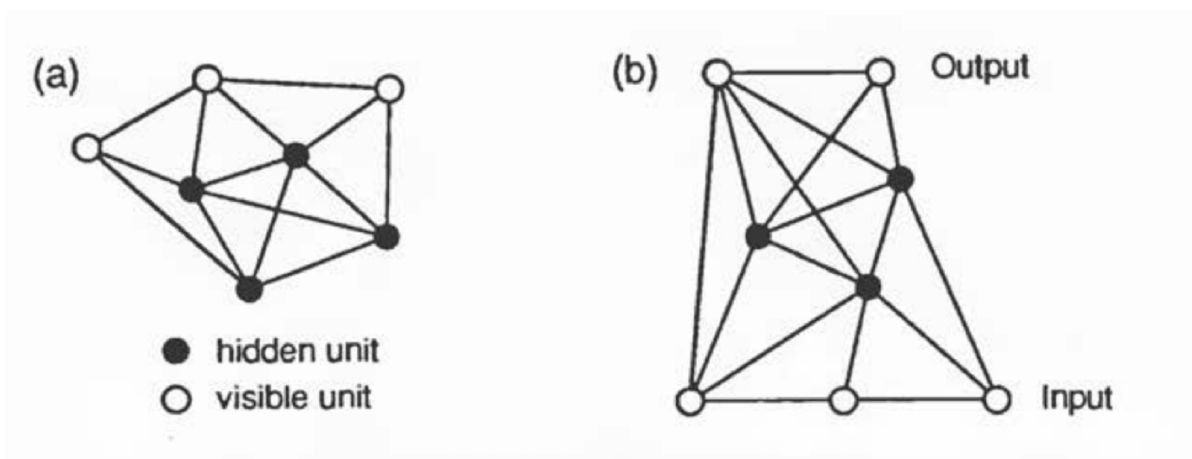
Redes Recurrentes

Maquina de Boltzmann



Hinton y Sejnowski 1983

S_i : variable aleatoria (-1 o $+1$)

$$w_{ij} = w_{ji}$$

$$\text{Pro}(S_i = +1) = g(h_i)$$

$$h_i = \sum_j w_{ij} S_j$$

$$g(h) = \frac{1}{1 + e^{2\beta h}}$$

$$\beta = 1/\cancel{T}$$

función de Llapunov

$$H(\{S_i\}) = -\frac{1}{2} \sum w_{ij} S_i S_j$$

H llega a un mínimo local si

$$S_i(t+\Delta t) = \text{signo}(h_i)$$

h_i

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h=0
for j=1 to N
    h = h + w(i,j)*S(j)
end
p = 1./(1+exp(2*BETA*h))
z = random(0,1)
if z<p entonces
    S(i)=+1
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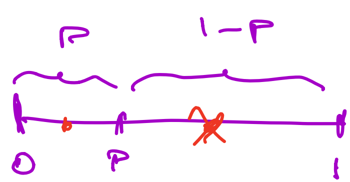

sino
$S(i) = -1$

end

$1-p$

El sistema evoluciona de forma tal que recorre las diferentes configuraciones con una prob. independiente de t.

$$P(\{S_i\}, t) \longrightarrow P(\{S_i\})$$

estado estacionario

Además vimos

$$P(\{S_i\}) \propto e^{-\beta H(\{S_i\})}$$

$$\sum_{\{S_i\}}^{2^N} P(\{S_i\}) = 1$$

$$P(\{S_i\}) = \frac{1}{Z} e^{-\beta H(\{S_i\})}$$

$$Z = \sum_{\{S_i\}}^{2^N} e^{-\beta H(\{S_i\})} \longrightarrow \text{función partición}$$

Supongamos que tengo una función de $\{S_i\}$: $\quad X(\{S_i\})$

$$\langle X \rangle = \sum_{\{S_i\}}^{2^N} P(\{S_i\})\, X(\{S_i\})$$

$\to$ nuevo N

Divido la red en N neuronas visibles y las pongo el índice α para identificarlas las configuraciones $(\alpha = 1, 2, \ldots, 2^N)$ y K neuronas invisibles y les pongo el índice γ para identificar las configuraciones $(\gamma = 1, 2, \ldots, 2^K)$

Tenemos ahora $\quad 2^N \cdot 2^K \quad 2^{(N+K)}$ config.

$$\{S_i\} = \underbrace{\{S_i^\alpha\}}_{\text{visible}} \cup \underbrace{\{S_i^\gamma\}}_{\text{invisible}}$$

$\Downarrow$

$K + N = \mathcal{N}$

$\downarrow$

otro N ojo

¿Cuál es la prob. de tener las visibles en el estado α?

"

configuración

$$P_\alpha = \sum_\gamma P_{\alpha\gamma}$$ $P_{\alpha\gamma}$: prob. conjunta

$$= \sum_\gamma \frac{e^{-\beta H_{\alpha\gamma}}}{Z}$$

$$Z = \sum_{\alpha\gamma} e^{-\beta H_{\alpha\gamma}}$$ $H_{\alpha\gamma} = -\frac{1}{2}\sum_{ij} w_{ij} S_i^{\alpha\gamma} S_j^{\alpha\gamma}$

Defino una función costo

$$E = \sum_\alpha R_\alpha \log\left(\frac{R_\alpha}{P_\alpha}\right)$$

R_α : prob. deseada de α

$$E \geq 0$$

$$\log(x) \geq 1 - \frac{1}{x}$$

$$E = \sum_\alpha R_\alpha \log\left(\frac{R_\alpha}{P_\alpha}\right) \geq \sum_\alpha R_\alpha\left(1 - \frac{P_\alpha}{R_\alpha}\right) = \sum_\alpha (R_\alpha - P_\alpha) \geq$$

$$E \geq \sum_\alpha R_\alpha - \sum_\alpha P_\alpha = 1 - 1 = 0$$

$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} = \eta \frac{\partial}{\partial w_{ij}} \left(\sum_\alpha R_\alpha \log\left(\frac{P_\alpha}{R_\alpha}\right)\right)$$

$$= \eta \sum_\alpha R_\alpha \frac{\partial}{\partial w_{ij}} \log\left(\frac{P_\alpha}{R_\alpha}\right)$$

$$= \eta \sum_\alpha R_\alpha \cdot \frac{R_\alpha}{P_\alpha} \cdot \frac{1}{R_\alpha} \frac{\partial P_\alpha}{\partial w_{ij}}$$

$$\frac{\partial P_\alpha}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \left[\frac{e^{-\beta H}}{Z} \right] = \frac{\partial}{\partial w_{ij}} \left[\frac{e^{-\beta\left(-\frac{1}{2}\sum_i \sum_j w_{ij} S_i S_j\right)}}{Z} \right]$$

$$= \frac{e^{-\beta H}}{Z} \cdot \underset{\substack{\downarrow \\ w_{ij}}}{\left(-\left(-\beta \frac{S_i S_j}{2}\right)\right)} + \underset{w_{ji}}{\left(-\left(-\beta \frac{S_i S_j}{2}\right)\right)}$$

$$\frac{\partial P_\alpha}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \left\{ \sum_\gamma P_{\alpha\gamma} \right\}$$

$$= \frac{\partial}{\partial w_{ij}} \left\{ \sum_\gamma \frac{e^{-\beta \overset{w_{ij}}{H_{\alpha\gamma}}}}{\underset{w_{ij}}{Z}} \right\}$$

$$= \beta \sum_\gamma \frac{e^{-\beta H_{\alpha\gamma}}}{Z} S_i^{\alpha\gamma} S_j^{\alpha\gamma}$$

$$-\left(\sum_{\gamma} e^{-\beta H_{\alpha\gamma}}\right)\frac{1}{Z^2}\frac{\partial Z}{\partial w_{ij}}$$

$$= \beta \sum_{\gamma} \frac{e^{-\beta H_{\alpha\gamma}}}{Z} S_i^{\alpha\gamma} S_j^{\alpha\gamma} - \frac{\left(\sum_{\gamma} e^{-\beta H_{\alpha\gamma}}\right)\beta \sum_{\lambda\mu} e^{-\beta H_{\lambda\mu}} S_i^{\lambda\mu} S_j^{\lambda\mu}}{Z^2}$$

$$= \beta \left[\sum_{\gamma} S_i^{\alpha\gamma} S_j^{\alpha\gamma} P_{\alpha\gamma} - P_\alpha \langle S_i S_j \rangle\right]$$

$$\Delta w_{ij} = \eta\beta \left[\sum_{\alpha} \frac{R_\alpha}{P_\alpha} \sum_{\gamma} S_i^{\alpha\gamma} S_j^{\alpha\gamma} P_{\alpha\gamma} - \sum_{\alpha} R_\alpha \langle S_i S_j \rangle\right]$$

$$= \eta\beta \left[\langle S_i S_j \rangle_{fijo} - \langle S_i S_j \rangle_{libre}\right]$$

$$\langle S_i S_j \rangle_{fijo} = \sum_{\alpha\gamma} R_\alpha P_{\gamma|\alpha} P_\alpha$$