Supplementary information

Inverse transition in the two dimensional dipolar frustrated ferromagnet

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Here we include the expressions of the variational mean field free energies per unit volume $f \equiv F/L^2$ used in the work.

Stripes free energy for $n_{max} = 3$, used in the calculation of the phase diagram Fig.3

$$\begin{aligned} f_s[m_0,\ldots,m_3,k_0] &= 2\,m_0^2 A(0) + \sum_{n=1}^{3} A(nk_0)\,m_n^2 - 2\,h\,m_0 + \frac{u}{2}\left[8m_0^4 + 24m_0^2m_1^2 + 3m_1^4 + 24m_0m_1^2m_2 + \right. \\ &+ \; 24m_0^2m_2^2 + 12m_1^2m_2^2 + 3m_2^4 + 4m_1^3m_3 + 48m_0m_1m_2m_3 + 12m_1m_2^2m_3 + 24m_0^2m_3^2 + \\ &+ \; \left. 12m_1^2m_3^2 + 12m_2^2m_3^2 + 3m_3^4 + \right] \qquad (1) \end{aligned}$$

Bubbles free energy for $n_{max} = 3$, used in the calculation of the phase diagram Fig.3

$$\begin{aligned} f_b[m_0,\ldots,m_3,k_0] &= 18\,m_0^2 A(0) + 3\sum_{n=1}^{3} A(nk_0)\,m_n^2 - 6\,h\,m_0 + \frac{u\,6^4}{4}\left[m_0^4 + m_0^2m_1^2 + \frac{2}{9}m_0m_1^3 + \frac{5}{72}m_1^4 + \frac{1}{3}m_0m_1^2m_2 \right. \\ &+ \; \frac{1}{9}m_1^3m_2 + m_0^2m_2^2 + \frac{2}{9}m_1^2m_2^2 + \frac{2}{9}m_0m_2^3 + \frac{5}{72}m_2^4 + 24m_1^3m_3 + \frac{2}{3}m_0m_1m_2m_3 + \frac{1}{9}m_1^2m_2m_3 + \\ &+ \; \left. \frac{1}{6}m_1m_2^2m_3 + m_0^2m_3^2 + \frac{1}{6}m_1^2m_3^2 + \frac{1}{9}m_1m_2m_3^2 + \frac{1}{6}m_2^2m_3^2 + \frac{2}{9}m_0m_3^3 + \frac{5}{72}m_3^4 \right] \qquad (2) \end{aligned}$$

Stripes and bubbles free energies for $n_{max} = 1$

$$f_s[m_0,m_1] = 2\,m_0^2 A(0) + A(k_m)\,m_1^2 - 2\,h\,m_0 + \frac{u}{2}\left[8m_0^4 + 24m_0^2m_1^2 + 3m_1^4\right] \qquad (3)$$

$$f_b[m_0,m_1] = 18\,m_0^2 A(0) + 3A(k_m)\,m_1^2 - 6\,h\,m_0 + \frac{u\,6^4}{4}\left[m_0^4 + m_0^2m_1^2 + \frac{2}{9}m_0m_1^3 + \frac{5}{72}m_1^4\right] \qquad (4)$$

The one mode phase diagram from the inset of Fig.3 (compare with Fig.2 of Ref.[7] and Fig.4 of Ref.[13]) is obtained from these equations (references from the manuscript). They exactly map into Eqs.(8) and (9) of Ref.[7] with the replacement: $m_0 = m_{s0}/2$, $m_1 = m_s/2$ and $m_0 = m_{B0}/6$, $m_1 = m_B/2$ into Eqs.(3)

and (4) respectively. With an appropriated rescaling of the variables they also map into Eqs.(21) and (22) of Ref.[13].

Stripes free energy at zero field (only modes with odd n contribute) for $n_{max} = 9$

$$\begin{aligned}
f_s[m_1, \ldots, m_9, k_0] \;=\; & A(k_0)\, m_1^2 + \cdots + A(9k_0)\, m_9^2 + \frac{u}{2}\left[3m_1^4 + 4m_1^3 m_3 + 12m_1^2 m_3^2 + 3m_3^4 + 12m_1^2 m_3 m_5 + \right. \\
& + 12m_1 m_3^2 m_5 + 12m_1^2 m_5^2 + 12m_3^2 m_5^2 + 3m_5^4 + 12m_1 m_3^2 m_7 + 12m_1^2 m_5 m_7 + \\
& + 24m_1 m_3 m_5 m_7 + 12m_3 m_5^2 m_7 + 12m_1^2 m_7^2 + 12m_3^2 m_7^2 + 12m_5^2 m_7^2 + 3m_7^4 + 4m_3^3 m_9 + \\
& + 24m_1 m_3 m_5 m_9 + 12m_1 m_5^2 m_9 + 12m_1^2 m_7 m_9 + 24m_1 m_3 m_7 m_9 + 24m_3 m_5 m_7 m_9 + \\
& \left. + 12m_5 m_7^2 m_9 + 12m_1^2 m_9^2 + 12m_3^2 m_9^2 + 12m_5^2 m_9^2 + 12m_7^2 m_9^2 + 3m_9^4\right] \qquad (5)
\end{aligned}$$