18/10/18

Resumen

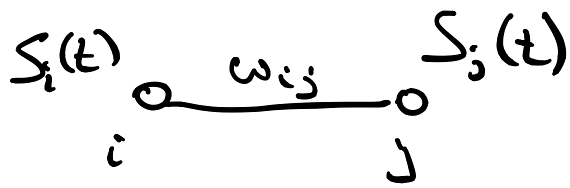
$$P \propto e^{-\frac{E}{k_B T}}$$

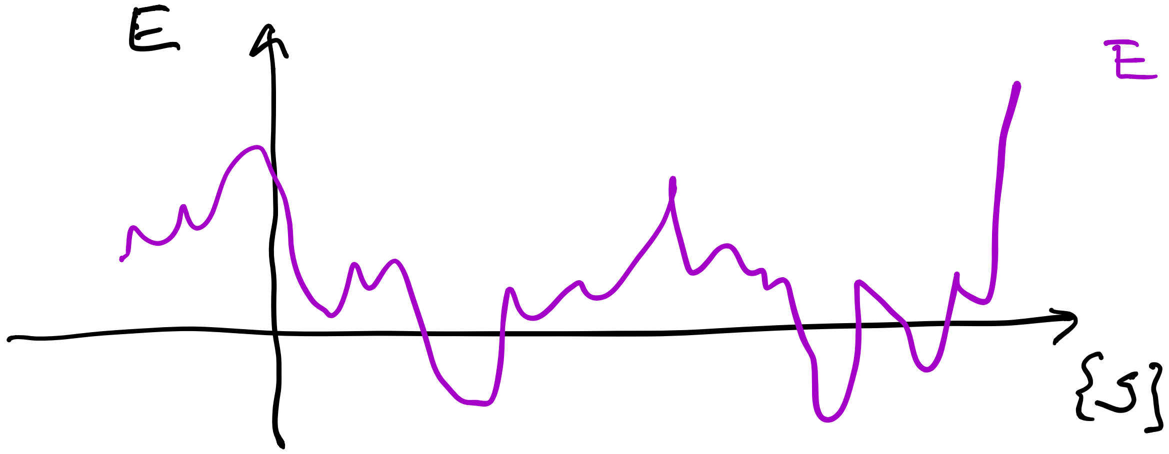
$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{P} \xi_i^{\mu} \xi_j^{\mu}$$

Función Energía

$> 0 \quad < 0$

$$H(t) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \, S_i(t) S_j(t)$$





$$\mathbb{R}^N \longrightarrow \mathbb{R}$$

$$\vec{S} \longrightarrow E$$

Veamos que no puede aumentar

$$E(t+1) \le E(t)$$

Asumimos dinámica secuencial

Supongamos que entre t a $t+\frac{1}{N}$

solo di vuelta la neurona k

$$S_k\left(t+\frac{1}{N}\right) = -S_k(t)$$

$$S_j\left(t+\frac{1}{N}\right) = S_j(t)$$

$$\downarrow$$

$$E\left(t+\frac{1}{N}\right) - E(t) = -\frac{1}{2}\sum_{ij} w_{ij}\, S_i\left(t+\frac{1}{N}\right) S_j\left(t+\frac{1}{N}\right)$$

$$+\frac{1}{2}\sum_{ij} w_{ij}\, S_i(t)\, S_j(t)$$

$$= -S_k\left(t+\frac{1}{N}\right)\sum_{j\neq k} w_{kj}\, \overbrace{S_j\left(t+\frac{1}{N}\right)}^{=\,S_j(t)}$$

$$+\, S_k(t)\sum_{j\neq k} w_{kj}\, S_j(t)$$

$$= \left[-S_k\left(t+\frac{1}{N}\right) + S_k(t) \right] \overbrace{\sum_{j \neq k} w_{kj} S_j(t)}^{h_k(t)}$$

$$= 2 S_k(t)\, h_k(t)$$

Nota

$S_k(t+\frac{1}{N}) = $ s' no $h_k(t)$

$$= -2 S_k\left(t+\frac{1}{N}\right) h_k(t)$$

$$= -2 \,\text{signo}(h_k(t))\, h_k(t)$$

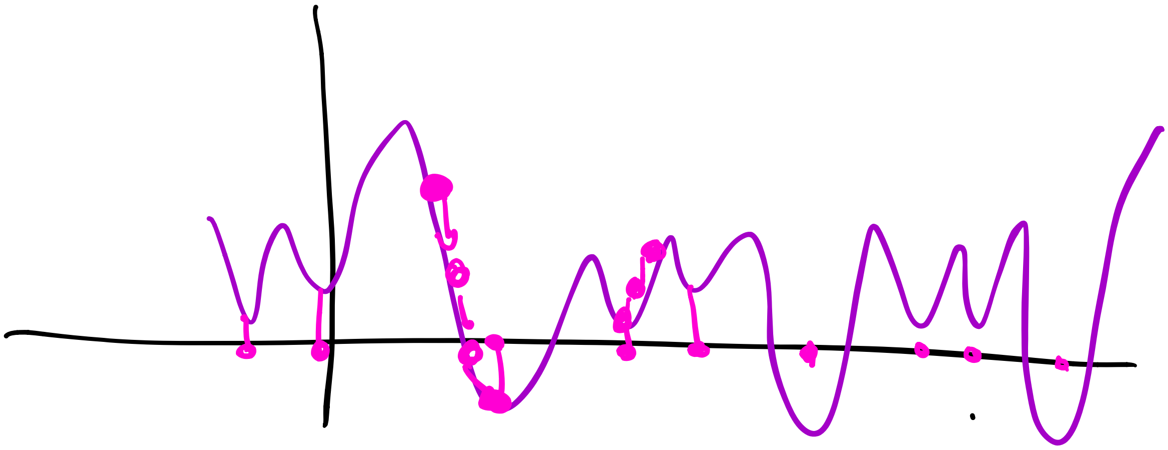
$$= -|h_k(t)| \leq 0$$

Resumiendo

$$E\left(t+\frac{1}{N}\right) - E(t) \leq 0$$

Luego, $E_w(\{S\})$ es una función de Lyapunov.

Para $t \to \infty$ el sistema estará en algún mínimo local de E



Una forma alternativa de llegar a Hopfield

Imaginemos que queremos guardar una única configuración $\{\xi_i^1\}$ ($\mu=1$)

defino la distancia de Hamming

$$d^1 = \frac{1}{4N}\sum_{i=1}^{N}\left(S_i - \xi_i^1\right)^2$$

$$= \frac{1}{4N}\sum_{i=1}^{N}\left(\overbrace{S_i^2}^{=1} - 2 S_i \xi_i^1 + \overbrace{(\xi_i^1)^2}^{=1}\right)$$

$$= \frac{1}{4N}\sum_{i=1}^{N}\left(2 - 2 S_i \xi_i^1\right)$$

$$= \frac{\cancel{2}}{2\cancel{4}N}\sum_{i}\left(1 - S_i \xi_i^1\right)$$

$$= \frac{1}{2N} \overbrace{\sum_i 1}^{=N} - \frac{1}{2N} \sum S_i \xi_i^1$$

$$= \frac{1}{2} - \frac{1}{2} m^1$$

$$d^1 = \frac{1}{2}(1 - m^1)$$

$$m^1 = \frac{1}{N} \sum_i S_i \xi_i^1$$

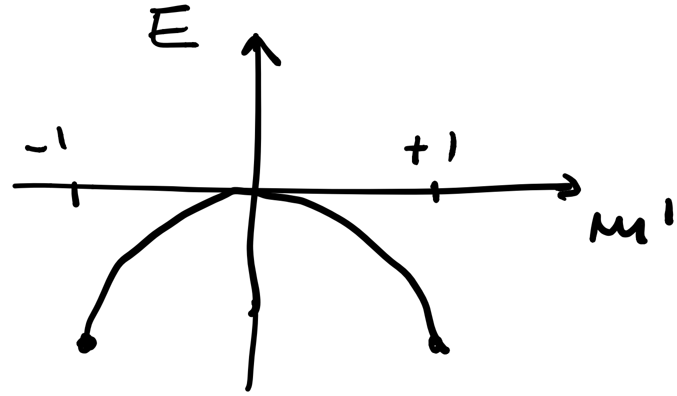
Si $m^1 = 1$ $\quad d^1 = 0$

$m^1 = 0$ $\quad d^1 = \frac{1}{2}$

$m^1 = -1$ $\quad d^1 = 1$

Construyo la función con un mínimo en ξ_i^1

$$E = -\frac{N}{2}(m^1)^2$$

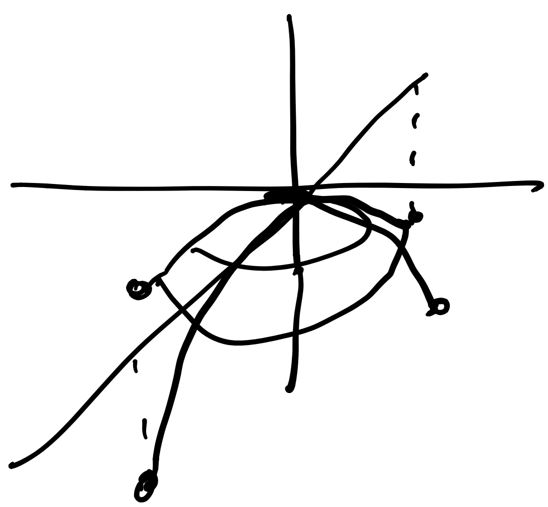


¿Y si tengo 2?

$$E = -\frac{N}{2}\left[(m^1)^2 + (m^2)^2\right]$$

Si tengo P

$$E = -\frac{N}{2}\sum_{\mu=1}^{P}(m^\mu)^2$$



$$E = -\frac{N}{2}$$

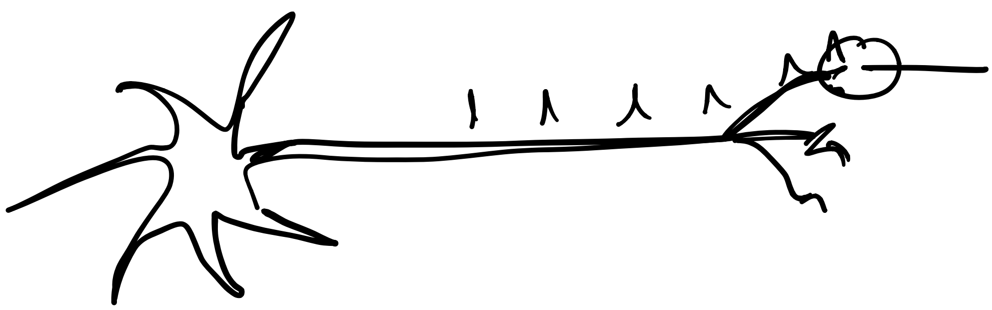
$$E = -\frac{N}{2}\sum_{\mu=1}^{P}\left(\frac{1}{N}\sum_{i=1}^{N} S_i(t)\,\xi_i^\mu\right)\left(\frac{1}{N}\sum_{j} S_j(t)\,\xi_i^\mu\right)$$

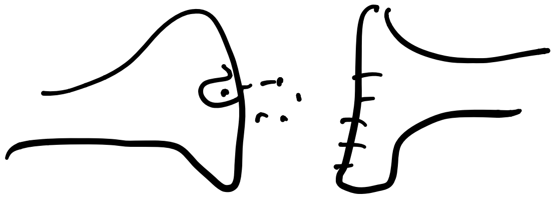
$$= -\frac{\cancel{N}}{2}\frac{1}{N^{\cancel{2}}} \sum_i \sum_j S_i(t)\, S_j(t) \sum_\mu \xi_i^\mu \xi_j^\mu$$

$$= -\frac{1}{2}\sum_{i,j} S_i(t)\, S_j(t) \underbrace{\left(\frac{1}{N}\sum_\mu \xi_i^\mu \xi_j^\mu\right)}_{w_{ij}}$$

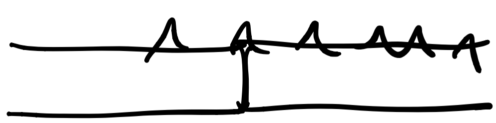
Si $i=j$ $\quad \frac{1}{N}\sum_{\mu}^{p} \overbrace{\xi_i^\mu \xi_i^\mu}^{=1} = \frac{p}{N}$

<u>Un breve retorno a la biología</u>





sinapsis químicas



sinapsis eléctrica

En lugar de hablar de el valor de $\underline{h_i = \sum w_{ij} S_j}$ en forma determinista, suponemos que es una variable aleatoria

el valor medio $\bar{h}_i = \sum_{j \neq i}^{N} w_{ij} S_j$

y defino un ancho o varianza de la distribución δ

Supongo una distribución gaussiana

$$P(h_i = h) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(h-\bar{h})^2}{2\delta^2}}$$

$$P(S_i(t+1) = 1) = \int_0^{\infty} \frac{e^{-\frac{(h-\bar{h})^2}{2\delta^2}}}{\sqrt{2\pi\delta^2}} dh$$

Cambio de variable

$$u = \frac{h - \bar{h}}{\sqrt{2\sigma^2}} \qquad du = \frac{dh}{\sqrt{2\sigma^2}}$$

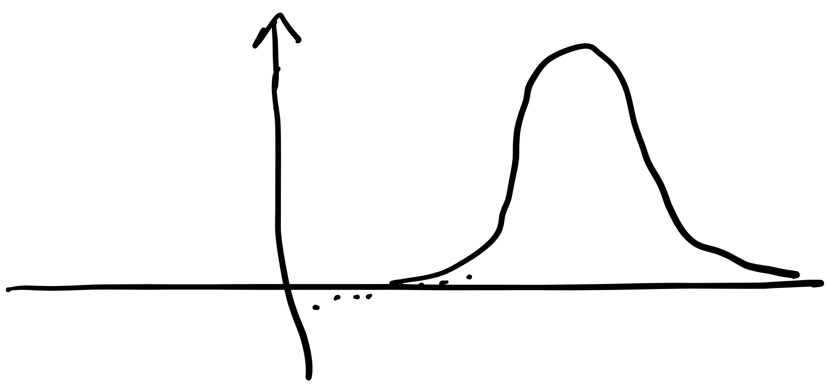
$$P(S_i(t+1)=1) = \int_{\frac{\bar{h}}{\sqrt{2\sigma^2}}}^{\infty} \frac{e^{-u^2}}{\sqrt{\pi}}\, du$$

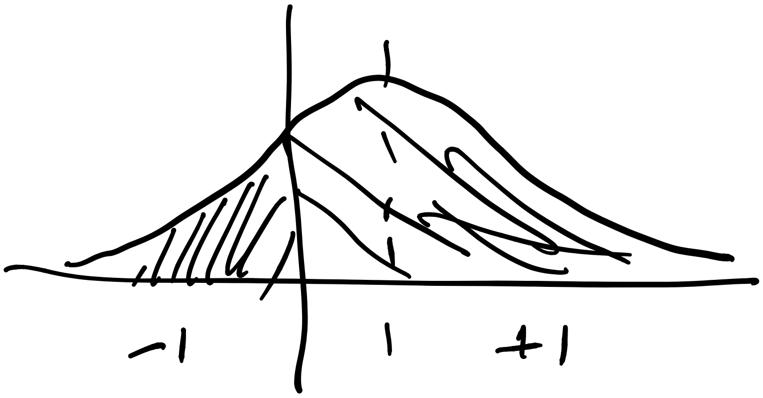
$$= \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\bar{h}}{\sqrt{2}\,\sigma}\right)\right]$$

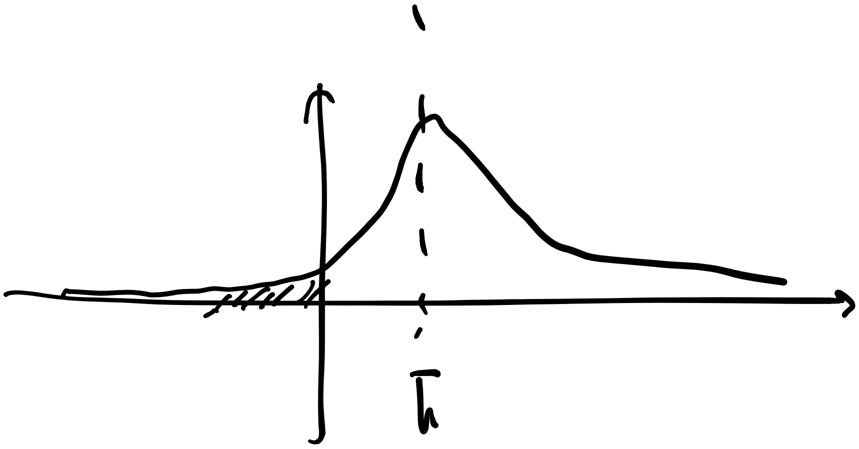
$$P(S_i(t+1)=-1) = 1 - P(S_i(t+1)=+1)$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\bar{h}}{\sqrt{2}\,\sigma}\right)\right]$$

calcolo $\bar{h} = \sum w_{ij} S_j$







Observación

$$\tanh\left(\frac{h}{2\sqrt{2}\,\delta}\right) \simeq \operatorname{erf}\left(\frac{h}{\sqrt{2}\,\delta}\right)$$

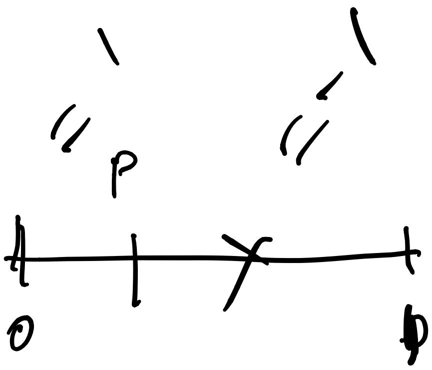
$$P(S_i = 1) = \frac{1}{2}\left[1 + \tanh\left(\frac{h}{T}\right)\right]$$

$$P(S_i = -1) = \frac{1}{2}\left[1 - \tanh\left(\frac{h}{T}\right)\right]$$

$$T = 2\sqrt{2}\,\delta$$

Programa tipo

```
h:=0
for j:=1 to N
    h:= h + W[i][j]* S[j]
end
```



Antes

~~S[i]:= signo(h)~~

Ahora

```
P= 0.5*[1. + tanh(h/T)]
q= rand(iseed)
if (q<P) then
    S[i]:= +1
else
    S[i]:= -1
end if
```