El modelo de Hopfielfd 2

Tenemos N neuronas binarias

$$S_i(t+\Delta t) = \text{signo}\left(h_i(t)\right)$$

$$= \text{signo}\left(\sum_{j\neq i} w_{ij} S_j(t)\right)$$

$N(N-1)$ sinapsis siendo que

$$W_{ij} = W_{ji}$$

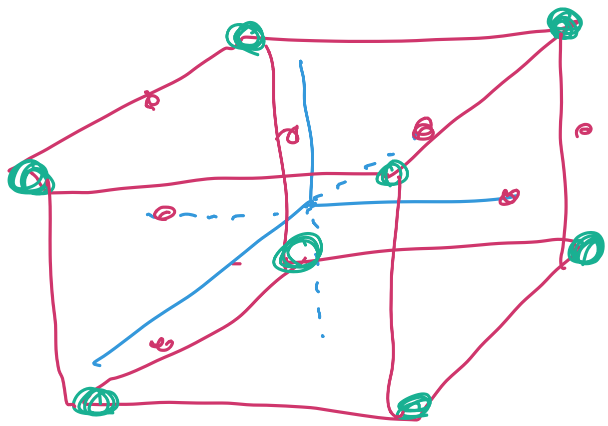
$\frac{N(N-1)}{2}$ sinapsis independientes

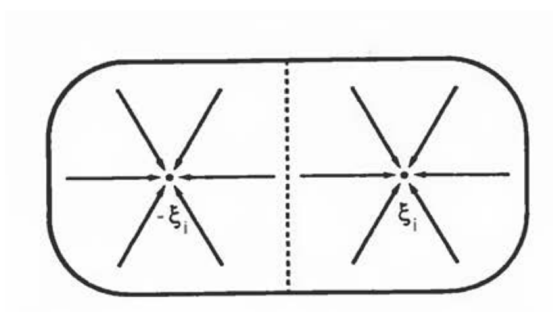
Para 1 memoria:

$$W_{ij} = W_{ji} = \frac{1}{N} \xi_i^1 \xi_j^1$$

$S_i = \pm 1$ $\qquad i = 1, \ldots, N$

$\xi_i^{\ell} = \pm 1$



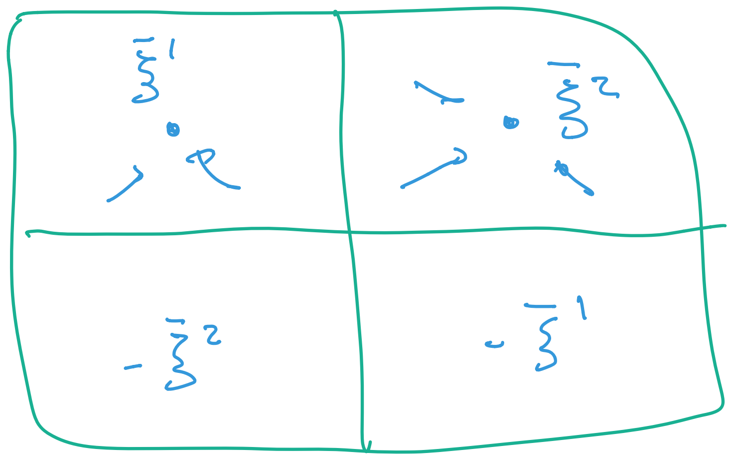


Mattis

¿Cómo puedo almacenar más patrones?

Si son $p = 2$ patrones

$$W_{ij} = \frac{1}{N}\left(\xi_i^1 \xi_j^1 + \xi_i^2 \xi_j^2\right)$$



van Hemmen

Si $p > 2$

$$W_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_i^{\mu} \xi_j^{\mu}$$

¿ Si $S_i(0) = S_i^0$ ¿dónde estará en $S_i(1)$?

Análisis de estabilidad

Sea $S_i(0) = \xi_i^{\nu} \quad \forall i$

$$S_i(1) = \text{signo}\left(h_i(t) \right)$$

$$= \text{signo}\left(\sum_{j \neq i} W_{ij} S_j(0) \right)$$

$$= \text{signo}\left(\sum_{j \neq i} \left(\frac{1}{N} \sum_{\mu=1}^{p} \xi_i^{\mu} \xi_j^{\mu} \right) \xi_j^{\nu} \right)$$

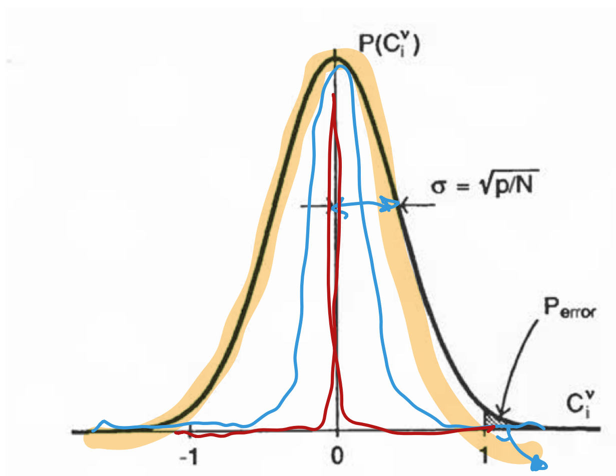
$$= \text{signo}\left(\xi_i^\nu + \frac{\xi_i^{\nu 2}}{N}\sum_i^N \sum_{\mu\neq\nu}^{P-1} \xi_i^\mu \xi_j^\mu \xi_j^\nu\right)$$

$$= \text{signo}\left(\xi_i^\nu \left(1 + \underbrace{\frac{\xi_i^\nu}{N}\sum_i \sum_{\mu\neq\nu} \xi_i^\mu \xi_j^\mu \xi_j^\nu}_{-C_i^\nu}\right)\right)$$

$$= \text{signo}\left(\xi_i^\nu\right) \text{ si } \left|\frac{1}{N}\sum_i^N \sum_{\mu\neq\nu}^{P-1} \xi_i^\mu \xi_j^\mu \xi_j^\nu\right| < 1$$

$$P\left(\{\xi_i^\mu\}_{i=1,\ \mu=1}^{N,\ P}\right) = \prod_\mu^P \prod_i^N p(\xi_i^\mu)$$

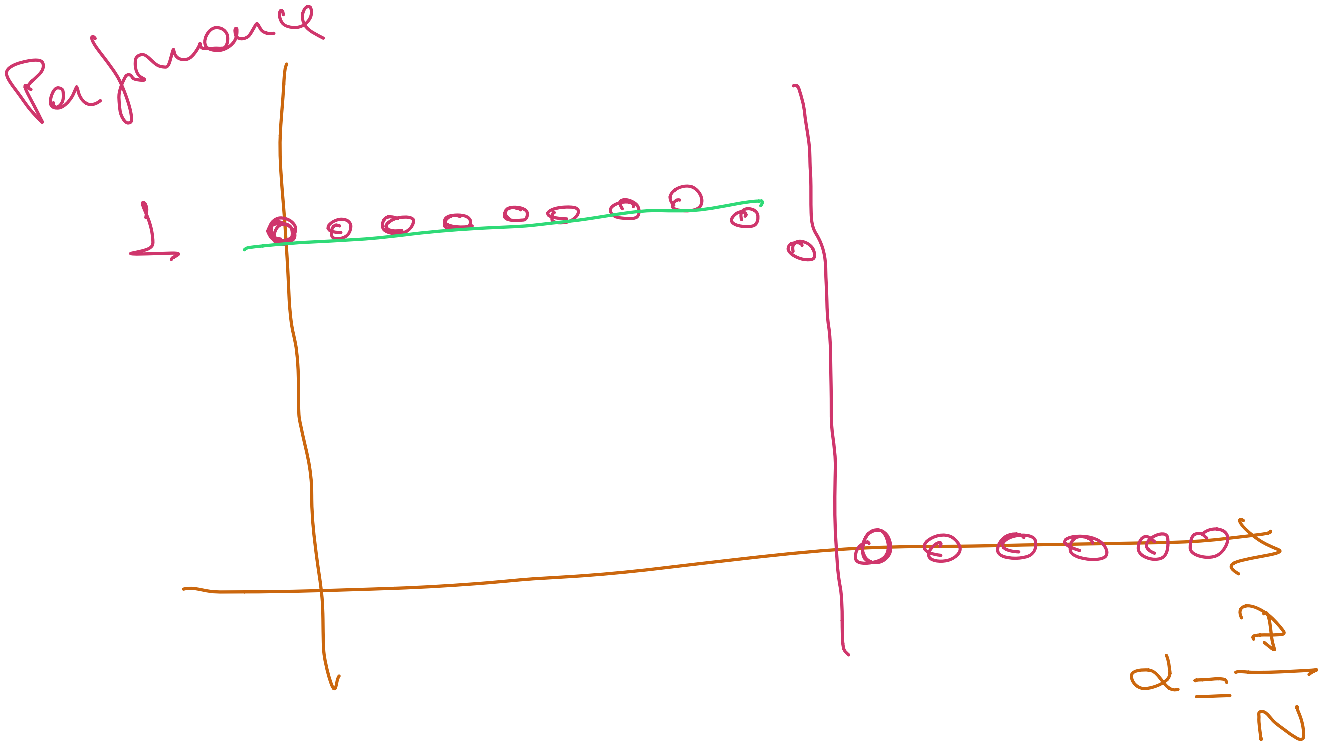
$$p(\xi_i^\mu) = \begin{cases} +1 & \text{es } \frac{1}{2} \\ -1 & \text{es } \frac{1}{2} \end{cases}$$



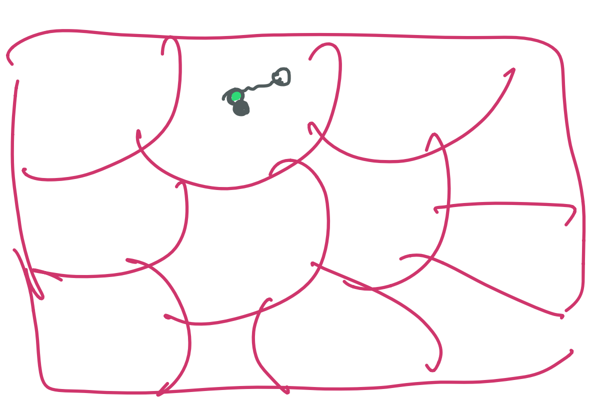
$$C_i^\nu = -\frac{\xi_i^\nu}{N}\sum_{j=1}^N \sum_{\mu=1}^{P-1} \xi_i^\mu \xi_j^\mu \xi_j^\nu$$

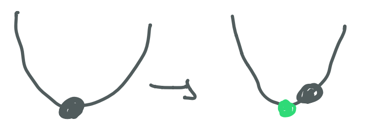
$$P_{error} = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{1}^{\infty} e^{\frac{-x^2}{2\sigma^2}}\, dx$$

ojo con $\frac{P}{N} \ll 1$



$\frac{P}{N} \simeq 0.138$





$$m^{\nu} = \frac{1}{N} \sum_{i=1}^{N} \xi_i^{\nu} \, \underline{S_i(\infty)}$$

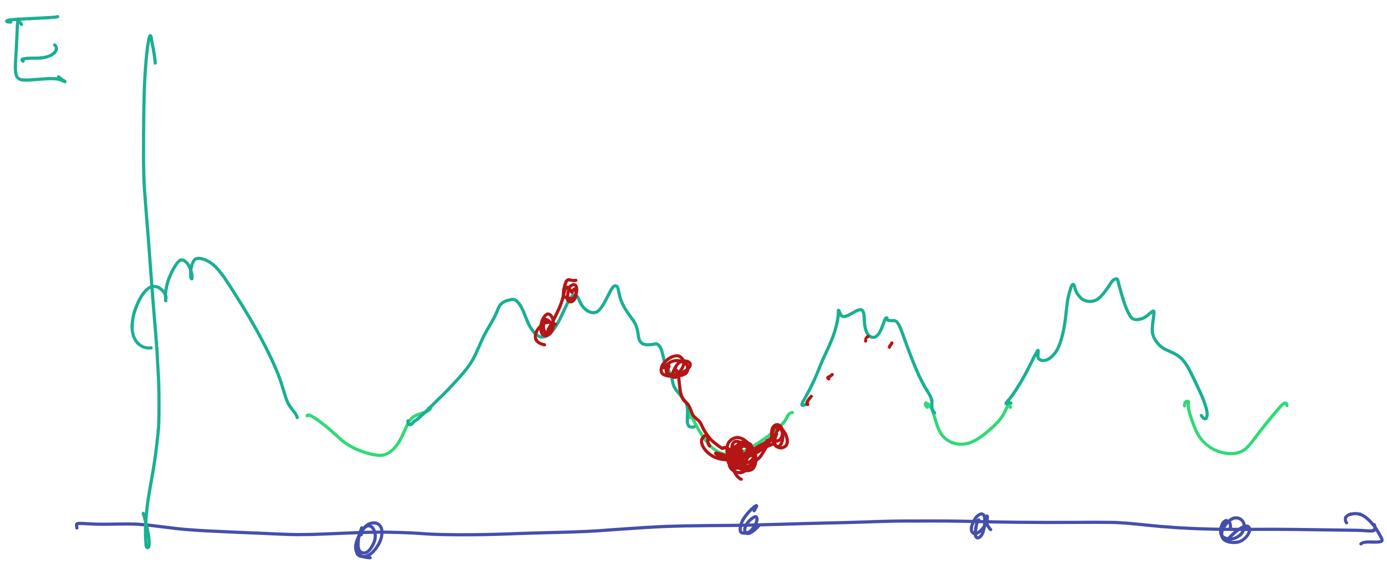
Si $m^{\nu} = 1$ llego al patrón

$m^{\nu} \lesssim 1$ estoy cerca del patrón

$$E(t) = -\frac{1}{2} \sum_{i}^{N} \sum_{j}^{N} w_{ij} \, S_i(t) \, S_j(t)$$

$$E(t+1) \leq E(t)$$

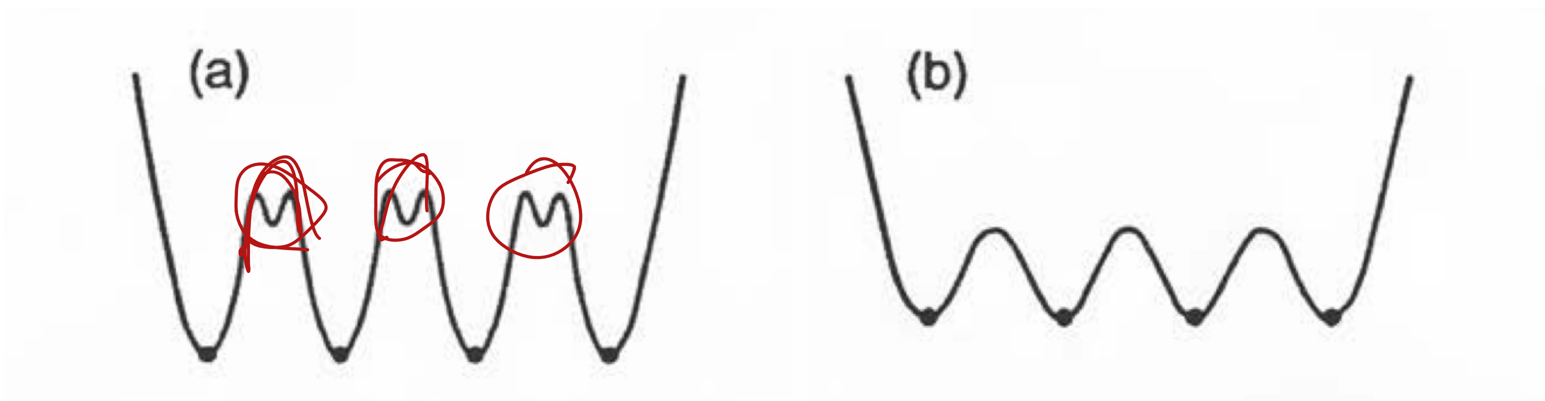
$$E \geq E_{min}$$



Amit, Gutfreund, Sompolinsky 1985

$$\frac{P}{N} \to 0$$

$$\frac{P}{N} > 0$$ 1987



Neurone probabilistico

$$\text{Prob}\left(S_i(t+1)=1\right) = \frac{1}{1+e^{-\frac{2h}{T}}}$$

$$\mathrm{Prob}\left(S_i(t+1) = -1\right) = 1 - \frac{1}{1 + e^{-2\frac{h_i}{T}}}$$