Tumor location and parameters estimation by termography

Juan P. Agnelli Andrés Barrea Cristina V. Turner *

Abstract

In non-invasive thermal diagnostics, accurate correlations between the thermal image on skin surface and interior human pathophysiology are often desired, which require general solutions for the bioheat equation. In this paper, a difference scheme for the Pennes equation with a space-dependent thermal physiological parameters and convective boundary condition on the top surface, was used to approximate the solution. Details computations indicated that the thermal states of biological bodies, reflecting physiology conditions, could be correlated to the temperature mapping recorded at the skin surface. The effect of the convective heat transfer coefficient, the temperature of the sourranding air, the metabolic rate, the blood perfusion rate in the tumor, the tumor size and number on the sensitivity of the thermography are investigated. Moreover the inverse problems consisting in the simultaneous estimation of unknow geometrical parameters (location size) of the tumor are solved. On the stage of numerical solution the pattern search algorithm coupled with the difference scheme has been applied.

Mathematical Subject Classification: 35K05, 35K60.

Keywords: Bioheat transfer; Thermal image; Inverse problems; Diagnosis criteria.

*FaMAF, U.N.Córdoba, CIEM-CONICET agnelli@famaf.unc.edu.ar, abarrea@famaf.unc.edu.ar, turner@famaf.unc.edu.ar

1 Introduction.

Breast cancer, which is common among women, is an international concern. There are about 200.000 cases a year, and it is estimated that there are more than 1 million women with undetected breast cancer. Among various techniques to detect breast cancer, infrared (IR) imaging has been widely used since the late $1950s$. In general, the temperature of the human body on the surface of the skin depends on the metabolic activity, the blood flow, and the surrounding temperature. Any abnormality in the tissue, such as the presence of a tumor, alters the normal temperature on the skin surface due to the increase metabolic activity and blood circulation. Therefore, abnormal skin temperature profiles are an indication of deceases such as mastitis, benign tumors, fibrocystic breast disease and cancer. Lawson [7] was the first to propose the use of thermography detection of breast cancer, when he observed that the local temperatures of the skin over a tumor were significantly higher (about $2-3$ degrees) than the normal skin temperatures. Lawson and Chughtai [8] established that the region temperature difference over an embedded tumor was due to convection effects associated with increased blood perfusion, and the increased metabolism around the tumor.

Thermography is a non-contact technique for early detection of breast cancer without the hazard of carcinogenic radiation. The infrared camera takes thermal images (pictures) that can detect subtle changes in breast physiology up to 10 years before a cancerous tumor can be detected by other sources such as mamography or skilled palpation. This is the most reliable form of early detecction though not often used by medical flield, it is becoming more readily available.

Thermography has 9% for both false positive and false negative readings. Thermography is painless and offers a view of the entire chest and underarm area. The infrared images show subtle temperature changes in the tissue.

Moreover this technique can be used on augmented breast and mastectomy since is no pressure applied and it is non-invasive way to evaluate lymphatic system.

Further aspect of IR imaging techniques of breast cancer and detection methods from infrared images are describes in detail in a recent book by Diakides and Bronzino, [9].

Recently, Mital and Scott [4] developed a method to determined parameters of an embedded heat source representing a tumor using IR.

The idea of this work is to use thermography- temperature profile in order to predict the location and size of the embedded tumor. These data can be found solving the inverse bioheat transfer problem.

In section 2 we will show the mathematical model proposed to simulate the heat transfer in a human body in a bidimensional domain. In Section 3 we use a difference scheme method of order two to discretize the continuous model for the Pennes equation with the mixed boundary conditions. In section 4 we show the simulations for the discrete system. In section 5 we present the inverse problem in order to obtain the tumor's parameters using an algorithm for optimization. Finally, in section 6 we give the results of the simulations with the parameters obtained and the corresponding percent relatives errors.

2 Mathematical model

A number of bioheat transfer equations for living tissues have been proposed since landmark paper by Pennes [1] appeared in 1948 in wich the perfusion heat source was introduced.

Pennes measured the tissue temperature profile in a human resting forearm and compared this with the governing differential equation presented by him. This equation describes a quantitative relation in a human tissue, and included the effect of blood flow in the tissue temperature on continuous basis. The equation includes the heat transfer by conduction through the tissue, the volumetric metabolic heat generation of the tissue and the volumetric blood perfusion rate whose strength was considered to be proportional to the arterial-venous temperature difference. The temperature of artery is approximated to the core temperature of the body and the the vein approximated to the local tissue temperature.

Pennes found that the temperature profile obtained by theoretical prediction by varying the blood perfusion term matched with the experimental data.

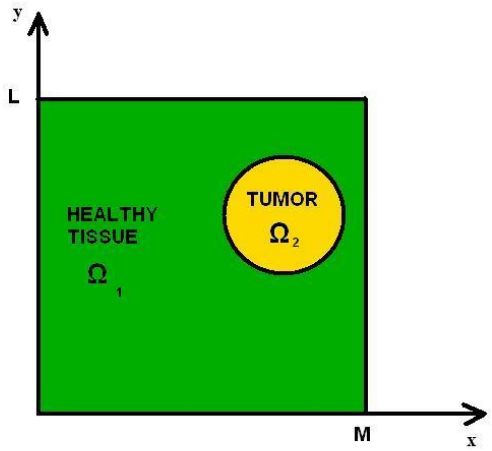
The Pennes bioheat transfer equation is based on the assumption that all heat transfer between the tissue and the blood occurs in the capillaries.

The Pennes bioheat transfer equation is widely used to solve the temperature distribution for thermal therapy [10],[11], [12], [13] and [14].

In this paper we use to model the steady-state Pennes equation,

$$\lambda_e \nabla^2 T_e(\mathbf{x}) + \mathbf{k_e}[\mathbf{T_b} - \mathbf{T_e}(\mathbf{x})] + \mathbf{Q_{me}} = \mathbf{0} \quad \mathbf{x} \in \Re^{\mathbf{2}}$$

where $e = 1, 2$ identifies the sub-domains of healthy tissue and tumor respectively, λ_e is the thermal conductivity, $k_e = G_{b_e} c_b$ is the perfusion coefficient (G_{b_e} is the blood perfusion rate, c_b is the volumetric specific heat blood), T_b is the constant blood temperature and Q_{me} is the constant metabolic heat source.



For the heat transfer at skin surface (top boundary) a convection condition was imposed:

$$-\lambda_1 \frac{\partial T_1(x,L)}{\partial n} = \alpha[T1(x,L) - T_a]$$

where α is a heat transfer coefficient and T_a is the ambient temperature. It is possible to consider another condition for the skin surface, that is the Stefan-Boltzman equation for the heat radiation, i.e.

$$-\lambda_1 \frac{\partial T_1(x,L)}{\partial n} = \alpha[T_1^4(x,L) - T_a^4],$$

moreover we can consider the evaporation on the skin surface, in that case we have to consider an evolution in time for the heat equation, since the humidity coefficient depends on time.

The other boundary conditions where prescribed as follows:

$$\begin{aligned} -\lambda_1 \frac{\partial T_1(x)}{\partial n} &= 0 \quad at \quad x = 0 \\ -\lambda_1 \frac{\partial T_1(x)}{\partial n} &= 0 \quad at \quad x = M \\ T &= T_b \quad at \quad y = 0, \end{aligned}$$

so a constant core temperature T_b was assumed at the bottom boundary.

3 Numerical method-Direct Problem

The embedded boundary method is a strategy for solving a partial differential equation such as the Poisson equation, $u_{xx} + u_{yy} = f(x,y)$, in a domain Ω which is bounded by a smooth but irregular curve $\partial\Omega$ [15], [16], [17] and [18]

The boundary problem exposed in the previous section is elliptic with mixed boundary conditions and discontinuous coefficients. In order to solve this problem more accurately we consider the coefficients as differentiable functions following the ideas in [19]. We assume that $\partial\Omega_1$ (the boundary of tumor) is defined as the zero-isoline (level set) of a C^∞ function $\Phi(x,y)$. It is convenient to rescale $\Phi(x,y)$ such that the level surface $\Phi = 1$ is a curve $\partial\Lambda$ which is contained in Ω and is exterior a Ω_1. The interior of boundary $\partial\Lambda$ is called Λ.

Now, we multiply the coefficients with appropriate C^∞ functions. The basic idea for extending λ_e, k_e and Q_{me} to C^∞ functions is to multiply each by a "window" function $\rho_e(x,y)$.

The function defined in all domain Ω is :

$$\rho_1 k_1 + \rho_2 k_2.$$

The $\rho_e(x,y)$ functions have the following properties:

1. $\rho_e(x,y) \equiv 1 \quad (x,y) \in \Omega - \Lambda (\text{or} \quad \Lambda)$
2. $\rho_e(x,y) \equiv 0 \quad (x,y) \in \Lambda (\text{or} \quad \Omega - \Lambda)$
3. $\rho_e \quad \text{is} \quad C^\infty$ everywhere.

Next, we consider the following equation:

$$\lambda(x)\nabla^2 T(x) + k(x)[T_b - T(x)] + Q_m(x) \quad = \quad 0, \quad x \in \Re^2$$

where $\lambda(x), k(x)$ and $Q_m(x)$ are C^∞ in all the domain Ω.
Using the following transformation:

$$T(x,y) = \Theta(x,y) + T_b - (T_b - T_a)\frac{y\alpha}{\lambda + \alpha L}$$

we rewrite the previous equation and the boundary conditions as

$$\begin{aligned} \lambda\nabla^2\Theta - k(x,y)\Theta + \tilde{Q}(x,y) &= 0 \\ -\lambda\frac{\partial\Theta}{\partial x} &= 0 \quad at \quad x = 0 \quad and \quad x = M \\ -\lambda\frac{\partial\Theta}{\partial y} &= \alpha\Theta \quad at \quad y = L \quad (\text{top boundary}) \\ \Theta &= 0 \quad at \quad y = 0 \end{aligned}$$

where $\tilde{Q} = Q + k(T_b - T_a)\frac{y\alpha}{\lambda+\alpha L}$, L is the height of the domain in the y direction and M the width of the domain in x direction.

To solve this system we proposed a finite difference method. We considered an equispaced grid on the x and y directions and we approximated the Laplacian by a second order scheme $\rightarrow$ x-direction $i = 1, ..., N_x \uparrow$ y-direction $j = 1, ..., N_y$,

$$(\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2}) \mid i,j = \frac{\Theta_{i,j+1} - 2\Theta_{i,j} + \Theta_{i,j-1}}{h^2} + \frac{\Theta_{i+1,j} - 2\Theta_{i,j} + \Theta_{i-1,j}}{h^2}$$

then the equation can be rewrite as

$$\lambda_{i,j}(\frac{\Theta_{i+1,j} - 2\Theta_{i,j} + \Theta_{i,j-1}}{h^2} + \frac{\Theta_{i,j+1} - 2\Theta_{i,j} + \Theta_{i,j-1}}{h^2}) - k_{i,j}\Theta_{i,j} + \tilde{Q}_{i,j} = 0.$$

and doing some elementary operations we conclude that

$$\Theta_{i,j} = \frac{\lambda_{i,j}}{4\lambda_{i,j} + h^2 k_{i,j}}(\Theta_{j+1,i} + \Theta_{j-1,i} + \Theta_{j,i+1} + \Theta_{j,i-1}) + \frac{h^2}{4\lambda_{i,j} + h^2 k_{i,j}}\tilde{Q}_{i,j} \qquad (3.1)$$

As the partial derivatives in the Laplacian equation were approximated by a second order finite difference scheme, the partial derivatives of the boundaries were also approximated by a second order.

We accomplished this by taking fictitious grid points outside the domain and then we replaced these fictitious points in the main equation. Here we describe each boundary condition,

The left lateral flux:

$$\frac{\partial \Theta}{\partial x} \mid_{1,j} = \frac{\Theta_{2,j} - \Theta_{0,j}}{2h} = 0 \quad \Rightarrow \Theta_{0,j} = \Theta_{2,j}$$

then replacing this into equation (1) we obtain:

$$\Theta_{1,j} = \frac{\lambda_{1,j}}{4\lambda_{1,j} + h^2 k_{1,j}}(\Theta_{1,j+1} + \Theta_{1,j-1} + 2\Theta_{2,j}) + \frac{h^2}{4\lambda_{1,j} + h^2 k_{1,j}} Q_{1,j} \quad j = 2, ..., N_y - 1. \quad (3.2)$$

The right lateral–flux:

$$\frac{\partial \Theta}{\partial x} \mid N_x, j = \frac{\Theta_{N_x+1,j} - \Theta_{N_x-1,j}}{2h} = 0 \quad \Rightarrow \Theta_{N_x+1,j} = \Theta_{N_x-1,j}$$

then replacing in equation (1), we obtain:

$$\Theta_{N_x,j} = \frac{\lambda_{N_x,j}}{4\lambda_{N_x,j} + h^2 k_{N_x,j}}(\Theta_{N_x,j+1} + \Theta_{N_x,j-1} + 2\Theta_{N_x-1,j}) + \frac{h^2}{4\lambda_{N_x,j} + h^2 k_{N_x,j}} Q_{N_x,j} \quad j = 2, ..., N_y - 1 \quad (3.3)$$

The bottom boundary condition is:

$$\Theta_{i,1} = 0 \quad i = 2, ..., N_x - 1 \quad (3.4)$$

The top convective boundary condition is:

$$\frac{\Theta_{i,N_y+1} - \Theta_{i,N_y-1}}{2h} = \frac{-\alpha}{\lambda_{i,N_y}} \Theta_{i,N_y} \quad (3.5)$$

therefore

$$\Theta_{i,N_y+1} = \frac{-2h\alpha}{\lambda i, N_y} \Theta_{i,N_y} + \Theta_{i,N_y-1} \quad (3.6)$$

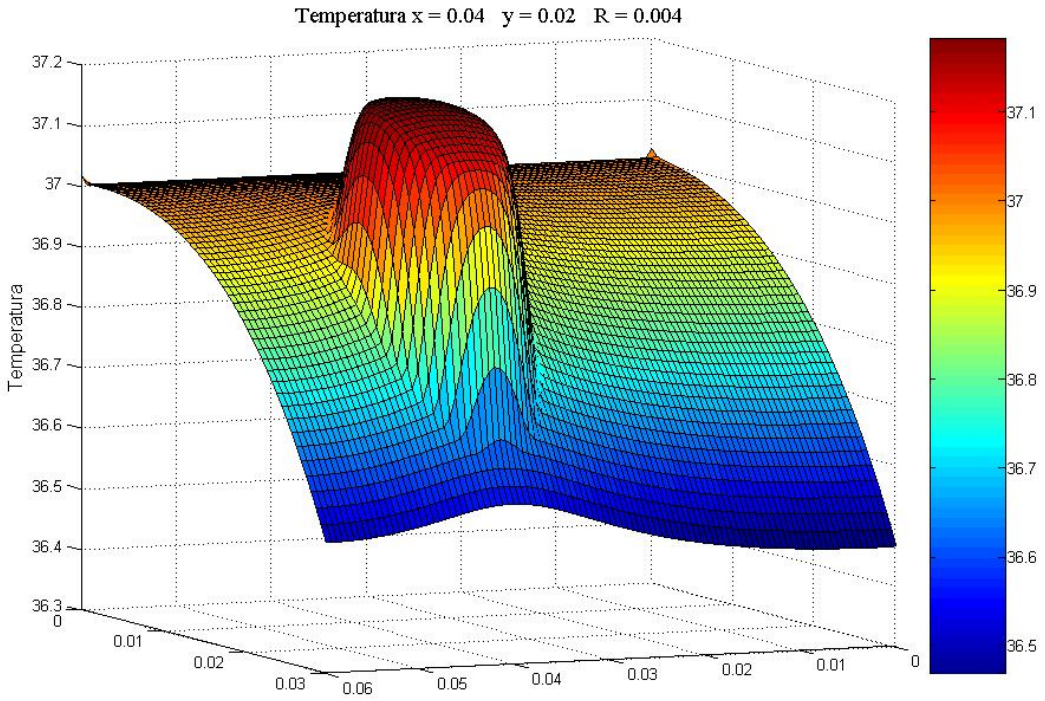
replacing this expression into equation (1) we get

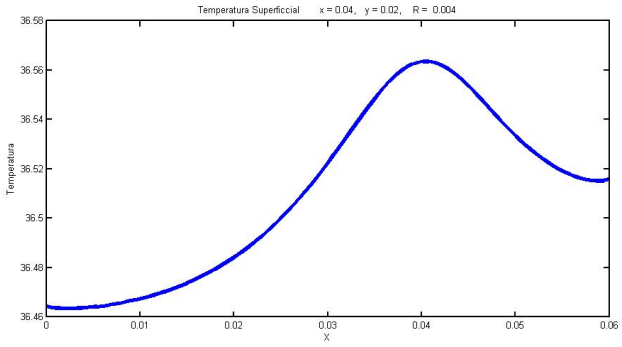
$$\Theta_{i,N_y} = \frac{\lambda_{i,N_y}}{4\lambda_{i,N_y} + h^2 k_{i,N_y} + 2h\alpha}(2\theta_{i,N_y-1} + \Theta_{i+1,N_y} + \Theta_{i-1,N_y}) + \frac{h^2}{4\lambda_{i,N_y} + h^2 k_{i,N_y} + 2h\alpha} Q_{i,N_y}, i = 2, ..., N_x - 1 \quad (3.7)$$

We define the sequence $\Theta_{i,j}^{n+1} = F(\Theta_{i,j}^n)$,, where F stands for the right hand sides corresponding to the equations (3.1), (3.2), (3.3),(3.4) and (3.6). Given an initial value $\Theta_{i,j}^0$, we compute the values of $\Theta_{i,j}^{n+1}$ throught the whole domain until $|\Theta_{i,j}^{n+1} - \Theta_{i,j}^n| < \epsilon$,, for a convenient ϵ.

4 Simulations

Here we present some of the simulations. The first picture shows the surface temperature of a piece of skin with an embeded tumor. The second graphic is the temperature of the skin as a function of x.





We could consider the following biologic parameters for the simulations above

$\lambda_1 = 0.5[W/mK], \lambda_2 = 0.75[W/mK], k_1 = 1998.1[W/m^3K], k_2 = 7992.4[W/m^3K], Q_{m1} = 720[W/m^3], Q_{m2} = 7200[W/m^3], T_b = 37C, T_a = 25C$

Here we can observe that the top surface's temperature decreases owing to the convective boundary condition on the top surface. The presence of the tumor, a temperature source, produces a temperature increase inside the domain which arrives to the top of domain.

The computational time is really fast, (order of seconds). The algorithm speed is not trivial since we will be using this method (FDM) for the inverse problem, where we have to compute several times (around 5000 times) this procedure.

5 Inverse Problem

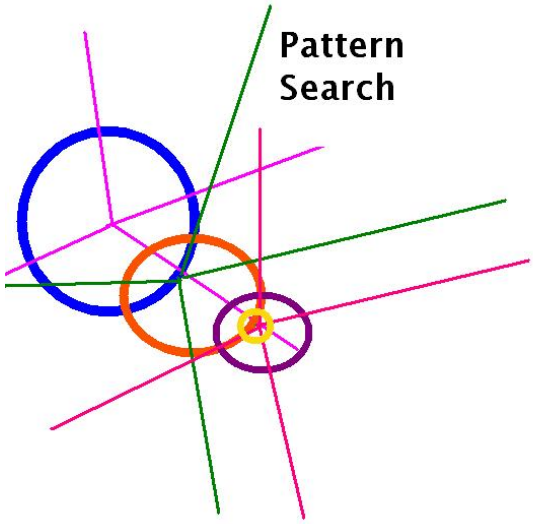
The estimation methodology involves the use of the finite difference (FDM) method and the use of an optimization algorithm (pattern search). The FDM method is used to map

the relationship of tumor location and tumor size to the temperature profile over the idealized bioheat transfer model (direct problem).

Optimization algorithm is used to estimate the tumor parameters (depth, width and size) by minimizing a fitness function involving the temperature profiles obtained from simulated or clinical data to those obtained by FD. The fitness function relates the data from the observed thermographic temperature profile to the temperature profile for a given set of estimated parameters. It is defined as:

$$E = \|T_{obs} - T_{num}(x, y, R)\|_2$$

where T_{obs} is the observed temperature and T_{num} is the estimated skin temperature by FDM using the parameters (x, y, R).



Original Data			Numerical Data			Relative Error		
x	y	R	x	y	R	x	y	R
0.040	0.020	0.003	0.040	0.019	0.0031	0	5	3.333
0.020	0.015	0.004	0.019	0.017	0.0035	5	13.333	12.500
0.033	0.012	0.0028	0.033	0.011	0.0030	0	8.333	6.111
0.022	0.019	0.0027	0.0221	0.0185	0.0028	0.454	2.631	3.703

Here we can observe that this methodology to obtain the tumors parameters is a very good approximation, but it has the problem that a smaller size tumor near to the top domain will give us a similar profile temperature, than a bigger near the bottom domain.

This situation can be improve using more information about the measurements of the temperature. That means, we can propose a new methodology that will take into account the temperature at the top domain with two different exterior temperature,

$$E = \|T_{obs1} - T_{num1}(x, y, R)\|_2 + \|T_{obs2} - T_{num2}(x, y, R)\|_2.$$

6 Conclusions

A methodology using difference scheme method for the Pennes equation and the pattern search algorithm was used to estimate the breast tumor parameters based on surface temperature profile that may be obtained by infrared thermography. The methodology is demonstrated with a simple model in order to illustrate the details of the procedures involved. For the cases studied, results from simulations shows that it is possible to determine the depth, width and diameter from the surface temperature data with good accuracy for the simplified 2D model.

However, the estimation methodology is general and can be applied even to realistic breast geometry. This may increase the computation time but the methodology can be easily extended. We show here a picture of a domain to be considered, obtained from the work of Mital, [4].

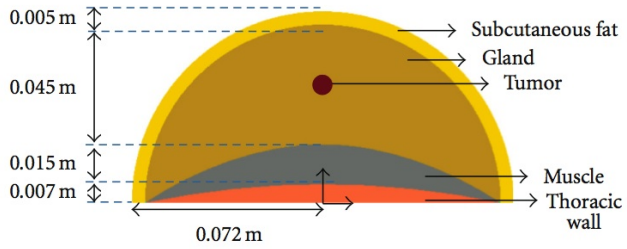


FIGURE 3: A 2D model of the breast based on Sudershan et al. [6].

One of the problems that we have is that for the algoritm it is difficult to distinguish the temperature profile of a bigger radious or a bigger vertical coordinate of the center of the tumor. The different skin temperature profiles are very similar. In order to solve this problem we proposed to use two thermography profile, the first at the ambient temperature and the second one using a cooling procedure.

Acknowledgments

The work of the authors was partially supported by grants from CONICET, SECYT-UNC. The authors want to thank Gustavo Santacruz and Sara González for the motivation to connect this work with the Boro Neutronic Capture Therapy that they are using at CNEA and Roffo Hospital.

References

[1] **H. Pennes** Analysis of tissue and arterial blood temperature in the resting human forearm J. Appl. Physiol. 1, 1948, 93-122.

[2] **M. Paruch, E. Majchrzak.** Identification of tumor region parameters using evolutionary algorithm and multiple reciprocity boundary element method. Enginnering Applications of Artificial Intelligence. 20, 2007, 647-655.

[3] **Z. Deng, J. Lui.** Mathematical modelling of temperature over skin surface and its implementation in thermal disease diagnostics. Computers in Biology and Medicine. 34, 2004, 495-521.

[4] **M. Mital, and Elaine P. Scott.** Thermal Detection of Embedded Tumors using Infrared Imaging, ASME Journal of Biomechanical Engineering, vol. 129 (1), pp. 33-39, 2007.

[5] **J. Dennis, V. Torczon.** Derivative free pattern search method for multidisciplinary design problems. Proceedings of AIAA, 94, 2001,1-11.

[6] **H. Qi, Nicholas A. Diakides.** Infrared imaging in Medicine. CRC Press, Boca Raton, Fla, USA, 2007.

[7] **R. N. Lawson** Implications of surface temperatures in the diagnosis of breast cancer. Canadian Medical Asociation Journal, vol. 75, 309-310, 1956.

[8] **R. N. Lawson-M.S. Chugtai** Breast cancer and body temperatures. Canadian Medical Asociation Journal, vol. 88, 68-70, 1963.

[9] **N. A. Diakides-J.D. Bronzino** Medical infrared imaging. CRC Press, Boca Raton, Fla, USA, 2007.

[10] **R. Chopra, J. Wachsmuth, M. Burtnyk, M.A. Haider and M.J. Bronskill**, Analysis of factors important for transurethral ultrasound prostate heating using MR temperature feedback, Phys Med Biol 51 (2006), pp. 827-844.

[11] **S.C. Jiang, N. Ma, H.J. Li and X.X. Zhang**, Effects of thermal properties and geometrical dimensions on skin burn injuries, Burns 28 (2002), pp. 713-717.

[12] **E.Y.K. Ng and L.T. Chua**, Comparison of one-and two-dimensional programmes for predicting the state of skin burns, Burns 28 (2002), pp. 27-34.

[13] **C. Chen and L.X. Xu**, Tissue temperature oscillations in an isolated pig kidney during surface heating, Ann Biomed Eng 30 (2002), pp. 1162-1171.

[14] **E.H. Liu, G.M. Saidel and H. Harasaki**, Model analysis of tissue responses to transient and chronic heating, Ann Biomed Eng 31 (2003), pp. 1007-1048.

[15] **L. Badea and P. Daripa**, On a Fourier method of embedding domains using an optimal distributed control. Numer. Alg. 30 (2003), pp. 199-239.

[16] **M. Elghaoui and R. Pasquetti**, Mixed spectral-boundary element embedding algorithms for the NavierStokes equations in the vorticitystream function formulation. J. Comput. Phys. 153 (1996), pp. 82-100.

[17] **M. Elghaoui and R. Pasquetti**, A spectral embedding method applied to the advectiondiffusion equation. J. Comput. Phys. 125 (1996), pp. 464-476.

[18] **M. Garbey and D. Tromeur-Dervout**, A new parallel solver for the nonperiodic incompressible NavierStokes equations with a Fourier method: application to frontal polymerization. J. Comput. Phys. 145 (1998), pp. 316-331.

[19] **J. Boyd**, Fourier embedded domain methods: extending a function defined on an irregular region to a rectangle so that the extension is spatially periodic and C^∞ App. Math. and Computation 2 (2005), pp. 591-597.