to get a wa[illegible]g time, possibility b).

y 1:

data $q = q(t), \theta_0 = \theta(r)$, and $b = b(t)$ verify the following conditions:

$< q(t) \leq q_0,\ 0 < t \leq t_0;$

$< \beta_0 \leq \theta_0(r) \leq \beta_1,\ \theta_0'(r) \geq 0, \qquad r_1 \leq r \leq r_2;$

$(t) \geq \beta_1,\ \dot{b}(t) \geq 0, t > 0;$

exists a "waiting time" $T_\omega > 0$ for Problem (Ps) and its expression is given by $= min(t_0, T_W^*)$ where

$$\begin{cases} +\infty \quad if & 1 - \frac{\beta_0 k}{r_1 q_0} \leq 0 \\ \frac{r_1^2}{\alpha}\left(H_1^{-1}(1 - \frac{\beta_0 k}{r_1 q_0})\right)^2 \ if & -\frac{\beta_0 k}{r_1 q_0} + 1 > 0. \end{cases} \tag{2.3}$$

H_1^{-1} is the inverse function of H_1 where $H_1(x) = exp(x^2)erfc(x)$.

n to prove the result it is sufficient to show that $\theta(r,t) \geq 0$ for $r_1 \leq r \leq r_2$ and $0 \leq t \leq T_\omega$.

1.

waiting time T_ω increases with the parameter β_0 (the lower bound for the initial tem-

Property 1 implies that the model of heat conduction, given by problem Ps, under the n (i-iii) is only valid for $t < T_\omega$.

emperature θ, the solution of Problem Ps, is a non decreasing function of variable r, by un principle, ie. $\theta_r(r,t) \geq 0$ for all $r_1 \leq r \leq r_2, t > 0$., when the following conditions

$(r) \geq 0, \qquad r_1 \leq r \leq r_2;$

$(t) \geq 0, \qquad t > 0;$

$= q(t) > 0, \quad t > 0.$

ed.

The temperature θ (be written as $\theta = \theta_\infty + U_0 + \frac{2}{k}U_1$, where U_0 and U_1 satisfy the following problems:

Problem Pu_0	Problem Pu_1
$\alpha(U_{0_{rr}} + \frac{2}{r}U_{0_r}) = U_{0_t}$	$\alpha(U_{1_{rr}} + \frac{2}{r}U_{1_r}) = U_{1_t}$
$U_0(r,0) = \theta_0(r) - b \leq 0$	$U_1(r,0) = -r_1^2\left(\frac{1}{r_2} - \frac{1}{r}\right) > 0$
$U_{0_r}(r_1,t) = 0$	$U_{1_r}(r_1,t) = 0$
$U_0(r_2,t) = 0$	$U_1(r_2,t) = 0.$

By the maximun principle $U_0 \leq 0$ and $U_1 \geq 0$. Using [11] the solution U_1 can be written down as:

$$U_1(r,t) = \frac{1}{r}\sum_{m=1}^{\infty} \frac{R(\beta_m r)}{N(\beta_m)} e^{-\alpha\beta_m^2 t} \int_{r_1}^{r_2} \rho(r_1^2\left(\frac{1}{\rho} - \frac{1}{r_2}\right)R(\beta_m \rho)d\rho, \qquad r_1 < r < r_2,\ t > 0 \tag{2.4}$$

where
$$R(\beta_m r) = \beta_m\ cos(\beta_m(r - r_1)) + \frac{1}{r_1}sin(\beta_m(r - r_1)), \tag{2.5}$$

$$\frac{1}{N(\beta_m)} = \frac{2}{(\beta_m^2 + \frac{1}{r_1^2})(r_2 - r_1) + \frac{1}{r_1}}; \tag{2.6}$$

with $\beta_m > 0$ (for $m \in N$) are the solutions of the equation:

$$tg(\beta_m(r_2 - r_1)) = -\beta_m r_1, \text{ and } \beta_m > 0. \tag{2.7}$$

By some computation and taking into account (2.7) the function R can be written only in sine terms as:

$$R(\beta_m r) = -\sqrt{\beta_m^2 + \frac{1}{r_1^2}}\ \sin\ (\beta_m(r_2 - r)), \tag{2.8}$$

and then we can compute the integral

$$\int_{r_1}^{r_2} \rho r_1^2\left(\frac{1}{\rho} - \frac{1}{r_2}\right) R(\beta_m\rho)d\rho = \frac{r_1^2}{\beta_m}\sqrt{\beta_m^2 + \frac{1}{r_1^2}}\ cos(\beta_m(r_1 - r_2)). \tag{2.9}$$

Therefore U_1 can be written as

$$U_1(r,t) = -\frac{r_1^2}{r}\sum_{m=1}^{\infty}\left(\frac{\beta_m^2 + \frac{1}{r_1^2}}{N(\beta_m)}\right)e^{-\alpha\beta_m^2 t}\,\frac{sin(\beta_m(r_2 - r))cos(\beta_m(r_2 - r_1)}{\beta_m} \tag{2.10}$$

for $r_1 < r < r_2, t > 0$ and then we have

$$U_1(r_1,t) = -\sum_{m=1}^{\infty}\frac{e^{-\alpha\beta_m^2 t}}{N(\beta_m)}, t > 0. \tag{2.11}$$

Theorem 2.

Let θ be the solution of Problem Ps, with the initial and boundary data satisfying the following hypotheses:

i) $\theta_0(r) \leq b(0)$, $r_1 \leq r \leq r_2$;

ii) $\theta_0'(r) \geq 0$, $r_1 < r < r_2$;

iii) $b(t) > 0$, $\dot{b}(t) > 0$, $t > 0$;

iv) $q(t) > 0$, $t > 0$.

there exists a curve $q = Q(t)$ in the first quadrant of the plane (q,t) such that if $q > Q(t)$, $> t^*$, we have a phase change process for the material where t^* is defined by

$$t^* = \frac{1}{\alpha\beta_1^2} log\Big(\frac{\sum_{m=1}^{\infty}\frac{1}{N(\beta_m)}}{r_1^2(\frac{1}{r_1}-\frac{1}{r_2})}\Big). \tag{2.12}$$

any $\gamma = \frac{r_2}{r_1} \in (1, \gamma_0]$ with a suitable $\gamma_0 > 1$.

ce $\theta_r \geq 0$ we will only check the temperature θ at $r = r_1$, that is

$$\theta(r_1,t) \leq b + \frac{q}{k}\Big(\sum_{m=1}^{\infty}\frac{e^{-\alpha\beta_1^2 t}}{N(\beta_m)} + r_1^2(\frac{1}{r_2} - \frac{1}{r_1})\Big),$$

$U_0(r_1,t) \leq 0$, and β_m is increasing with m.

refore we have $\theta(r_1,t) \leq 0$ when

$$q \geq Q(t) = \frac{bk}{r_1^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right) - e^{-\alpha\beta_1^2 t}\sum_{m=1}^{\infty}\frac{1}{N(\beta_m)}}$$

t^*, where t^* is the value that makes

$$r_1^2\Big(\frac{1}{r_2}-\frac{1}{r_1}\Big) + e^{-\alpha\beta_1^2 t^*}\sum_{1}^{\infty}\frac{1}{N(\beta_m)} = 0,$$

(2.12).

e value t^* is great than zero if and only if $H_2(\gamma) > 1$, where $H_2(x)$ is defined by the following ession

$$H_2(x) = 2\frac{\sum_{m=1}^{\infty}\frac{1}{1+\frac{\lambda_m^2}{x(x-1)}}}{x-1};$$

$$\lambda_m = \beta_m(r_2 - r_1);\ \gamma = \frac{r_2}{r_1}.$$

er some mathematical manipulation we can conclude that $H_2(\gamma) > 1$ for some $(1, \gamma_0]$ with an adequate constant $\gamma_0 > 1$.

Remark 2.

i) If we consider the (t,q) plane and we define the following set $S = \{(t,q)/q > Q(t), t \geq t^*\}$, in the first quadrant then we have a two-phase problem for all $(t,q) \in S$, and $\frac{r_2}{r_1} \in (1, \gamma_0]$.

ii) We can obtain an upper estimation of the time t^* defined by (2.12). Since $\beta_m \geq \left(\frac{2m-1}{2(r_2-r_1)}\right)\pi$ we can deduce

$$\sum_{m=1}^{\infty}\frac{1}{\left(\beta_m^2 + \frac{1}{r_1^2}\right)(r_2 - r_1) + \frac{1}{r_1}} \leq \sum_{m=1}^{\infty}\frac{4(r_2 - r_1)}{\left(2m-1\right)^2\pi^2} = \frac{r_2 - r_1}{2}.$$

Then the upper bound for t_Q is given by the following inequality

$$t_Q \leq \frac{1}{\alpha\beta_1^2} log\Big(\frac{r_2}{r_1}\Big). \tag{2.13}$$

Heat Conduction Problems in Spherical Coordinates With a Source Term

Let θ be the solution of the following heat conduction problem with a constant source term g in spherical coordinates:

Problem P_g:

$\theta_t - \alpha(\theta_{rr} + \frac{2}{r}\theta_r) = \frac{g}{\rho c}$, $r_1 < r < r_2, t > 0$;

$\theta(r,0) = \theta_0(r)$, $r_1 < r < r_2$;

$k\theta_r(r_1,t) = q > 0$, $t > 0$; $\theta(r_2,t) = b > 0$, $t > 0$.

The steady-state solution for the problem Pg is given by [8]

$$\begin{aligned}\theta_\infty(r) &= -\frac{g}{6k}r^2 - \Big(\frac{g}{3k}r_1^3 + \frac{q}{k}r_1^2\Big)\frac{1}{r} + b + \frac{g}{6k}r_2^2 + \frac{q}{k}\frac{r_1^2}{r_2} + \frac{g}{3k}\frac{r_1^3}{r_2}\\ &= b + \frac{g}{k}\left[\frac{-r^2}{6} + \frac{r_2^2}{6} - \frac{r_1^3}{3r} + \frac{r_1^3}{3r_2}\right] + \frac{q}{k}\left[\frac{-r_1^2}{r} + \frac{r_1^2}{r_2}\right], \quad r_1 < r < r_2.\end{aligned} \tag{2.14}$$

Property 3:

We obtain the following properties in order to determine the sign of the steady-state temperature θ_∞ (i.e. the phase of the material).

(i) If $q > 0$, then the function θ_∞ is of non-constant sign (there are two-phases within the material) if and only if $\theta_\infty(r_1) < 0$, that is $q > Q_\infty^g = \frac{kr_2}{r_1(r_2-r_1)}\left[b + \frac{g}{k}\left(\frac{r_1^3}{3r_2} + \frac{r_2^2}{6} - \frac{r_1^2}{2}\right)\right]$ and $g > g_0$ with $g_0 = \frac{bk}{-\frac{r_1^2}{2}+\frac{r_2^2}{6}+\frac{r_1^3}{3r_2}} > 0$.

$q < 0$, then θ_∞ if of non-constant sign if and only if $g < 0$ and $\theta_\infty(r_m) < 0$, where r_m is the solution of the equation $\theta'_\infty(r_m) = 0$ in (r_1, r_2), where $r_m = \sqrt[3]{r_1^3 + 3\frac{q}{gr_1}}$ and $\theta_\infty(r_m) < 0$ $(\frac{r_2}{r_1})^3 > 1 + \frac{3q}{gr_1}$.

ark 3

r all $r_2 > r_1$ we have that $\frac{r_1^2}{2} + \frac{r_2^2}{6} + \frac{r_1^3}{3r_2} > 0$.

ow we shall consider the possibility of the occurence of the phase-change process in the evolu- blem Pg. From the above results it is natural to think that we can expect to have a change in the evolution case if $q \geq Q^g(t) \geq Q^g_\infty > 0$ for $t > t_{Q,g}$, with $t_{Q,g} > 0$.

m 4.

$\theta(x,t)$ be the solution of the problem Pg with the hypotheses of the Theorem 2 then there curve $q = Q^g(t)$ in the (q,t) plane (for each positive g) such that if $q > Q^g(t)$ and $t \geq t^*$, a change of phase for a suitable $t^* > 0$

temperature θ is given by $\theta = \theta_\infty + u_0 + \frac{q}{k}u_1 + \frac{g}{k}u_2$, where u_0 satisfies problem Pu e initial data $u_0(r,0) = \theta_0(r) - b \leq 0, r_1 < r < r_2$; u_1 satisfies problem Pu with the data $u_1(r,0) = \frac{r^2}{6} + \frac{r_1^3}{3r} - \left(\frac{r_1^3}{3r_2} + \frac{r_2^2}{6}\right)$; and u_2 satisfies the problem Pu with the initial data $= r_1^2\left[\frac{1}{r} - \frac{1}{r_2}\right]$.

an compute the temperature at r_1 and we get

$$\theta(r_1,t) = u_0(r_1,t) + \frac{g}{k}u_1(r_1t) + \frac{q}{k}u_2(r_1t) +$$

$$-\frac{g}{6k}r_1^2 - \left(\frac{g}{3k}r_1^3 + \frac{q}{k}r_1^2\right)\frac{1}{r_1} + b + \frac{g}{6k}r_2^2 + \frac{gr_1^3}{3kr_2} + \frac{qr_1^2}{kr_2} \leq$$

$$\leq \frac{q}{k}\left[\sum_{m=1}^{\infty}\frac{e^{-\alpha\beta m^2 t}}{N(\beta m)} - r_1 + \frac{r_1^2}{r_2}\right] + b + \frac{g}{k}\left[-\frac{r_1^2}{2} + \frac{r_2^2}{6} + \frac{r_1^3}{3r_2}\right]$$

$$\leq \frac{q}{k}\left[e^{-\alpha\beta_1^2 t}\sum_{m=1}^{\infty}\frac{1}{N(\beta_m)} - r_1\frac{(r_2 - r_1)}{r_2}\right] + b + \frac{g}{k}\left[-\frac{r_1^2}{2} + \frac{r_2^2}{6} + \frac{r_1^3}{3r_2}\right] \leq 0,$$

ng $t > t^* = t_Q$ such that

$$q \geq k\frac{b + \frac{g}{k}\left[\frac{r_2^2}{6} - \frac{r_1^2}{2} + \frac{r_1^3}{3r_2}\right]}{(-e^{-\alpha\beta_1^2 t})\sum_{m=1}^{\infty}\frac{1}{N(\beta_m)} + \frac{r_1}{r_2}(r_2 - r_1)}.$$

and t^* given by the solution of the equation $e^{-\alpha\beta_1^2 t^*}\sum_{m=1}^{\infty}\frac{1}{N(\beta m)} - \frac{r_1}{r_2}(r_2 - r_1) = 0$, that is

$$t^* = \frac{1}{\alpha\beta_1^2}log\left(\frac{\sum_{m=1}^{\infty}\frac{1}{N(\beta_m)}}{r_1^2(\frac{1}{r_1} - \frac{1}{r_2})}\right),$$

which is great than zero by the same argument used in the proof of the Theorem 2.

Conclusion

We have obtained sufficient condition on data for a heat conduction problem with mixed boundary conditions in order to estimate a phase-change process for a hollow sphere with radii $r_2 > r_1$ with or without a constant heat source within the domain.

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Nomenclature

c specific heat, $[J/Kg°K]$
k thermal conductivity, $[W/m°K]$
t time, $[s]$
r radial space variable, $[m]$
q heat flux on the face $r = r_1$, $[Kg/s^3]$
b temperature on the face $r = r_2$, $[°K]$

Greek symbols

ρ mass density, $[Kg/m^3]$
θ temperature, $[°K]$
θ_0 initial temperature, $[°K]$
$\gamma = \frac{r_2}{r_1} > 1$ adimensional constant.
$\alpha = \frac{k}{\rho k}$ thermal diffusivity, $[m^2/s]$

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EFFECT OF DARCY, FLUID RAYLEIGH AND HEAT GENERATION PARAMETERS ON NATURAL CONVECTION IN A POROUS SQUARE ENCLOSURE: A BRINKMAN-EXTENDED DARCY MODEL

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ABSTRACT

A Pressure-velocity solution for natural convection for fluid saturated heat generating porous medium in a square enclosure is analysed by finite element method. The numerical solutions obtained for wide range of fluid Rayleigh number, Ra_f, Darcy number, Da, and heat generating number, Q_d. The justification for taking these non-dimensional parameters independently is to establish the effect of individual parameters on flow patterns. It has been observed that peak temperature occurs at the top central part and weaker velocity prevails near the vertical walls of the enclosure due to the heat generation parameter alone. On comparison, the modified Rayleigh number used by the earlier investigators[4,6], can not explain explicitly the effect of heat generation parameter on natural convection within an enclosure having differentially heated vertical walls. At higher Darcy number, the peak temperature and peak velocity are comparatively more, resulting in better enhancement of heat transfer rate. © 1999 Elsevier Science Ltd