Why spontaneous emission?

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This paper is a discussion of that perennial question, "Why does an excited atom radiate?" A satisfactory physical picture emerges when proper account is taken of the interplay between radiation reaction and the (quantum-mechanical) zero-point fluctuations of the radiation field. The fluctuation-dissipation connection between these two effects is therefore emphasized.

The intense atom glows
A moment, then is quenched in a most
cold repose.
—P. B. Shelley, *Adonais*

I. THE QUESTION

Professor Feynman has described an experience familiar to many of us[1]:

You might wonder what [my father] got out of it all. I went to MIT. I went to Princeton. I came home, and he said, "Now you've got a science education. I always wanted to know something that I have never understood; and so, my son, I want you to explain it to me." I said, "Yes."

He said, "I understand that they say that light is emitted from an atom when it goes from one state to another, from an excited state to a state of lower energy."

I said, "That's right."

"And light is a kind of particle, a photon, I think they call it."

"Yes."

"So if the photon comes out of the atom when it goes from the excited to the lower state, the photon must have been in the atom in the excited state."

I said, "Well, no."

He said, "Well, how do you look at it so you can think of a particle photon coming out without it having been there in the excited state?"

I thought a few minutes, and I said, "I'm sorry; I don't know. I can't explain it to you."

He was very disappointed after all these years and years of trying to teach me something, that it came out with such poor results.

The question of why or how an atom radiates would likely elicit a similar response in most physicists even today. But some progress has been made in the past ten years. In this article I will describe recent developments in the simplest way I can. To set a proper context for the discussion I will begin in Sec. II by emphasizing the "relevance" of the subject, and follow in Sec. III with some of its history. Sections IV and V are devoted to two possible interpretations of spontaneous emission, and in Sec. VI the two are, to some extent, made one. We close in Sec. VII with some details.

II. THE RELEVANCE

Spontaneous emission is ultimately responsible for most of the light around us. We would not be here without it.

Consider a thermal source of radiation. The atoms in such a source radiate by both spontaneous and stimulated emission. The rates of spontaneous and stimulated emission are A and $B\rho(\nu)$, respectively, where A and B are the Einstein coefficients for spontaneous and stimulated emission and $\rho(\nu)$ is the Planck spectral energy density. The ratio of these rates for a transition with Bohr frequency ν is

$$\frac{A}{B\rho(\nu)} = e^{h\nu/kT} - 1 \tag{2.1}$$

since $\rho(\nu) = (A/B)(e^{h\nu/kT} - 1)^{-1}$.[2] If the Sun is regarded as a thermal source at $T = 6000$ K, this ratio is about 400 at $\lambda = 4000$ Å and about 30 at $\lambda = 7000$ Å. Thus, to the extent that the Sun is an ideal blackbody radiator, most of its visible output is due to spontaneous emission.

Spontaneous emission is so ubiquitous that there are many names associated with what is basically the same thing. If the atoms (or molecules) are excited by some means other than by heating, the spontaneous emission is called luminescence. Fireflies are luminescent. And there are different names associated with luminescence, depending specifically on how excited atoms are produced (electroluminescence, chemiluminescence, etc.) If the excitation is effected by the absorption of radiation, the spontaneous emission is called fluorescence. Sometimes the molecules have a metastable level and continue to fluoresce long after the exciting radiation is turned off. This is called phosphorescence. Figurines that magically glow in the dark are phosphorescent.

Lasers, of course, produce light by stimulated emission. However, when a laser is turned on the photons that first do the stimulating are themselves the result of spontaneous emission.

III. SOME HISTORY

Hertz's experiments of 1887 confirmed that oscillating charges radiate. In Lorentz's theory of light and matter[3] atomic radiation was attributed to the oscillation of atomic electrons. There was no way to understand why they would radiate at only certain frequencies. The emission and absorption frequencies of an atom were simply inserted into the theory through "spring constants" associated with the (unexplained) binding of electrons. All this changed in 1913 with the advent of the Bohr theory of the hydrogen atom.

Bohr recognized a nonclassical element in spontaneous emission, for to him "spontaneous" meant "acausal." It was (and is) impossible to predict exactly when an excited atom will make a quantum jump and emit a photon.

Further nonclassical aspects of spontaneous emission were uncovered by Einstein in 1917. In particular, Einstein inferred that an atom must recoil upon spontaneous emission. This recoil cannot be understood classically, because

classically the field radiated by an atom carries no linear momentum.[4] According to Einstein, "Outgoing radiation in the form of spherical waves does not exist,"[5] for if an atom radiated a classical spherical wave it could not recoil. Using thermodynamical arguments, Einstein derived the ratio A/B of his coefficients for spontaneous and stimulated emission.

The first person to derive the A coefficient directly from first principles was Dirac,[6] who used the newly formulated quantum theory of radiation. No one has ever presented a satisfactory theory of spontaneous emission within the framework of classical electromagnetism.[7] The explanation of phenomena as diverse as the glow of fireflies and the start-up of a laser requires quantum electrodynamics.

But what sort of physical picture does quantum electrodynamics provide for spontaneous emission? In the extensive literature relating to this question, two answers can be found.

The older answer is to associate spontaneous emission with the classically familiar *radiation reaction*. Dirac in his paper had written that "The present theory, since it gives a proper account of spontaneous emission, must presumably give the effect of radiation reaction on the emitting system... ."[6] Such an interpretation is unmistakable also in a paper published by Landau in the same year,[8] and in an earlier paper by van Vleck that uses the Correspondence Principle.[9]

Later a different answer emerged. Research on the Lamb shift led to the idea that this shift of an energy level—and also the "natural linewidth" due to spontaneous emission—could be attributed to the quantum-mechanical zero-point fluctuations of the electromagnetic field. In a well-known paper published in 1948, Welton[10] wrote that spontaneous emission "can be thought of as forced emission taking place under the action of the fluctuating field."

Gradually it came to be recognized that these two points of view are essentially the same. Before discussing this new perspective, it is worthwhile to discuss more carefully the two "separate" notions of radiation reaction and the zero-point field.

IV. RADIATION REACTION

Consider an oscillating electric dipole moment $\mathbf{p} = \mathbf{p}_0 \cos\omega_0 t$. It is well known that the dipole radiates electromagnetic energy at the rate

$$W = \frac{2}{3c^3}\left(\frac{d^2\mathbf{p}}{dt^2}\right)^2 = \frac{2p_0^2\omega_0^4}{3c^3}\cos^2\omega_0 t \rightarrow \frac{p_0^2\omega_0^4}{3c^3} \tag{4.1}$$

when we average the rapidly oscillating $\cos^2\omega_0 t$ over a few cycles.

Imagine a charge e with displacement $\mathbf{x} = x_0\cos\omega_0 t$ from an infinitely massive particle of opposite charge. This forms an oscillating dipole with $\mathbf{p}_0 = e\mathbf{x}_0$, and the radiation rate is given by (4.1). Since

$$\epsilon = m\omega_0^2 x_0^2 \tag{4.2}$$

is the total energy of the oscillating charge of mass m, we have

$$\frac{d\epsilon}{dt} = -\frac{p_0^2\omega_0^4}{3c^3} = -\left(\frac{e^2\omega_0^2}{3mc^3}\right)\epsilon \tag{4.3}$$

for the rate at which ϵ decreases due to radiation.

Suppose we want to apply (4.3) to an electron in an atom. If the electron can make a transition from one energy level E_2 to another E_1, we identify ω_0 as the transition frequency $(E_2 - E_1)/\hbar$ and write

$$R_r = \frac{e^2\omega_0^2}{3mc^3} \tag{4.4}$$

as the radiative decay rate of the upper level. However, our derivation has been classical. To get the predictions of quantum mechanics we must "weight" (4.4) by the transition oscillator strength, i.e.,[11]

$$R_r\frac{e^2\omega_0^2}{3mc^3}f = \frac{e^2\omega_0^2}{3mc^3}\left(\frac{2md^2\omega_0}{e^2\hbar}\right) = \frac{2d^2\omega_0^3}{3\hbar c^3}, \tag{4.5}$$

where d is the transition dipole matrix element.

Unfortunately this is *half* the Einstein A coefficient for the spontaneous emission rate; we will return to this point later.

Suppose we calculate the electric field at the position of the oscillating (point) charge. A rather involved calculation leads to the result[12]

$$\mathbf{E}_{RR} = \frac{2e}{3c^3}\frac{d^3\mathbf{x}}{dt^3} - K\frac{d^2\mathbf{x}}{dt^2} \tag{4.6}$$

for this so-called *radiation reaction field*. As a result of being acted upon by this field, the energy of our dipole is changing at the rate

$$\begin{aligned}\frac{d\epsilon}{dt} &= e\mathbf{E}_{RR}\cdot\frac{d\mathbf{x}}{dt} = \left(\frac{2e^2}{3c^3}\frac{d^3\mathbf{x}}{dt^3} - Ke\frac{d^2\mathbf{x}}{dt^2}\right)\cdot\frac{d\mathbf{x}}{dt}\\ &= \left(\frac{2e^2\omega_0^3}{3c^3}\mathbf{x}_0\sin\omega_0 t + Ke\omega_0^2\mathbf{x}_0\cos\omega_0 t\right)\cdot\\ &\quad -(\omega_0\mathbf{x}_0\sin\omega_0 t)\\ &\rightarrow -\left(\frac{e^2\omega_0^4}{3c^3}\right)x_0^2 = -\left(\frac{e^2\omega_0^2}{3mc^3}\right)\epsilon,\end{aligned} \tag{4.7}$$

where the arrow again indicates that we have averaged over a few cycles of oscillation.

The results (4.3) and (4.7) are identical. This means that we can attribute the loss of energy of our oscillating charge to the influence of its own "self-field," i.e., the radiation reaction field (4.6). And so, by making the classical-to-quantum correspondence (4.5), we are led to the idea that spontaneous emission is caused by the radiation reaction field of an atomic electron, except that our answer for the decay rate is off by a factor $\frac{1}{2}$.

V. VACUUM FIELD

According to quantum mechanics a harmonic oscillator of frequency ω has a zero-point energy $\frac{1}{2}\hbar\omega$. Similarly the electromagnetic field, according to quantum mechanics, has a zero-point energy $\frac{1}{2}\hbar\omega$ per mode of frequency ω. But there are $\omega^2/\pi^2c^3\,d\omega$ modes of the field, per unit volume of space, in the frequency range $(\omega, \omega + d\omega)$.[13] Thus there is an electromagnetic zero-point energy

$$\frac{\omega^2}{\pi^2c^3}(\tfrac{1}{2}\hbar\omega)(V\,d\omega) = \frac{\hbar\omega^3}{2\pi^2c^3}(V\,d\omega) = \rho(\omega)V\,d\omega \tag{5.1}$$

in the frequency range $(\omega, \omega + d\omega)$ in the volume V. $\rho(\omega)$ is the "zero-point spectrum" of the vacuum electromagnetic field.

Does this field have any effect on an atom? According to

Einstein the rate of stimulated emission in a (broadband) field of energy spectrum $\rho(\omega)$ is $B\rho(\omega_0)$, where

$$B = 4\pi^2 d^2/3\hbar^2 \tag{5.2}$$

is the B coefficient for stimulated emission. Thus *the stimulated emission rate due to the zero-point field is*

$$B\left(\frac{\hbar\omega_0^3}{2\pi^2c^3}\right) = \frac{2d^2\omega_0^3}{3\hbar c^3} \equiv R_{vf} \tag{5.3}$$

for a transition with dipole matrix element $\mathbf{d}$ and frequency ω_0. Interestingly enough, $R_{vf} = R_r$.

The B coefficients for stimulated emission and absorption are the same (if we ignore degeneracy factors). Why is there no spontaneous *absorption* from the zero-point field?

VI. A PARTIAL UNIFICATION

It is no accident that $R_r = R_{vf}$ nor that R_r and R_{vf} are separately equal to half the Einstein A coefficient for spontaneous emission. As a first step in understanding this, we will show that the radiation reaction and zero-point fields share a rather intimate relationship.

To establish this relationship it is convenient to consider a "free" electron, acted upon only by the vacuum field and its own radiation reaction field. The Heisenberg equation of motion for the electron coordinate operator $x(t)$ is formally the same as its classical counterpart:

$$\begin{aligned} m\frac{d^2x}{dt^2} &= e[E_0(t) + E_{RR}(t)] \\ &= eE_0(t) + \frac{2e^2}{3c^3}\frac{d^3x}{dt^3} - Ke\frac{d^2x}{dt^2}. \end{aligned} \tag{6.1}$$

Here $E_0(t)$ is the electric field operator associated with the vacuum or zero-point electric field. (Actually it is the x component of this field, because we are considering for simplicity only the motion of the electron in the x direction.) We write (6.1) as

$$m_0\frac{d^2x}{dt^2} = eE_0(t) + \frac{2e^2}{3c^3}\frac{d^3x}{dt^3}, \tag{6.2}$$

where

$$m_0 = m + Ke \tag{6.3}$$

is the observed electron mass, m is its "bare" mass, and Ke is its "electromagnetic mass."

Unfortunately the electromagnetic mass turns out to be infinite. In our nonrelativistic theory it is linearly divergent; in the relativistic theory it is logarithmically divergent. We ignore this difficulty and assume that somehow the bare and electromagnetic masses add up to give a finite number $m_0 \simeq 9.1\times10^{-28}$ g. This procedure is called *mass renormalization*. Present-day theoretical physics is founded on the "principle" of renormalization.

Physicists have tried for a long time to get rid of this infinity. The Wheeler–Feynman absorber theory,[14] for instance, led to a radiation reaction field having the d^3x/dt^3 term, but not the term contributing to electromagnetic mass. More recently it has been argued that a fully quantum-mechanical theory of radiation reaction gives $K \equiv 0$, although the relativistic version has not been worked out.[15] In any case it is sobering to keep in mind the dissenting view of Dirac on renormalization:[16] "This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it!"

We will assume that (6.2) is correct. But if it is correct it must be consistent with the basic ideas of quantum mechanics. In particular, the solution of the operator equation (6.2) for x, and the corresponding solution for the linear momentum p, must satisfy the commutation rule $(x, p) = i\hbar$. We obtain[17]

$$[x(t), p(t)] = 4i\gamma\pi^2c^3\int_0^\infty \frac{d\omega\,\rho(\omega)}{\omega^3[1+\gamma^2\omega^2]} = i\hbar, \tag{6.4}$$

and so the equation of motion (6.2) preserves the commutation rule. In (6.4) $\rho(\omega)$ is the zero-point field energy spectrum associated with $E_0(t)$, and we have defined $\gamma = 2e^2/3mc^3 \simeq 6.3\times10^{-24}$ s.

Now it is not difficult to see that the term $\gamma^2\omega^2$ in the denominator of (6.4) stems from the d^3x/dt^3 term in the radiation reaction field appearing in (6.2). Furthermore, the energy spectrum $\rho(\omega)$ of the zero-point field is proportional to ω^3 [Eq. (5.1)], and so we get a cancellation of ω^3 factors in (6.4), and are left with just

$$\int_0^\infty \frac{d\omega}{1+\gamma^2\omega^2} = \frac{\pi}{2\gamma}. \tag{6.5}$$

If $\rho(\omega)$ were to be proportional to any other power of ω we would not be able to preserve the x–p commutator!

What we have here is an example of the fluctuation-dissipation theorem.[18] If there is a radiation reaction field there *must* be a zero-point field, and vice versa. Furthermore, the spectrum of the zero-point field goes as the *third* power of ω *because* the radiation reaction field goes as the *third* derivative of x.[17] This is the intimate connection between radiation reaction and the zero-point field, and it is the reason for the equality $R_r = R_{vf}$.

The "derivations" of R_r and R_{vf} given earlier were somewhat heuristic. A quantum-mechanical calculation, however, largely supports these results. The calculation can be done in such a way as to reveal explicitly the separate contributions to spontaneous emission of the radiation reaction and zero-point fields. For an atom in an excited state the contributions add up to give the spontaneous emission rate

$$R_r + R_{vf} = A, \tag{6.6}$$

the Einstein coefficient. For an atom in the ground state, however, the two contributions add "destructively" to give a decay rate $R_{vf} - R_r = 0$. This latter result explains, at least quantum mechanically, why there is no spontaneous absorption from the zero-point field.[19]

The key to this interplay of radiation reaction and zero-point fields is the fluctuation-dissipation connection between them. Just as the Nyquist–Johnson voltage fluctuations in an electric circuit are related to the resistance, so too are the fluctuations of the zero-point field related to the radiative resistance (i.e., radiation reaction). In both examples we have explicit relations between the dissipative force and the frequency spectrum of the concomitant fluctuating force.

R_r and R_{vf} differ from the Einstein A coefficient by a factor of $\frac{1}{2}$. This factor, which plays a rather important role in our discussion, has a long history in radiation theory, especially in the embryonic stages of quantum theory. For instance, in 1913 Einstein and Stern found that the Planck spectrum could be derived if it was assumed that a dipole oscillator has a zero-point energy $\frac{1}{2}\hbar\omega_0$.[4] From the present perspective, their classical approach failed to account for

the zero-point energy of the *field*, which is $\frac{1}{2}\hbar\omega_0$ for a field mode of frequency ω_0. This adds to the zero-point energy $\frac{1}{2}\hbar\omega_0$ of a dipole oscillator to give the "total" zero-point energy, $\hbar\omega_0$, that Einstein and Stern ascribed entirely to material oscillators.[4] In quantum theory both contributions appear and are joined in a natural way, and Eq. (6.6) is one example of this.

VII. SOME DETAILS

To forestall the objections of experts, I should mention some details concerning this view of spontaneous emission that ties together the two older points of view. This new perspective was "discovered" independently and practically simultaneously by physicists at three different institutions.[20,21] These investigations were stimulated largely by some work of Ackerhalt *et al.*,[22] which provided a quantum-mechanical basis for the explanation of spontaneous emission as a radiation reaction effect. More recent elaborations and insights are due to Fain[23] and Dalibard *et al.*[24] Sciama[25,26] has used the fluctuation-dissipation connection between $\mathbf{E}_{RR}$ and $\mathbf{E}_0$ in discussing the thermodynamics of black holes.

The Heisenberg-picture derivation of the spontaneous emission rate involves products of commuting atomic operators and field operators. If we use a symmetrical ordering of these operators we obtain $A = R_r + R_{vf}$, i.e., Eq. (6.6). A "normal" ordering, however, gives $A = 2R_r$, while a still different ordering gives $A = 2R_r - R_{vf}$, etc. Of course we always get the same expression A for the spontaneous emission rate, because the operators we are reordering do in fact commute, but our *interpretation* of the origin of spontaneous emission may change. Again this is a consequence of the intimate, fluctuation-dissipation connection between radiation reaction and the zero-point, vacuum electromagnetic field. Dalibard *et al.*[24] have argued that a symmetrical ordering is preferable in some ways.

There are many physical effects attributable to the zero-point electromagnetic field. These include van der Waals and Casimir-type forces.[27] The question arises whether these other effects can, like spontaneous emission, be regarded as consequences of radiation reaction. It turns out that they can.[28] The theory is essentially no different from that for spontaneous emission. However, I know of no earlier work in which van der Waals or Casimir forces were attributed to radiation reaction. As noted by Jaynes,[29] it has taken us a long time to appreciate the fluctuation-dissipation theorem for the radiation field.

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