19/10/18

3º Clase Hopfield

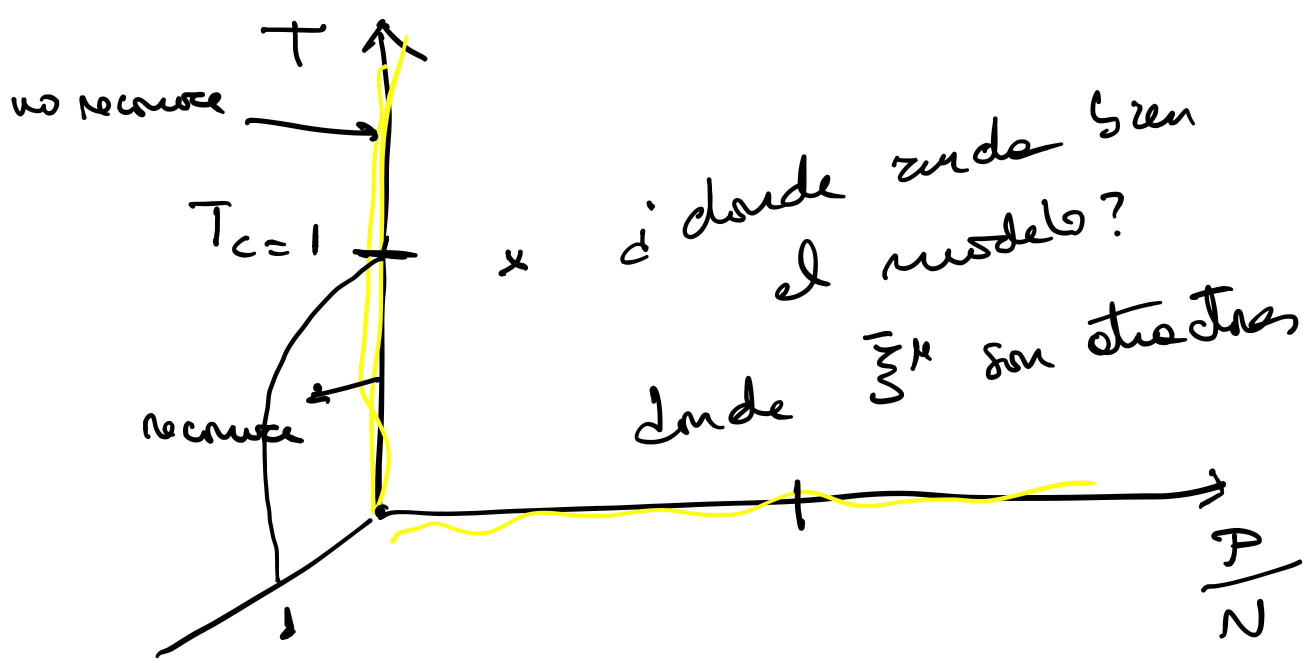
Ayer mostramos que la regla de Hopfield se puede deducir de la función energía

$$E_w(\{S\}) = -\frac{1}{2}\sum_i \sum_j w_{ij} S_i S_j$$

$$w_{ij} = \frac{1}{N}\sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu$$

ξ^μ : configuración almacenada ($\mu = 1 \ldots p$)

Tenemos 2 parámetros hacemos un diagrama de fase



P : es el # de memorias

guardamos PN bits

calculo $N(N-1)$ acoplamientos W_{ij}

una medida del esfuerzo

$$\frac{\#\text{ bits}}{\#\text{ acoplamientos}} = \frac{PN}{N(N-1)} \simeq \frac{P}{N}$$

si $N \gg 1$

T : pseudo temperatura (ruido)

$$E = -\frac{1}{2}\sum_i E_i$$

Dinámica estocástica

$$P^+ \equiv P\left(S_i\left(t+\frac{1}{N}\right)=+1\right) = \frac{1}{2}\left[1+\tanh\left(\frac{h}{T}\right)\right] = P^+$$

$$P^- \equiv P\left(S_i\left(t+\frac{1}{N}\right)=-1\right) = \frac{1}{2}\left[1-\tanh\left(\frac{h}{T}\right)\right] = P^-$$

Si $T=0$ $\quad S_i\left(t+\frac{1}{N}\right) = \text{signo}(h)$

$T=\infty$ $\quad P^+ = P^- = \frac{1}{2}$

Al final calculo los

$$m^\mu = \frac{1}{N} \sum_i \xi_i^\mu S_i(t=\infty)$$

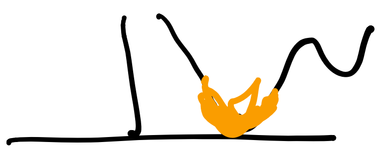
la red anda bien si en $t=\infty$

$$m^\mu \approx 1$$

Elegir las configuraciones aleatorias

$$P(\xi_i^\mu = 1) = \frac{1}{2}$$
$$P(\xi_i^\mu = -1) = \frac{1}{2}$$

Tengo que calcular $\langle m^\mu \rangle$ promedio



$$\langle m^\mu \rangle_T = \left\langle \frac{1}{N} \sum_i \xi_i^\mu S_i \right\rangle_T = \frac{1}{N} \sum_i \xi_i^\mu \langle S_i \rangle_T$$

$$\langle S_i \rangle_T = +1\, P(S_i = 1) + (-1) P(-1)$$

$$\rightarrow = \cancel{\tfrac{1}{2}} + \frac{1}{2}\tanh\left(\frac{h}{T}\right) - \cancel{\tfrac{1}{2}} + \frac{1}{2}\tanh\left(\frac{h}{T}\right)$$

$$\rightarrow = \tanh\left(\frac{h}{T}\right)$$

$$= \tanh\left(\frac{\sum w_{ij}\langle S_j\rangle_t}{T}\right)$$

$$\langle S_1\rangle = \tanh\left(\frac{w_{11}\langle S_1\rangle + w_{12}\langle S_2\rangle + \dots + w_{1N}\langle S_N\rangle}{T}\right)$$

$$\vdots$$

$$\langle S_N\rangle = \tanh\left(\frac{w_{N1}\langle S_1\rangle + w_{N2}\langle S_2\rangle + \dots + w_{NN}\langle S_N\rangle}{N}\right)$$

$$\langle m^\mu\rangle_t = \frac{1}{N}\sum \xi_i^\mu \langle S_i\rangle$$

$$\langle m^\nu\rangle = \frac{1}{N}\sum_i \xi_i^\nu \langle S_i\rangle$$

$$= \frac{1}{N}\sum_i \xi_i^\nu \tanh\left(\frac{1}{T}\sum_{j=1}^{N} w_{ij}\langle S_j\rangle\right)$$

$$= \frac{1}{N}\sum_i \xi_i^\nu \tanh\left(\frac{1}{T}\sum_j \sum_\mu \frac{\xi_i^\mu \xi_j^\mu}{N}\langle S_j\rangle\right)$$

$$= \frac{1}{N}\sum_i \xi_i^\nu \tanh\left(\frac{1}{T}\sum_\mu \xi_i^\mu \underbrace{\frac{1}{N}\sum \xi_j^\mu \langle S_j\rangle}\right)$$

$$\boxed{\langle m^\nu \rangle = \frac{1}{N}\sum_i \xi_i^\nu \tanh\left(\frac{1}{T}\sum_\mu \xi_i^\mu \langle m^\mu\rangle\right) \qquad \nu = 1 \ldots P}$$

Una solución buena

$$\overline{\langle m \rangle} = (\langle m_i^1 \rangle, \langle m^2 \rangle, \ldots \langle m^P \rangle)$$

$$\cong (0, 0, \ldots \underset{\substack{\uparrow \\ \nu}}{\pm 1}, \ldots 0)$$

Hay $2p$ buenas soluciones

Supongamos $P = 3$

$(1, 0, 0)$

$(-1, 0, 0)$

$(0, 1, 0)$

$(0, -1, 0)$

$(0, 0, 1)$

$(0, 0, -1)$

$$\begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix}$$

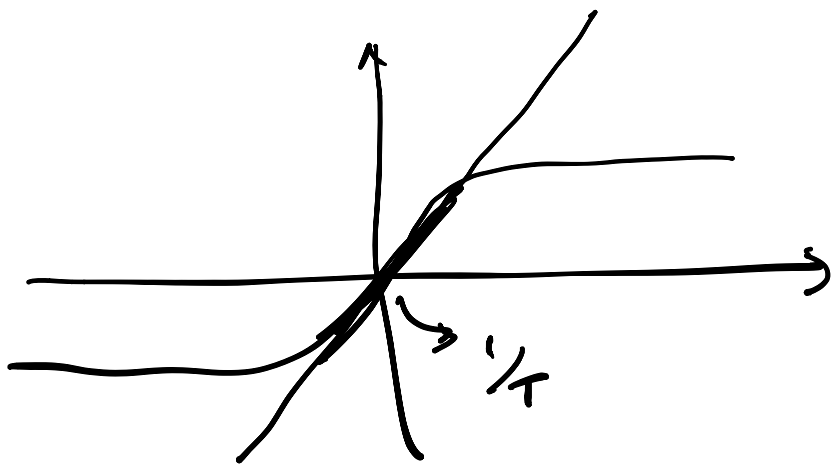
Supponiamo $\bar{m} = (0, 0, m^\nu, 0 \dots 0)$

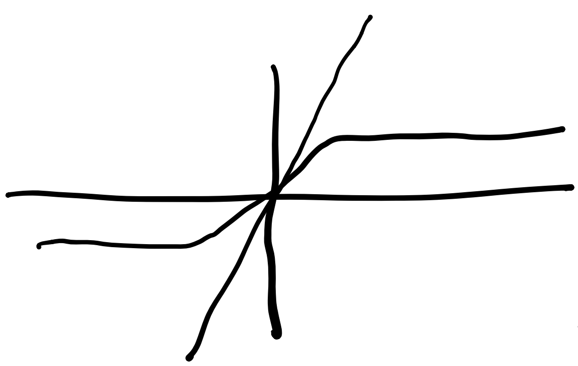
$$\langle m^\nu \rangle = \frac{1}{N} \sum_i \xi_i^\nu \tanh\left(\frac{1}{T} \xi_i^\nu \langle m^\nu \rangle\right)$$

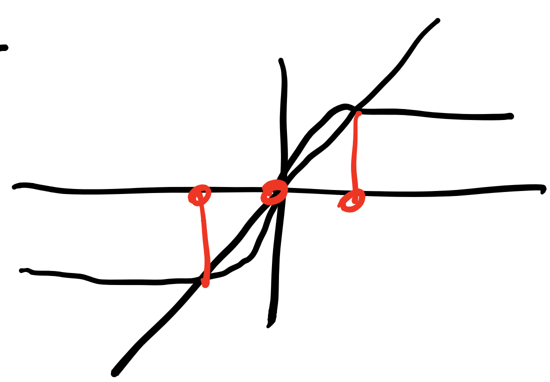
$$= \frac{1}{N} \sum_i \tanh\left(\frac{\xi_i^\nu \xi_i^\nu \langle m^\nu \rangle}{T}\right)$$

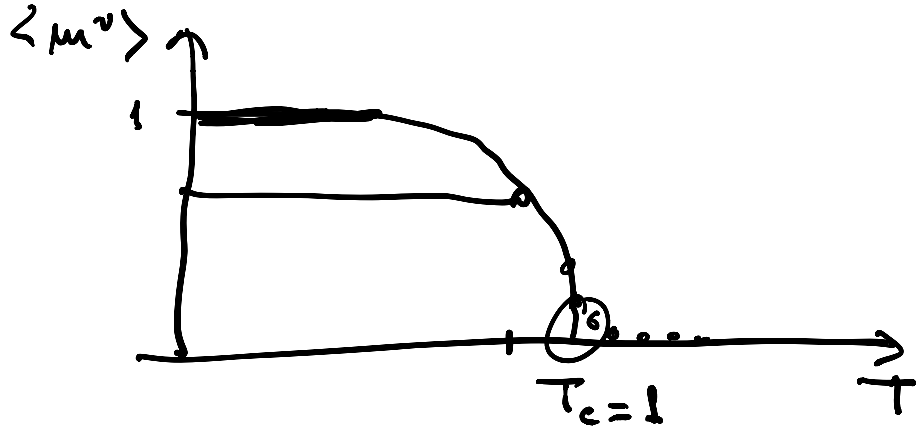
$$\boxed{\langle m^\nu \rangle = \tanh\left(\frac{\langle m^\nu \rangle}{T}\right)}$$

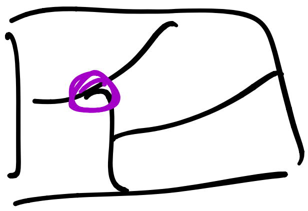
$$\tanh\left(\frac{x}{T}\right) = x$$











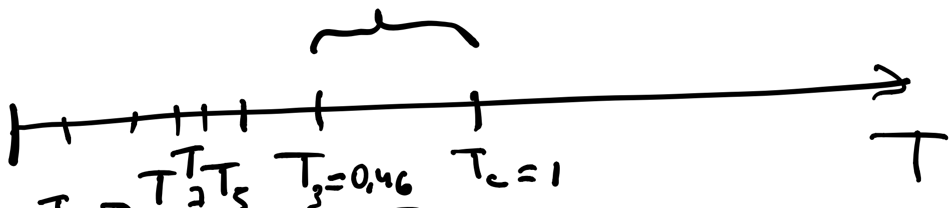
$N \gg 1$

$N \gg P$ $\qquad \frac{P}{N} \to 0$

Suposición $\bar{m} = (0, \dots m^\nu, \dots 0 \cdot m^\gamma, \dots)$

no $\exists$ estas soluciones

$\bar{m} = (0, \dots m^\nu, \dots m^\gamma, \dots m^\delta, \dots)$

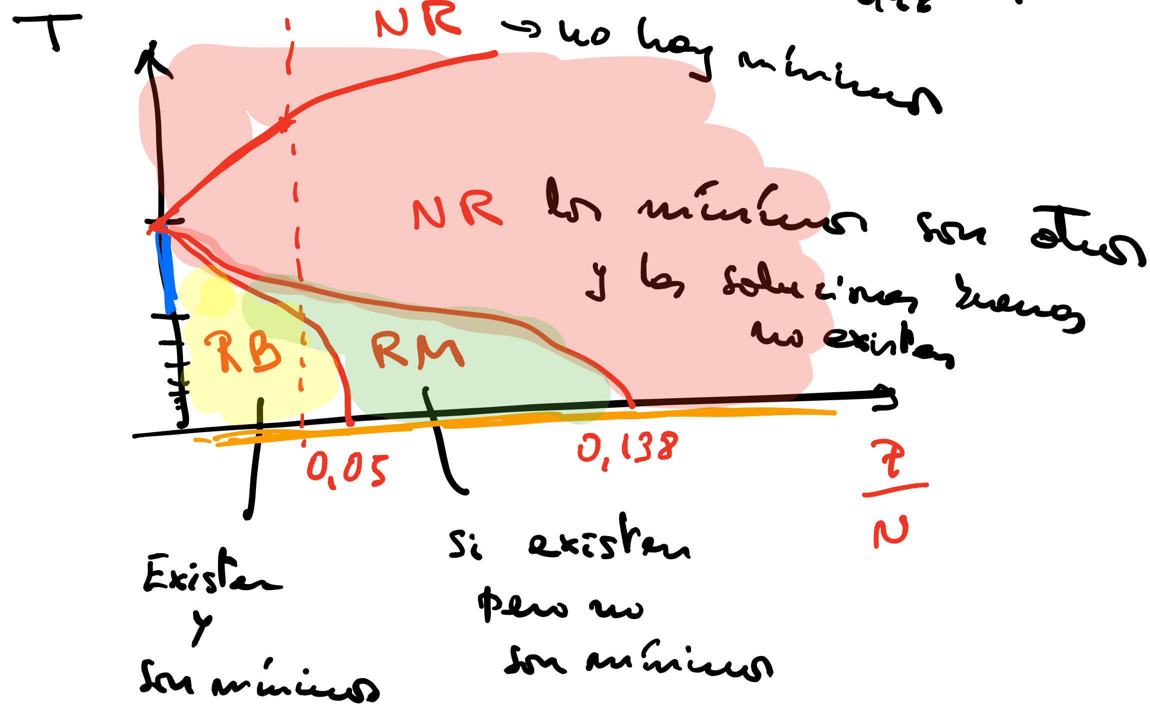


$\frac{P}{N} = 0$

$\langle m^1 \rangle = \tanh\left(\frac{1}{T}\langle m_1 \rangle + \langle m_2 \rangle + \langle m_2 \rangle\right)$

$\langle m^2 \rangle = \; - \; -$

$\langle m$



$T = 0$