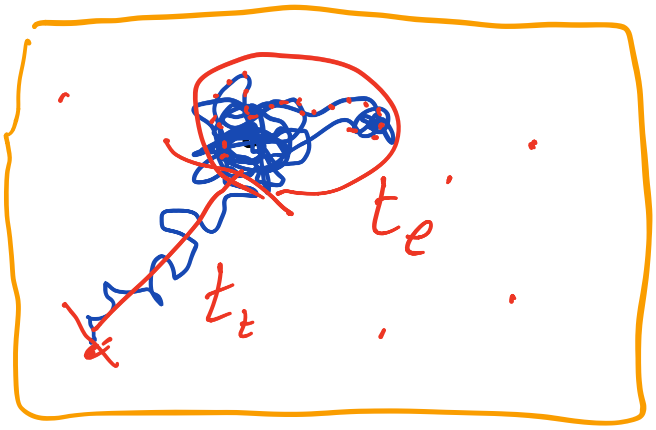
26/10/18



$A(\{S\})$

$$\langle A \rangle = \int_{t_t}^{t_t+t_e} \frac{A(t)\,dt}{t_e}$$

h

$$P(S=+1) = \frac{1}{1+e^{\overset{1/T}{\beta}\Delta E}}$$

q random entre 1 y 0

si $0 \le q \le p$ $\quad S_i = +1$

$p < q \le 1$ $\quad S_i = -1$

Máquina Boltzmann

. es una máquina probabilística

Supongamos que los acoplamientos son simétricos. $w_{ij} = w_{ji} = \frac{1}{N} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$

$$P(S_i=+1) = \frac{1}{2}\left[1+\tanh\left(\frac{h_i}{T}\right)\right]$$

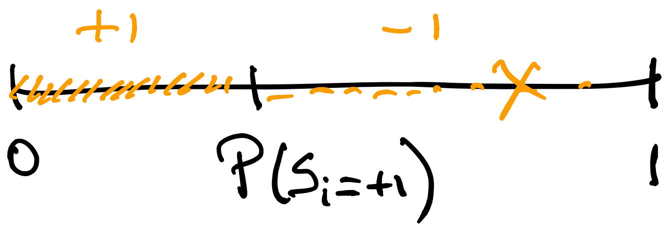
$$P(S_i=-1) = 1 - P(S_i=+1)$$

$$= \frac{1}{2}\left[1-\tanh\left(\frac{h_i}{T}\right)\right]$$

$$P(S_i) = \frac{1}{2}\left[1+\tanh\left(\frac{h_i S_i}{T}\right)\right]$$

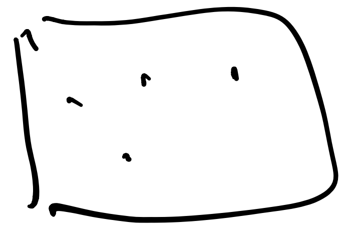
$$= \frac{1}{2}\left[1+S_i\tanh\left(\frac{h_i}{T}\right)\right]$$

i




```
for i:=1 to N
    h:=0
    for j=1 to N
        h:= h+ w[i][j]*S[j];
    end
    P+ := 0.5*[1. + Tanh(h/T)];
    q:= rand        (q on (0,1])
    if  (q ≤ P+) then
        S[i] := +1
```

```
        else
            S[i] := -1
        end
    end
```



conf. particular

¿ $P(\{S\})$?

$$\frac{\partial P(\{S\},t)}{\partial t} = \sum_{\{S'\}} T(S' \to S) P(S') - \sum_{\{S'\}} T(S \to S') P(S)$$

En el estado estacionario $\frac{\partial P}{\partial t} = 0$

o. toda la suma es cero

o hay un equilibrio de factores

$$\forall S', S \quad \left[T(S' \to S) P(S') - T(S \to S') P(S) \right] = 0$$

$$T(S' \to S) = \frac{1}{2}\left[1 + \mathrm{Tanh}\left(\frac{Sh}{T}\right)\right]$$

$$T(S \to S') = \frac{1}{2}\left[1 + \mathrm{Tanh}\left(\frac{S'h}{T}\right)\right]$$

$$\frac{P(S)}{P(S')} = \frac{T(S' \to S)}{T(S \to S')}$$

$$= \frac{\frac{1}{2}\left[1 + \tanh(S h/T)\right]}{\frac{1}{2}\left[1 + \tanh(S' h/T)\right]}$$

$$1 + \tanh(x) = 1 + \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{x} + \cancel{e^{-x}} + e^{x} - \cancel{e^{-x}}}{e^{x} + e^{-x}}$$

$$= \frac{2e^{x}}{e^{x} + e^{-x}} \qquad x = \frac{hS}{T}$$

$$\frac{P(S)}{P(S')} = \frac{e^{x}}{\cancel{e^{x} + e^{-x}}} \cdot \frac{1}{\frac{e^{+x'}}{\cancel{e^{x'} + e^{-x'}}}}$$

$$x' = -x$$

$$= e^{x - x'}$$

$$\frac{P(S)}{P(S')} = \frac{e^{\frac{hS}{T}}}{e^{\frac{hS'}{T}}}$$

Dame $\{S\}$ y te digo la probabilidad en el estado estacionario

$$P(S) \propto e^{\frac{hS}{T}}$$

$$\sum_{\{S\}} P(\{S\}) = Z$$ — funcion particion

$$P(S) = \frac{e^{\frac{hS}{T}}}{Z}$$

Ahora $\sum_{\{S\}} P(S) = 1$ $\qquad F = -k_B T \ln(Z)$

Moraleja: el sistema hace un paseo aleatorio entre todas las posibles conf.

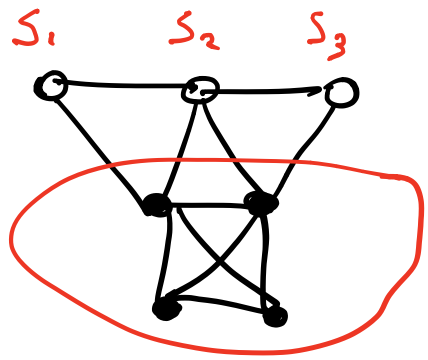
y la prob. de una conf. es $P(S) = \frac{e^{+Sh/T}}{Z}$

$$Z = \sum_{\{S\}} e^{+\frac{Sh}{T}}$$

1983 Hinton y Sejnowski

Divide la red

red visible
red oculta



o neurona visible
• neurona oculta

$\bar{S} = (S_1, S_2, S_3)$ visibles

$P=0.7$	+	+	+
$P=0.1$	−	+	+
$P=0.1$	+	+	−
$P=0.1$	+	−	+
0	−	−	+
0		⋮	

ocho

Tenemos N_T neuronas

K ocultas
N visibles

$$N_T = K + N$$

Los denotamos con dos índices

α recorre las visibles

β recorre las ocultas

$P_{\alpha\beta}$: prob de encontrar el sist. en $\{S_{\alpha\beta}\}$

$$P_{\alpha\beta} = P(\{S_{\alpha\beta}\}) = \frac{e^{-H_{\alpha\beta}/T}}{Z}$$

$$H_{\alpha\beta} = -\frac{1}{2} \sum_i \sum_j w_{ij} S_i^\alpha S_j^\beta$$

Ahora minimizar la función Entropía

Sea R_α la prob deseada con que las visibles aparecen

$$P_\alpha = \sum_\beta P_{\alpha\beta} = \sum_\beta \frac{e^{-H_{\alpha\beta}/T}}{Z}$$

Entropia $S_e = \sum_\alpha R_\alpha \log\left(\frac{R_\alpha}{P_\alpha}\right) \geq 0$

descenso por el gradiente en S_e

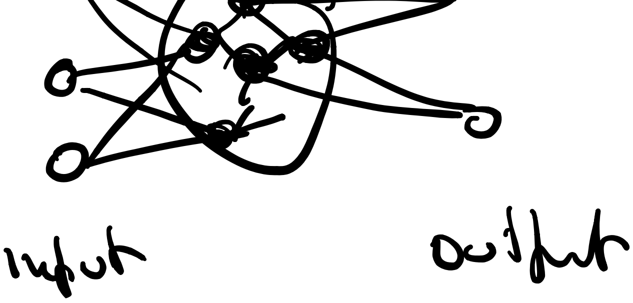
$$\Delta w_{ij} = -\eta \frac{\partial S_e}{\partial w_{ij}}$$

$$w_{ij}^{new} = w_{ij}^{old} + \Delta w_{ij}$$

$$\Delta w_{ij} = -\eta \frac{\partial S_e}{\partial w_{ij}} = \eta \sum_\alpha \frac{R_\alpha}{P_\alpha} \frac{\partial P_\alpha}{\partial w_{ij}}$$

$$\Delta w_{ij} = \frac{\eta}{T}\left[\sum_\alpha \sum_\beta \frac{R_\alpha}{P_\alpha} S_i^{\alpha\beta} S_j^{\alpha\beta} P_{\alpha\beta} - \sum_\alpha R_\alpha \langle S_i S_j \rangle\right]$$

$$= \frac{\eta}{T}\left[\langle S_i S_j \rangle_{clamped} - \langle S_i S_j \rangle_{libre}\right]$$



$$\Delta W_{ij} = \frac{\eta}{T}\left[\langle S_i S_j\rangle_{\substack{I,O \\ \text{clamped}}} - \langle S_i S_j\rangle_{\substack{I \\ \text{clamped}}}\right]$$

Practica 3

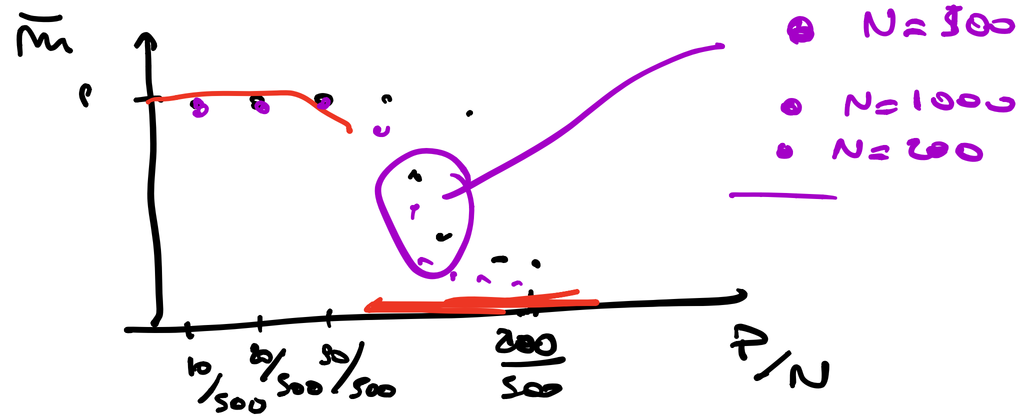
Hopfield $N = 500$ neuronas

Fijen P y evolucionen desde c/memoria

$$m^{\mu} = \frac{1}{N}\sum_i \xi_i^{\mu} S_i^{f}$$

$$\overline{m} = \frac{1}{P}\sum_{\mu=1}^{P} m^{\mu}$$

$P = 10, 20, 30, \ldots, 200$



$N = 1000$

$P = 20, 40, \dots \quad 400$

$N = 2000$

$P = 40, 80, \dots \quad 800$