BIFURCACIONES PITCHFORK

Supercríticas

Recordemos que cerca de la bifurcación tenemos un desarrollo en serie de Taylor en las dos variables x y r:

$$\dot{x} = f(x,r)$$
$$= \cancel{f(x^*,r_c)} + (x - x^*)\left.\frac{\partial f}{\partial x}\right|_{(x^*,r_c)} + (r - r_c)\left.\frac{\partial f}{\partial r}\right|_{(x^*,r_c)} + \tfrac{1}{2}(x - x^*)^2 \left.\frac{\partial^2 f}{\partial x^2}\right|_{(x^*,r_c)} + \cdots$$

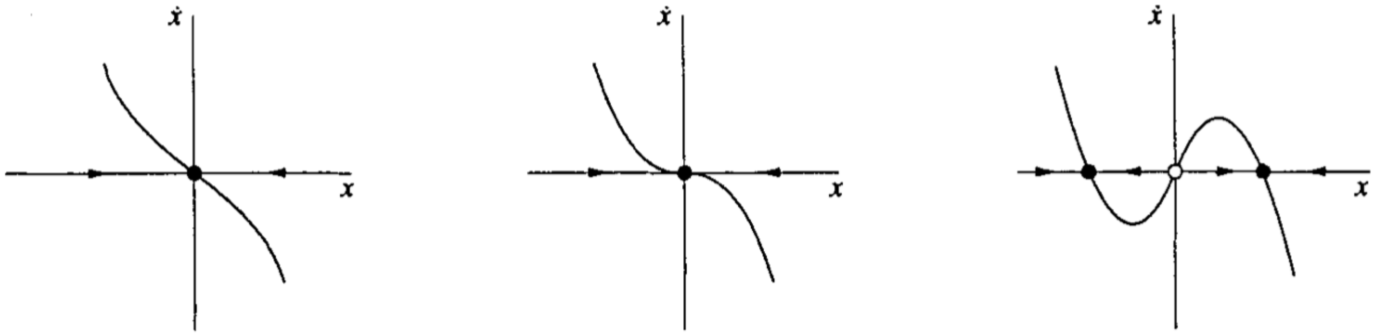
Si los términos que sobreviven son los siguientes

$$\dot{x} = r x - x^3$$

invarianza $x \to -x$

$$\dot{(-x)} = -(\dot{x}) = -\dot{x} = r(-x) - (-x)^3$$
$$-\dot{x} = -r x + x^3$$
$$\dot{x} = r x - x^3$$

volvemos a lo mismo

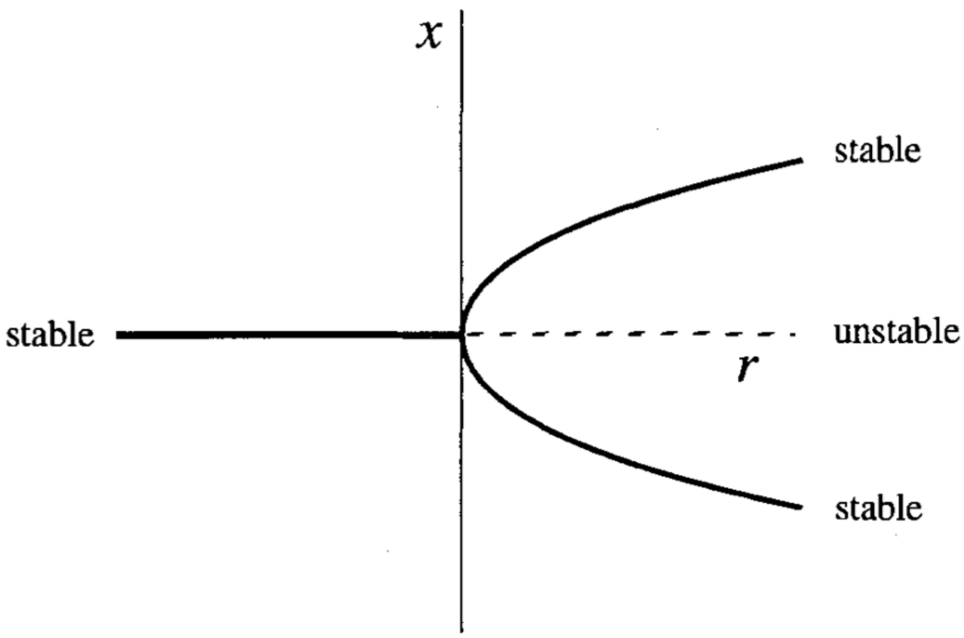


(a) $r<0$ (b) $r=0$ (c) $r>0$

critical slowing down → ya una perturbación no decae en forma exponencial sino ley de potencia

Para $r>0$, tenemos 2 raíces:

$$x^*_+ = \sqrt{r} \qquad x^*_- = -\sqrt{r}$$



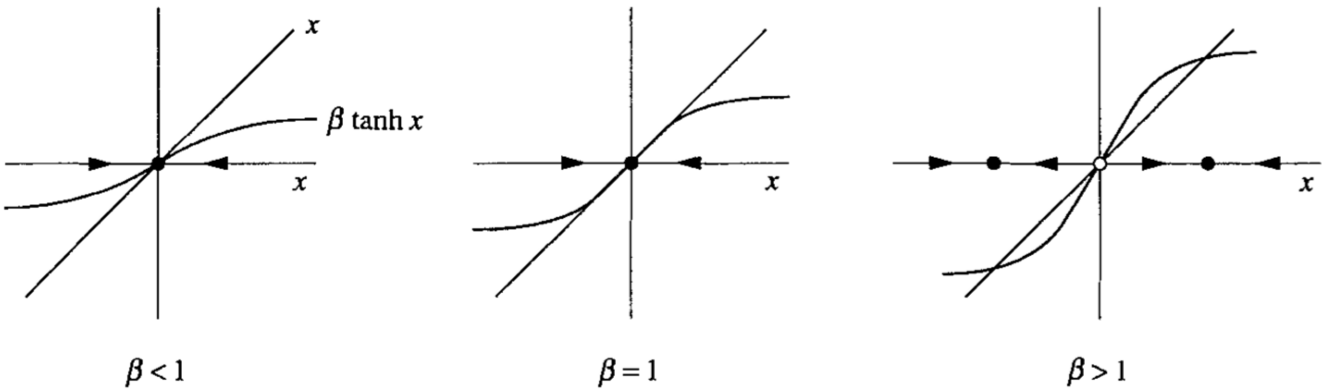
EJEMPLO

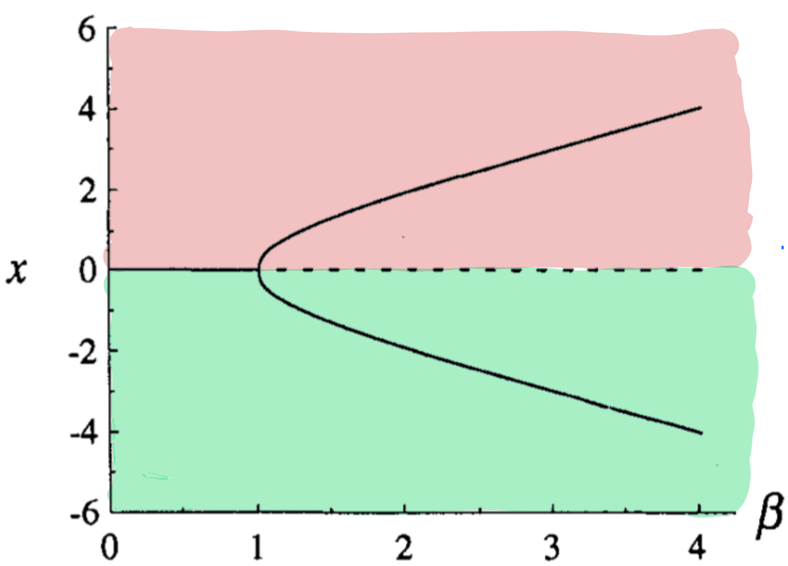
h - g

$$\dot{x} = -x + \beta \tanh(x) = f(x)$$

Ising

encontrar forma normal





Miremos la forma normal de la bifurcación pitchfork supercrítica

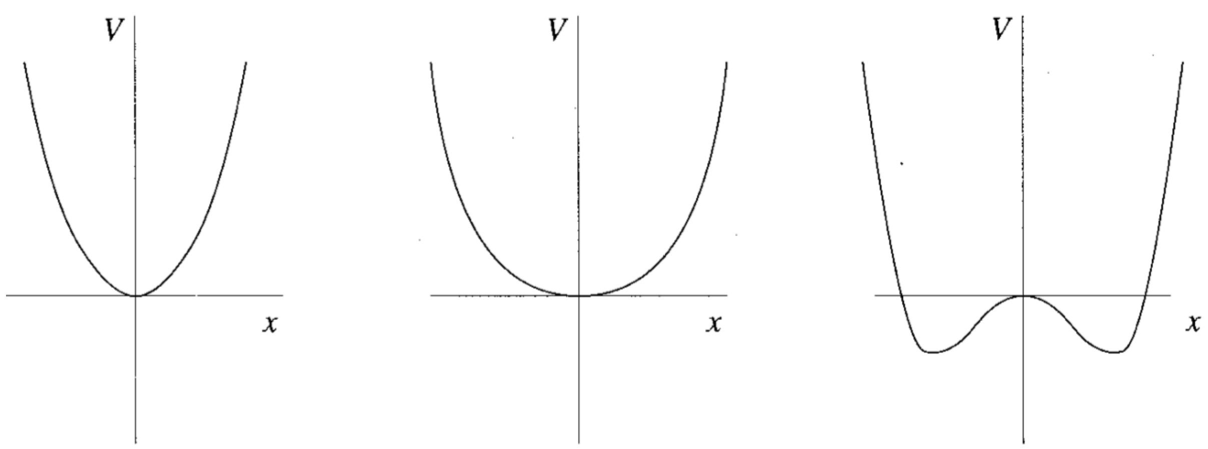
$$\dot{x} = rx - x^3$$

Podemos definir un potencial

$$V(x) = \frac{rx^2}{2} - \frac{1}{4}x^4$$

$$\frac{dV(x)}{dx} = rx - x^3$$

$$f(x) = -\frac{dV}{dx} = -rx + x^3$$

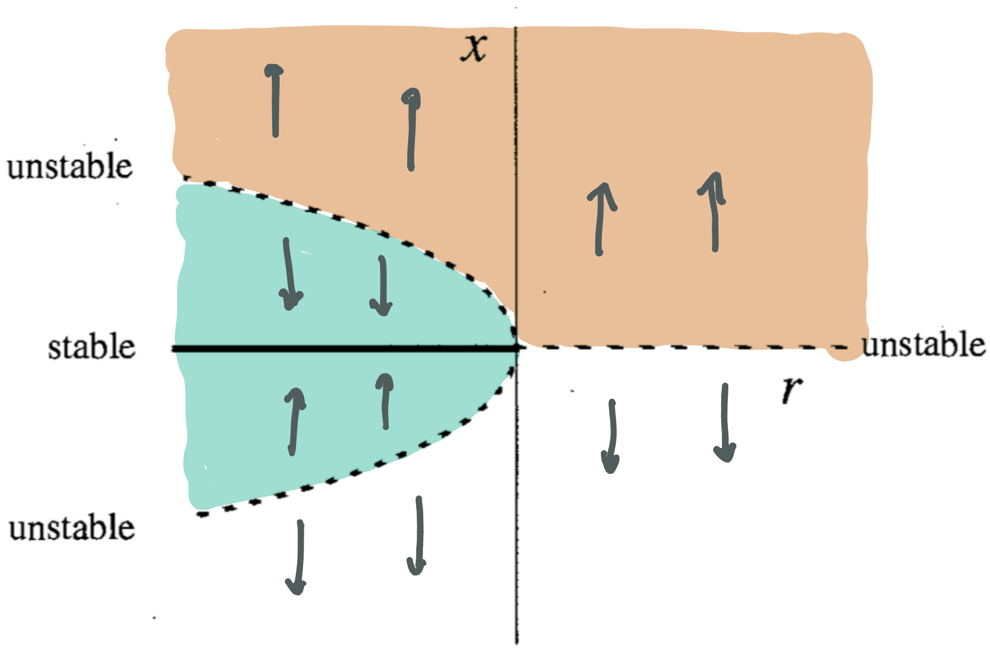


$r<0$ $r=0$ $r>0$

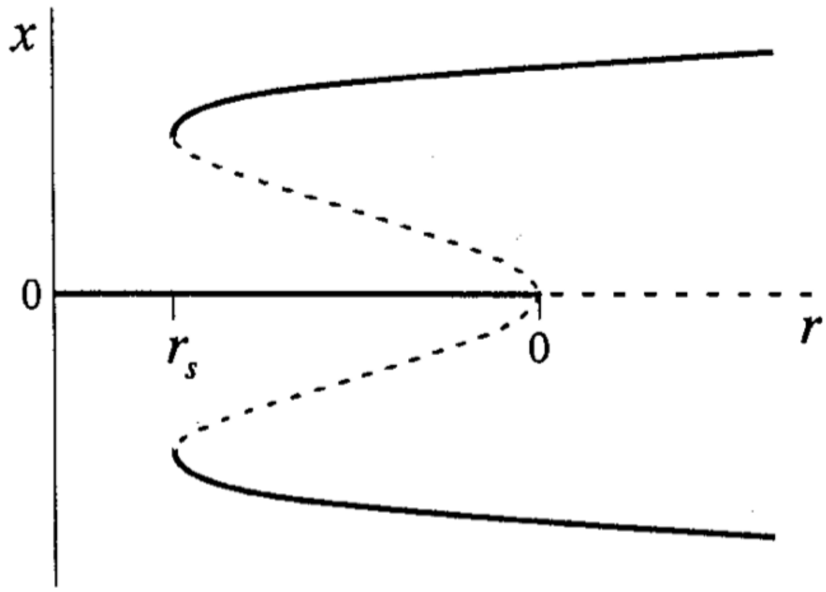
Subcríticas

$$\dot{x} = rx + x^3$$

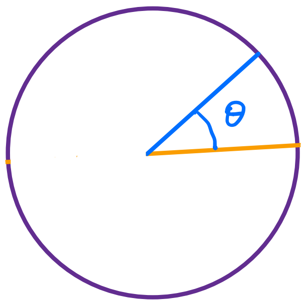
signo que desestabiliza



$$\dot{x} = rx + x^3 - x^5$$



FLUJO EN EL CÍRCULO



$$\dot{\theta} = f(\theta)$$

$\theta(t)$: ángulo

$\dot{\theta}(t)$: velocidad angular

De esta forma podemos tener un sistema 1D oscilante, aunque es claramente tramposo afirmar esto.

$$\dot{\theta} = \omega$$

$$\theta(t) = \omega t + \theta_0 \qquad \theta(t=0) = \theta_0$$

NO INVOLUCRA AMPLITUDES