$$\Delta(y) = y \otimes 1 + y_{-1} \otimes y_0$$

$$\Delta(x) = x \otimes 1 + x_{-1} \otimes x_0$$

$$\Rightarrow \Delta(yx) = yx \otimes 1 + \underbrace{y x_{-1} \otimes x_0}_{3} + \underbrace{y_{-1} x \otimes y_0}_{2} + y_{-1} x_{-1} \otimes y_0 x_0$$

$$\Delta((y_{-1} \cdot x) y_0) = (y_{-1} \cdot x) y_0 \otimes 1 + (y_{-1} \cdot x) y_{0-1} \otimes y_{00} + (y_{-1} \cdot x)_{-1} y_0 \otimes (y_{-1} \cdot x)_0 +$$

$$(y_{-1} \cdot x)_{-1} y_{0-1} \otimes (y_{-1} \cdot x)_0 y_{00}$$

$$= (y_{-1} \cdot x) y_0 \otimes 1 + \underbrace{(y_{-2} \cdot x) y_{-1} \otimes y_0}_{2} + \underbrace{(y_{-11} x_{-1} S(y_{-13})) y_0 \otimes y_{-12} \cdot x_0}_{3}$$

$$+ \underbrace{(y_{-11} x_{-1} S(y_{-13})) y_{0-1} \otimes (y_{-12} \cdot x_0) y_{00}}_{4}$$

$$\Delta(yx - (y_{-1} \cdot x) y_0) = (yx - (y_{-1} \cdot x) y_0) \otimes 1 + y_{-1} x_{-1} \otimes (y_0 x_0 - (y_{0-1} \cdot x_0) y_{00})$$

$$\Delta(-\langle y, x \rangle + y_{-1} x_{-1} \langle y_0, x_0 \rangle) \underset{5}{=} (-\langle y, x \rangle + y_{-1} x_{-1} \langle y_0, x_0 \rangle) \otimes 1 + y_{-1} x_{-1} \otimes (-\langle y_0, x_0 \rangle + y_{0-1} x_{0-1} \langle y_{00}, x_{00} \rangle$$

$$[\![y, x]\!] := yx - (y_{-1} \cdot x) y_0 - \langle y, x \rangle + y_{-1} x_{-1} \langle y_0, x_0 \rangle$$

$$\Rightarrow \Delta([\![y, x]\!]) = [\![y, x]\!] \otimes 1 + y_{-1} x_{-1} \otimes [\![y_0, x_0]\!]$$

Escaneado con CamScanner

(2)

$$(y_{-2} \cdot x)\, y_{-1} \otimes y_0 = (y_{-1_1} \cdot x)\, y_{-1_2} \otimes y_0 = y_{-1}\, x \otimes y_0$$

(4)

$$= y_{-4}\, x_{-1} \underbrace{S(y_{-2})\, y_{-1}}_{\varepsilon(y_{-1})} \otimes (y_{-3} \cdot x_0)\, y_0 = y_{-2}\, x_{-1} \otimes (y_{-1} \cdot x_0)\, y_0$$

$$= y_{-1}\, x_{-1} \otimes (y_{0_{-1}} \cdot x_0)\, y_{00}$$

(3)

$$= y_{-3}\, x_{-1}\, S(y_{-1})\, y_0 \otimes (y_{-2} \cdot x_0)$$

$$= (y_{-2_1} R^2)\, S(y_{-1})\, y_0 \otimes (y_{-2_2} R^1) \cdot x$$

$$= (R^2 y_{-2_2})\, S(y_{-1})\, y_0 \otimes (R^1 y_{-2_1}) \cdot x$$

$$= R^2 y_{-2}\, S(y_{-1})\, y_0 \otimes (R^1 y_{-3}) \cdot x$$

$$= R^2 y_0 \otimes (R^1 y_{-1}) \cdot x$$

$$= R^2 (S^{-1}(r^2) \cdot y) \otimes (R^1 r^1) \cdot x$$

$$= R^2_2 (S^{-1}(R^2_1) \cdot y) \otimes R^1 \cdot x$$

$$= (R^2_2\, S^{-1}(R^2_1)) \cdot y\, R^2_3 \otimes R^1 x$$

$$= y\, R^2 \otimes R^1 x$$

$$= y\, x_{-1} \otimes x_0$$

5

$$= -\langle y, x\rangle 1\otimes 1 + y_{-1,1} x_{-1,1} \otimes y_{-1,2} x_{-1,2} \langle y_0, x_0\rangle$$

$$= -\langle y, x\rangle 1\otimes 1 + y_{-1} x_{-1} \otimes \langle y_0, x_0\rangle - y_{-1} x_{-1} \otimes \langle y_0, x_0\rangle + y_{-2} x_{-2} \otimes y_{-1} x_{-1} \langle y_0, x_0\rangle$$

$$= \left(-\langle y, x\rangle + y_{-1} x_{-1} \langle y_0, x_0\rangle\right) \otimes 1 + y_{-1} x_{-1} \otimes \left(-\langle y_0, x_0\rangle + y_{0-1} x_{0-1} \langle y_{00}, x_{00}\rangle\right)$$

Obs

$$y_{-1} x_{-1} \langle y_0, x_0\rangle \overset{(H,R)}{=} \langle y_{-1,1}, x_{-1,1}\rangle \langle y_{-1,3}, S(x_{-1,3})\rangle x_{-1,2} y_{-1,2} \langle y_0, x_0\rangle$$

$$= \langle y_{-3}, x_{-3}\rangle \langle y_{-1}, S(x_{-1})\rangle x_{-2} y_{-2} \langle y_0, x_0\rangle$$

$$= \langle y_{-2}, x_{-3}\rangle \langle y_{0-1}, S(x_{-1})\rangle \langle y_{00}, x_0\rangle \cancel{x_{-2} y_{-1}}$$

$$= \langle y_{-2}, x_{-3}\rangle \left\langle S^{-1}\left(\langle R^1, S(x_{-1})\rangle R^2\right)\cdot y_0, x_0\right\rangle x_{-2} y_{-1}$$

$$= \langle y_{-2}, x_{-3}\rangle \langle x_{-1}\cdot y_0, x_0\rangle x_{-2} y_{-1}$$

$$\boxed{y_{-1} x_{-1} \langle y_0, x_0\rangle = \langle y_{-2}, x_{-3}\rangle \langle y_0, S(x_{-1})\cdot x_0\rangle x_{-2} y_{-1}}$$