Redes complejas: conceptos y aplicaciones

FaMAF, 31 de octubre de 2019

Contenido

Sistemas complejos

- Están compuestos de **muchos elementos y/o componentes y/o particulas.** Estos elementos **descriptos por su estado**: velocidad, posición, edad, espín, salud, masa, forma, etc. Los componentes pueden tener elementos estocásticos.
- La **interacción entre los elementos** puede ser específica. Que cosa interactua con que, cuando, y que tanto, es **descripto por la red de interacciones**
- **La naturaleza de las interacciones es variada** y no se restringe fuerzas elementales de la física, puede ser intercambio de mensajes u objetos, regalos, información, proyectiles, etc.
- Puede haber superposión de interacciones de magnitudes similares.
- Son general **sistemas fuera del equilibrio** que no necesariamente obedecen leyes de conservación.

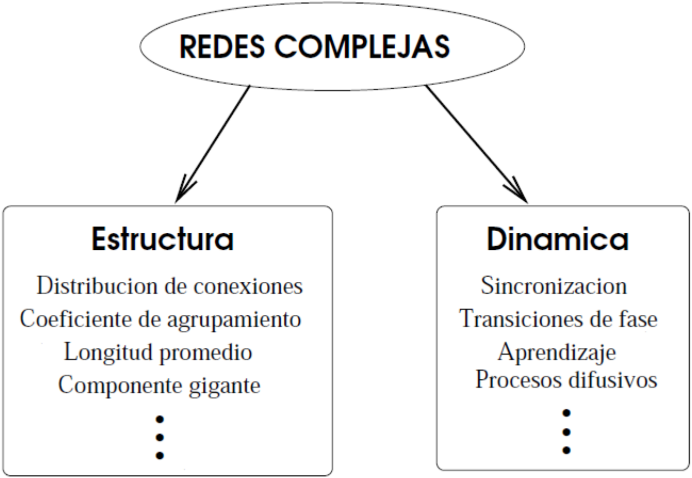
Sistemas complejos

- Son sistemas caóticos fuertemente dependientes de las condiciones iniciales que **evolucionan algorítmicamente** de forma no lineal.
- Tienen un estructura de fases muy rica con una gran variedad de macroestados que frecuentemente no pueden inferirse de las propiedades de los elementos que forman el sistema. Esto se denomina generalmente **emergencia**. En la física hay formas simples tales como por ejemplo la fase líquida que surge de la interacción de las moléculas de agua.
- Son **sistemas robustos** es decir que son resistentes a fallas.

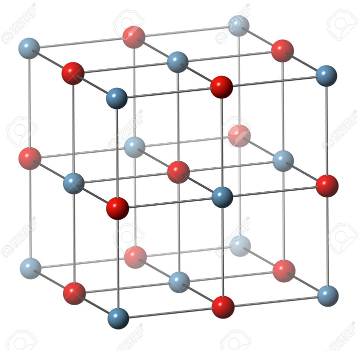
Introduction to the theory of complex systems, Stefan Thurner, Rudolf Hanel and Peter Klimek, Oxford 2018.

Understanding complexity, S. Kivelson and S. Kivelson, Nature Physics, 5, **14**, 426–427, (2018).

Redes complejas



Redes regulares

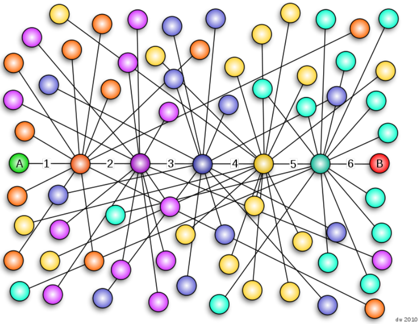


Características

- Nodos o vértices
- Conexiones
- Todos los nodos tienen el mismo grado (# de nodos grande)
- Simetrías
- $< l > \propto N^{1/d}$ cúbica $d = 3$

Grafo $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, donde $\mathcal{V}$ conjunto de nodos y $\mathcal{E}$ conjunto de conexiones. $N = |\mathcal{V}|$ número nodos y $M = |\mathcal{E}|$ número de conexiones.

Experimento Milgram



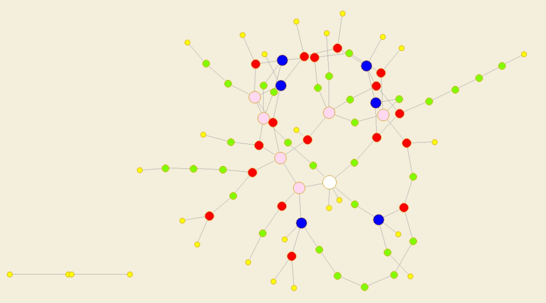
Características

- Dos zonas geográficas y sociales diferentes
- Información local
- $< l >= 6$ seis grados de separación

An Experimental Study of the Small World Problem Jeffrey Travers and Stanley Milgram, Sociometry Vol. 32, No. 4 (Dec., 1969), pp. 425-443

Red de Erdös Renyi (ER)

Con probabilidad p agrego una conexión entre dos nodos



$N = 100$, $M = 100$, $(p \sim 0.02)$
$< k >= 2$

Características

- Número medio de conexiones $< M >= p\frac{N(N-1)}{2}$
- Se pierden simetrías
- Los nodos tienen distinto grado k_i.
- $< k >= 2M/N \sim pN$ si $< k >\sim 1$ la red percola ($p \sim 1/N$).
- $< l >\propto \ln N$

Coeficiente de clustering

Definiciones

Clustering global

- $C_\Delta = \frac{3 \times \text{número de triangulos}}{\text{número de tripletes}}$

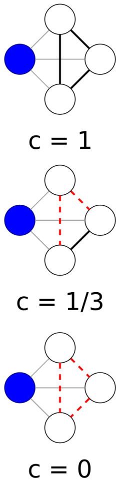
Clustering local

- $C_i = \frac{|\{e_{jk}: v_j, v_k \in \mathcal{V}_i, e_{jk} \in \mathcal{E}\}|}{k_i(k_i - 1)/2}$

Clustering global

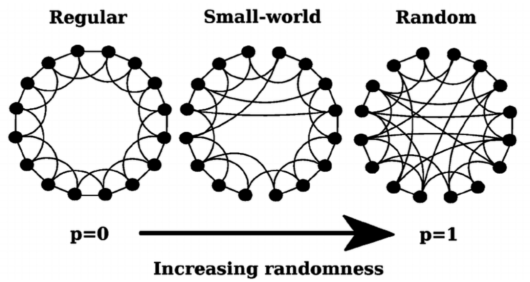
- $< C >= \frac{1}{N} \sum_{i=1}^{N} C_i$

Ejemplo C_i



Red de Watts-Strogatz (WS)

Partiendo de un grafo regular con probabilidad p hago una reconexión



Características

- $\#$ de conexiones no cambia $M = Nk/2$
- Cuando p es pequeño tiene un alto clustering o transitividad.
- Cuando p es grande tiende a una ER por lo tanto $< l > \propto \ln N$

Transitividad y distancia entre nodos en WS

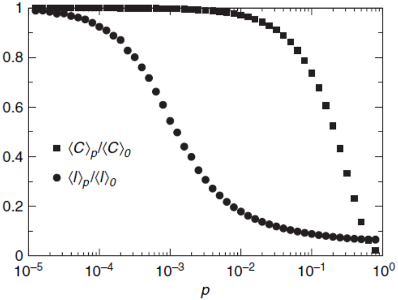
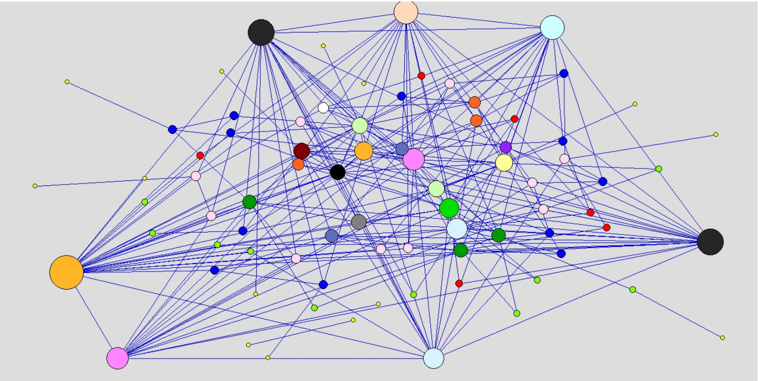


Fig. 3.2. Normalized clustering coefficient $\langle C\rangle_p/\langle C\rangle_0$ (squares) and average shortest path length $\langle \ell\rangle_p/\langle \ell\rangle_0$ (circles) as a function of the rewiring probability p for the Watts–Strogatz model. The results correspond to networks of size $N = 1000$ and average degree $\langle k\rangle = 10$, and are averaged over 1000 different realizations.

Red libre de escala (scale-free)



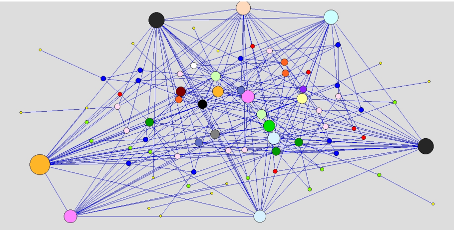
Preferential Attachment (Barabasi-Albert)

- Es un modelo generativo
- La red comienza con m_0 nodos (por ejemplo $m_0 = 1$)
- Se agregan nodos con $l \leq m_0$ conexiones fijas ($l = m_0 = 1$).
- La probabilidad con que un nodo v_i recibe las conexiones entrantes esta dada por

$$p_i = \frac{k_i}{\sum_i k_i}$$

https://vimeo.com/53071346

Ejemplo red libre de escala



Características

- Existencia de nodos muy conectados *Hubs*
- Distribución de grados heterogénea.
- El grado de clusterización depende de la distribución de grado.
- Son redes de mundo pequeño $< l > \propto \ln N$.

Redes homogéneas y heterogéneas

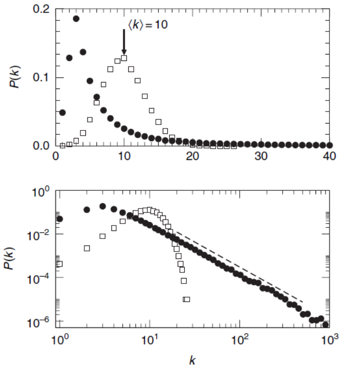


Fig. 2.8. Comparison of a Poisson and power-law degree distribution on a linear scale plot (top) and a double logarithmic plot (bottom). The two distributions have the same average degree $\langle k \rangle = 10$. The dashed line in the bottom figure corresponds to the power law $k^{-\gamma}$, where $\gamma = 2.3$.

Resumen propiedades estadísticas

Table 2: Analytical result of some basic measurements for the Erdős-Rényi,Watts-Strogatz and Barabási-Albert network models.

Measurement	Erdős-Rényi	Watts-Strogatz	Barabási-Albert
Degree distribution	$P(k) = \frac{e^{-\langle k \rangle}\langle k \rangle^k}{k!}$	$P(k) = \sum_{i=1}^{min(k-\kappa,\kappa)} \binom{\kappa}{i} (1-p)^i p^{\kappa-i} \frac{(p\kappa)^{k-\kappa-i}}{(k-\kappa-i)!} e^{-p\kappa}$	$P(k) \sim k^{-3}$
Average vertex degree	$\langle k \rangle = p(N-1)$	$\langle k \rangle = 2\kappa^\star$	$\langle k \rangle = 2m$
Clustering coefficient	$C = p$	$C(p) \sim \frac{3(\kappa-1)}{2(2\kappa-1)}(1-p)^3$	$C \sim N^{-0.75}$
Average path length	$\ell \sim \frac{\ln N}{\ln\langle k \rangle}$	$\ell(N,p) \sim p^\tau f(Np^\tau)^*$	$\ell \sim \frac{\log N}{\log(\log N)}$

$\star$ In WS networks, the value κ represents the number of neighbors of each vertex in the initial regular network (in Figure 6, $\kappa = 4$).

$*$ The function $f(u) =$ constant if $u \ll 1$ or $f(u) = \ln(u)/u$ if $u \gg 1$.

Tipos de redes

Redes biológicas

Redes sociales

Redes tecnológicas

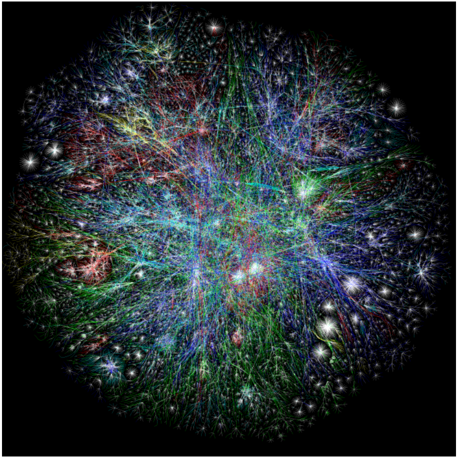
Redes virtuales

Tipos de nodos y conexiones

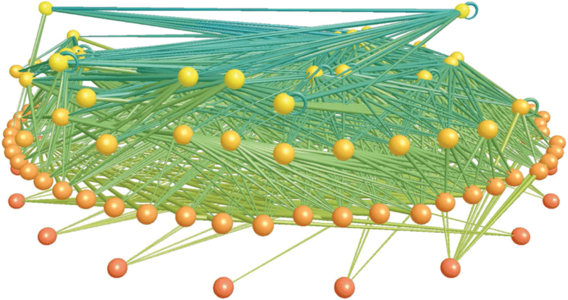
Network	Vertex	Edge
Internet	Computer or router	Cable or wireless data connection
World Wide Web	Web page	Hyperlink
Citation network	Article, patent, or legal case	Citation
Power grid	Generating station or substation	Transmission line
Friendship network	Person	Friendship
Metabolic network	Metabolite	Metabolic reaction
Neural network	Neuron	Synapse
Food web	Species	Predation

Table 6.1: Vertices and edges in networks. Some examples of vertices and edges in particular networks.

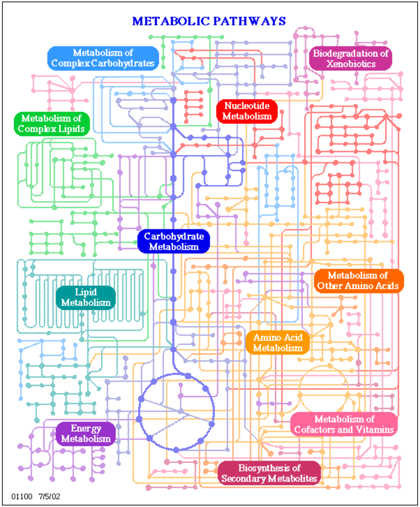
Internet (www.opte.org)



Cadenas tróficas



Red metabólica



Red de aeropuertos USA

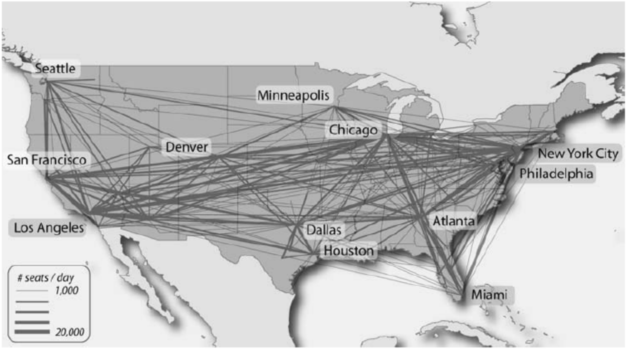


Fig. 2.6. Main air travel routes for the United States. Only the connections with more than 1000 available seats per day are shown. The presence of hubs is clearly visible. Figure courtesy of V. Colizza.

Internet Routers - AS

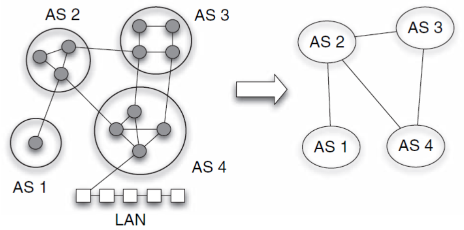


Fig. 2.3. Different granularity representations of the Internet. The hosts (squares) are included in the Local Area Networks (LAN) that connect to routers (shaded circles) and are excluded from the maps. Each Autonomous System is composed of many routers.

World Wide Web

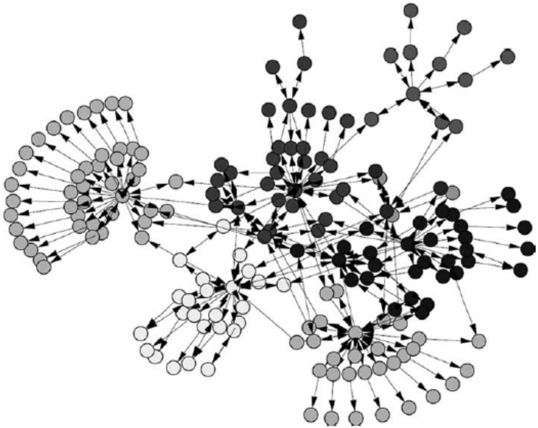
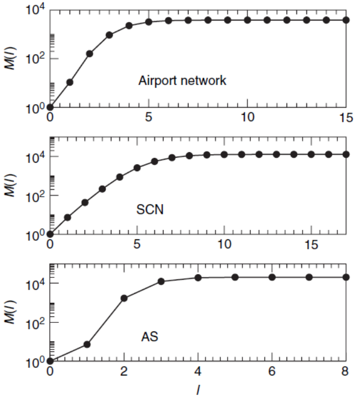


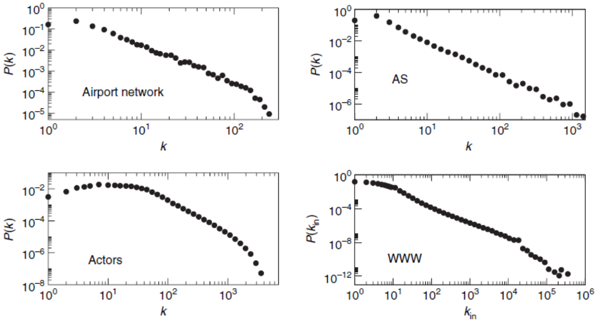
Fig. 2.4. Pages of a website and the directed hyperlinks between them. The graph represents 180 pages from the website of a large corporation. Different shades identify topics as identified with a community detection algorithm. Reprinted with permission from Newman and Girvan (2004). Copyright 2004 by the American Physical Society.

Numero de nodos vs distancia



Airport network: aeropuertos en todo el mundo. AS: internet a nivel de sistemas autonomos. SCN: Red de colaboraciones científicas.

Distribución de grados



Airport network: aeropuertos en todo el mundo. AS: internet a nivel de sistemas autonomos. Actores: Red de participación en peliculas. WWW: Red direccionada.

Betweenness

$$b_i = \sum_{i \neq j \neq k} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$$

$$< b >= \sum_b bP(b)$$

$$= \frac{1}{N} \sum_{i=1}^{N} b_i$$

$$< b >= (N-1)(< l > -1)$$

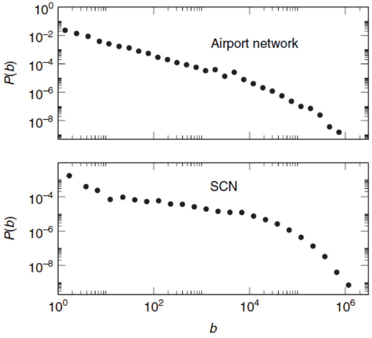
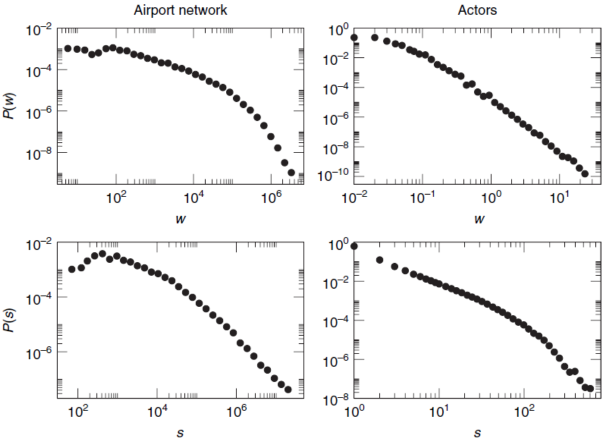


Fig. 2.9. Betweenness centrality distributions in the worldwide airport network (top, http://www.iata.org) and in the scientific collaboration network (SCN, bottom, http://www-personal.umich.edu/˜mejn/netdata/).

Distribución de Fuerzas y Pesos

$$s_i = \sum_{j \in \mathcal{V}_i} w_{ij}$$

$$w_i = s_i / k_i$$

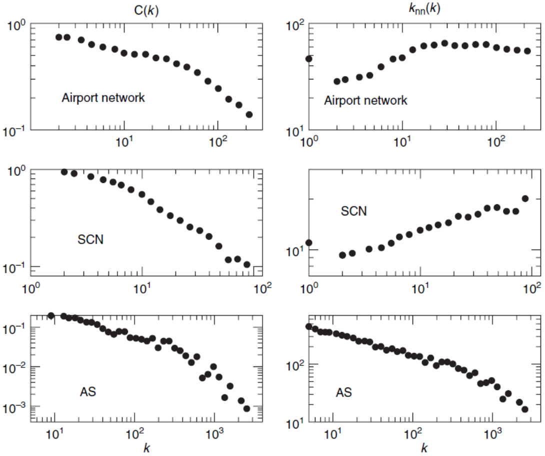


Parámetros característicos de redes reales

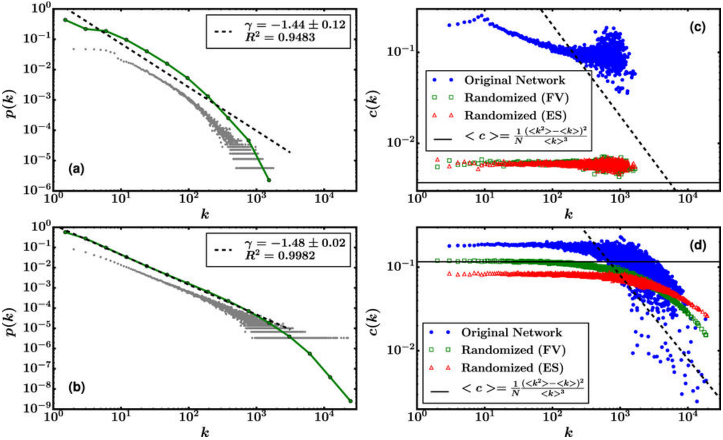
Variable x	Sample	$\langle x \rangle$	x_{max}	2σ	$\kappa = \langle x^2 \rangle / \langle x \rangle$
Degree k	WAN	9.7	318	41.4	53.8
	SCN	6.3	97	12.8	12.8
	Actors	80	3956	328	418
	AS	5.3	1351	70.8	242
In-degree k_{in}	WWW	24.1	378 875	843.2	7414.9
Betweenness b	WAN	6655	929 110	67 912	179 924
	SCN	37 087	3 098 700	235 584	411 208
	AS	14 531	9.3×10^6	439 566	3.3×10^6
Strength s	WAN	725 495	5.4×10^7	6×10^6	1.3×10^7
	SCN	3.6	91	9.4	9.8
	Actors	3.9	645	21	32

Networks included are the worldwide airport network (WAN), the scientific collaboration network (SCN, see `http://www-personal.umich.edu/˜mejn/ netdata/`), the Actors' collaboration network (`http://www.imdb.com/`), the map of the Internet at the AS level, obtained by the DIMES project in May 2005 (`http://www.netdimes.org`), and the map of the WWW collected in 2003 by the WebBase project (`http://dbpubs.stanford.edu: 8091/˜testbed/ doc2/WebBase/`). The quantity x_{max} is the maximum value of the variable observed in the sample. The parameter $\kappa = \langle x^2 \rangle / \langle x \rangle$ and the mean square root deviation $\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ estimate the level of fluctuations in the sample. The quantity 2σ is usually considered as a 95% confidence interval in measurements. It is also possible to appreciate that all heavy-tailed distributions show a maximum value of the variable $x_{max} \gg \langle x \rangle$.

Clustering $C(k)$ y valor medio vecinos cercanos $K_{nn}(k)$



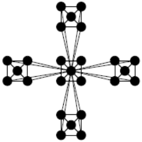
Redes de Ajedrecistas



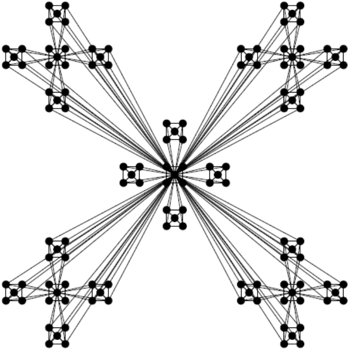
Nahuel Almeira, Ana L. Schaigorodsky, Juan I. Perotti, OVB Scientific Reports (7) 15186, (2017)

Modelo de red jerárquica

(a) n=0, N=5



(b) n=1, N=25



(c) n=2, N=125

Erzsebet Ravasz and Albert-Laszlo Barabasi Phisical Review E 67, 026112 (2003)

Evolución de Internet

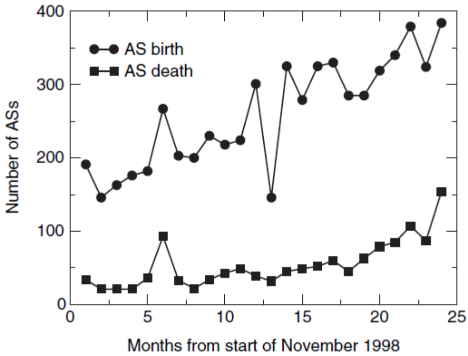
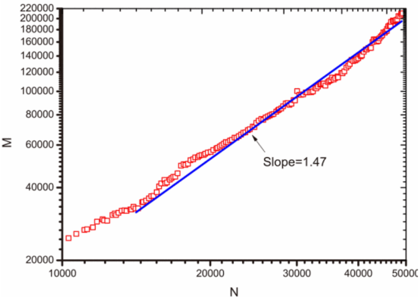
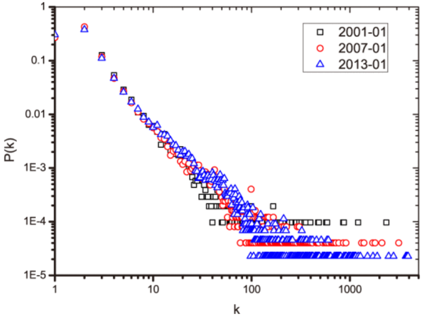


Fig. 2.13. Monthly number of new and dead autonomous systems in the period November 1998 to November 2000. Data from Qian *et al.* (2002).

Evolución de Internet





Dan Yang, Zhihai Rong, Proceedings of the 34th Chinese Control Conference (2015).

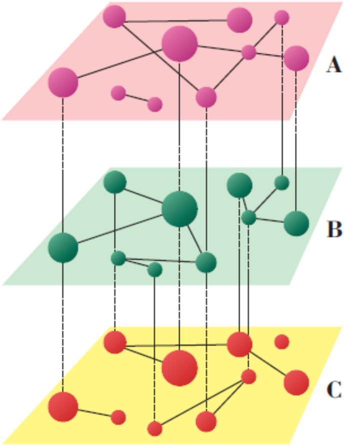
Red de redes

Redes Multiplex

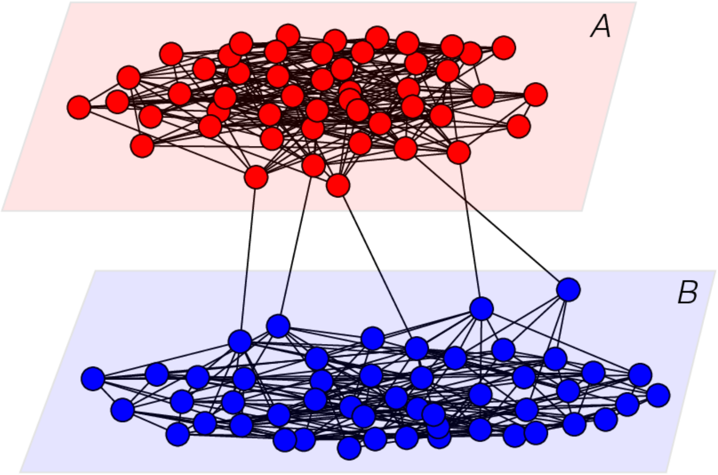
Redes Interactuantes

Redes Interdependientes

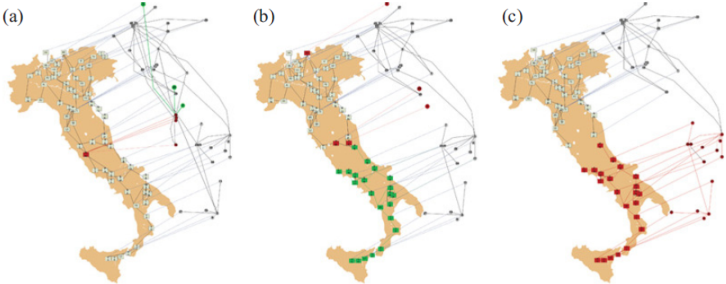
Redes multiplex



Redes interactuantes

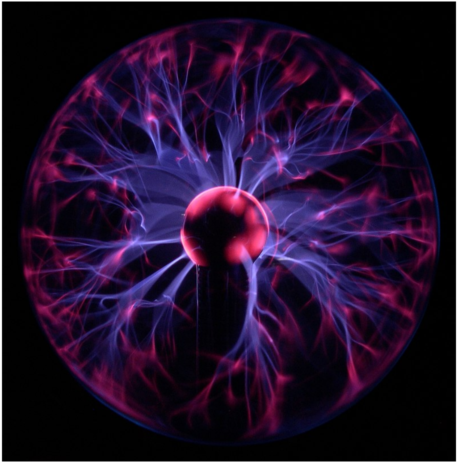


Redes interdependientes



Wikiquote

I think the next [21st] century will be the century of complexity.
Stephen Hawking



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- Networks An Introduction, M. Newman, Oxford University press, (2010).
- Characterization of Complex Networks: A Survey of measurements, L. da F. Costa and F. A. Rodrigues and G. Travieso and P. R. Villas Boas, Advances in Physics, 56, 167-242, (2007).
- Redes Complejas, M. Aldana, XIX Escuela de Verano en Física, Cuernavaca, Morelos, México, Julio 25-agosto 5, 2011