Resumen de Hopfield

Definimos una red con N neuronas binarias tipo Ising ($+1$ o -1), cuya dinámica está dada por una regla inspirada en las neuronas de McCullogh y Pitts pero sin umbrales.

$$S_i(t+\Delta t) = \text{signo}\left(h_i(t)\right)$$

$$= \text{signo}\left(\sum_{j \neq i}^{N} w_{ij} S_j(t)\right)$$

$$\{S(0)\} \longrightarrow \{S(\infty)\}$$

input output

dinámica secuencial $\Delta t = \frac{1}{N}$

" paralela $\Delta t = 1$

De entre las 2^N posibles configuraciones

$$\vec{S} = (S_1, \dots, S_N)$$

de la red, nosotros escogemos al azar con equiprobabilidad e independencia p configuraciones que deseamos que sean "atractores" de la dinámica

$$\vec{\xi}^{\mu} = (\xi_1^{\mu}, \xi_2^{\mu}, \dots, \xi_N^{\mu}) \quad \mu = 1 \dots p$$

Regla de Hopfield

$$W_{ij} = \begin{cases} \frac{1}{N} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu} & i \neq j \\ 0 & i = j \end{cases}$$

Anda bien si $\frac{P}{N} \lesssim 0.138$

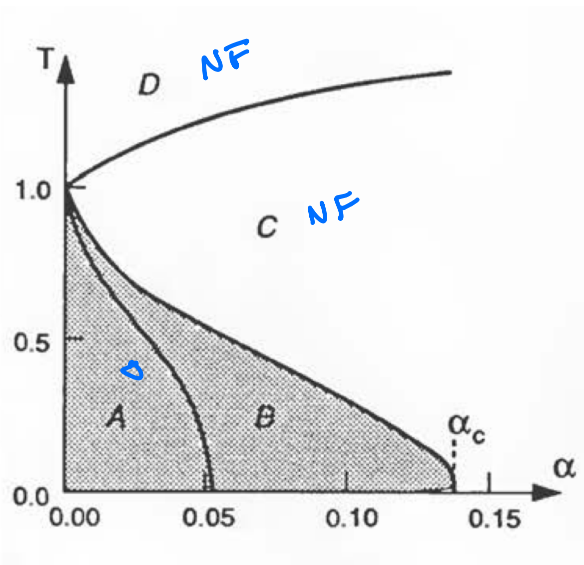
Podemos usar una dinámica estocástica

$$S_i(t+\Delta t) = \begin{cases} +1 & \text{con prob. } g(h_i) \\ -1 & \text{con prob } (1 - g(h_i)) \end{cases}$$

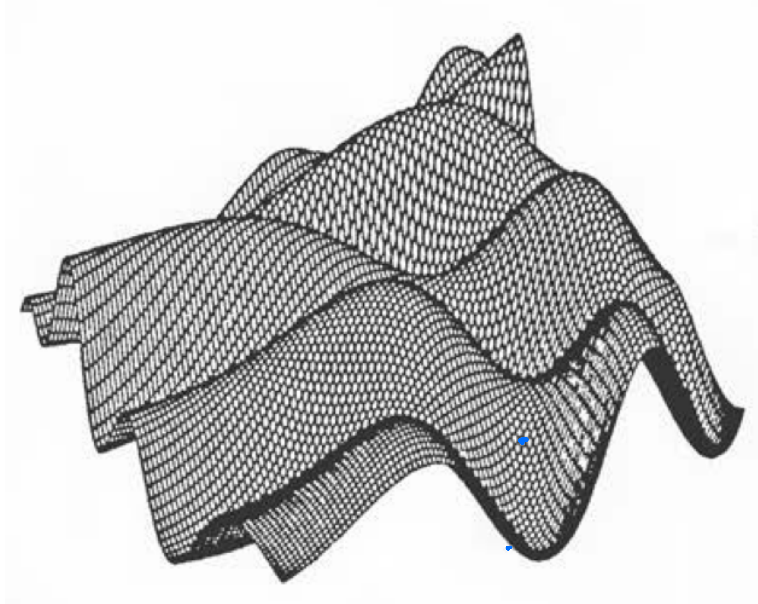
con
$$g(h) = \frac{1}{1 + e^{-\frac{2h}{T}}}$$

$T = 0$ determinista

$T \to \infty$ $S = +1$ y -1 con $p = \frac{1}{2}$



$= \frac{P}{N}$



Per ogni link $\qquad e_{ij}(t) = \underline{w_{ij} \, S_i(t) \, S_j(t)}$

$$E(t) = \sum_{\text{links}} e_{ij}(t)$$

$$= \sum_{i=1}^{N} \left[\sum_{\substack{j=1 \\ j \neq i}}^{N} w_{ij} \, S_i(t) \, S_j(t) \right]$$

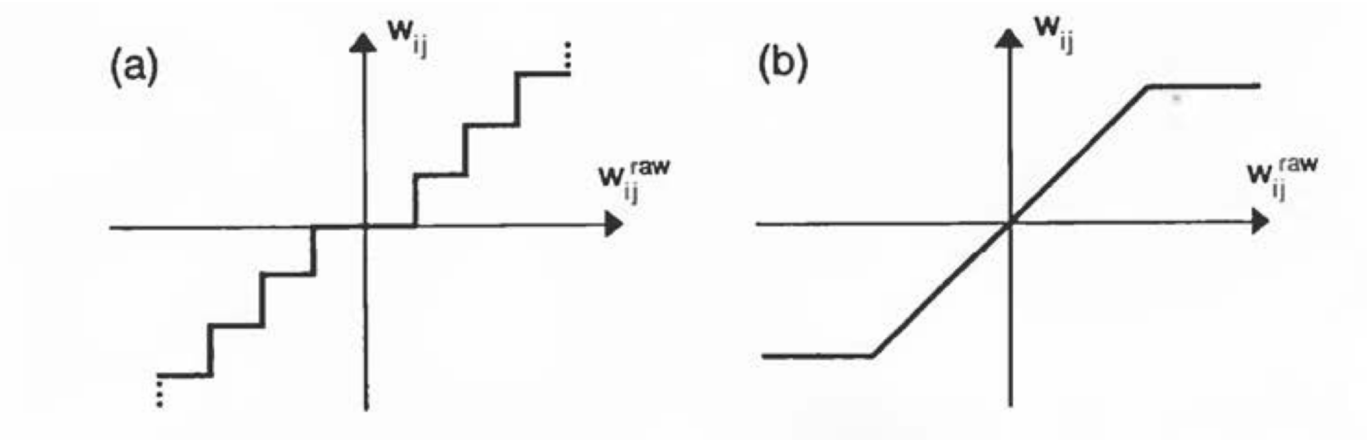
$$E(t+1) \leq E(t) \qquad T = 0$$

$$E = \sum_{i=1} \left[\sum_{\substack{j \neq i \\ j=1}}^{N} w_{ij} S_j(t) \right] S_i(t)$$

$$E = \sum_{i=1} h_i(t) S_i(t)$$

Variaciones del modelo de Hopfield

- Clipping



- Aprendizaje

$$w_{ij}^{nuevo} = w_{ij}^{anterior} + \gamma\, \xi_i^{\mu} \xi_j^{\mu} \qquad \mu = 1, \dots, p$$

- Redes sincrónicas $(\Delta t = 1)$

- Dilución débil

$$W_{ij} = \begin{cases} \frac{1}{N} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu} & \text{prob} = c \\ 0 & \text{prob} = 1 - c \end{cases}$$

$$C_{ij} = C_{ji} \quad \text{ó} \quad C_{ij} \neq C_{ji}$$

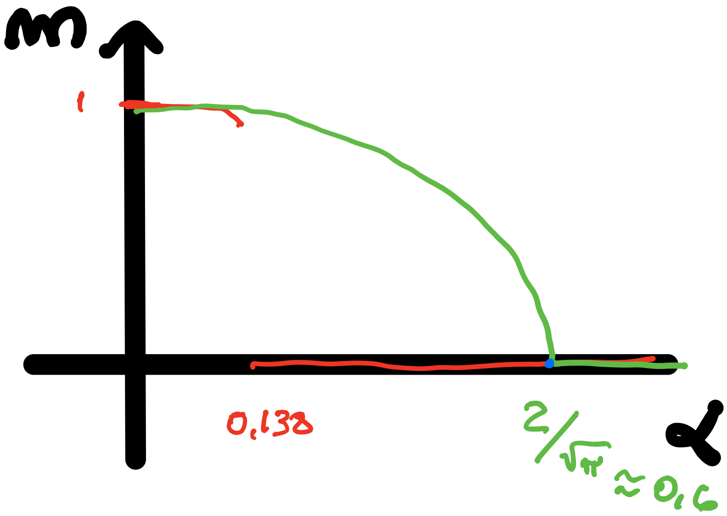
- Dilución fuerte

Derrida
Gardner
Zippelius

$$W_{ij} = \frac{1}{K} C_{ij} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$

$$\boxed{K \ll \ln(N)}$$

$$C_{ij} = \begin{cases} 1 & \text{con prob. } \frac{K}{N} \\ 0 & \text{" " } \left(1 - \frac{K}{N}\right) \end{cases}$$



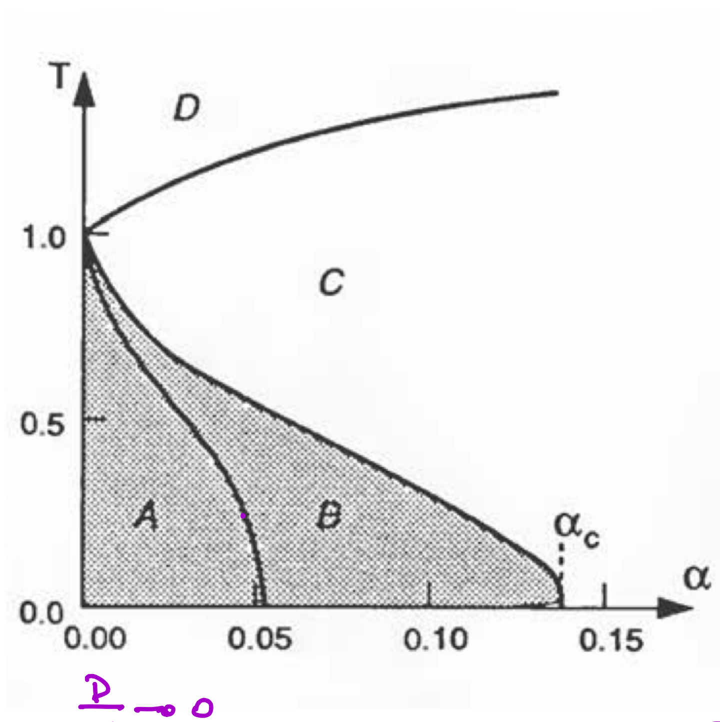
$$m = \frac{1}{N} \sum_{i=1} \underline{S(\alpha) \}^N}$$

0,6

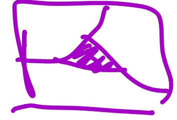
$$m = \operatorname{erf}\left(\frac{M}{\sqrt{2\alpha}} \right)$$

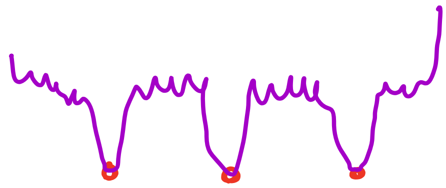
$$\alpha = \frac{K}{N}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt$$


T
D
C
A
B
1.0
0.5
0.0
0.00
0.05
0.10
0.15
α
α_c
$P \to 0$

N





A

B

$$F(T,N) \propto -\ln\left(Z(N,T)\right)$$

$$\lim_{N\to\infty} \frac{F(T,N)}{N} = f(T) = -\frac{1}{N}\ln\left(\underbrace{Z(T)}\right)$$

$$Z_{\{\xi\}}(T)$$

$$\langle\langle f\rangle\rangle = \frac{1}{N}\langle\langle \ln Z_{\{\xi\}}(T)\rangle\rangle_{\{\xi\}}$$

↑

truco de réplica   Replica trick

$$\ln(x) = \lim_{n\to 0}\left(\frac{1-x^n}{n}\right) \quad ¿?$$