REDES NEURONALES

2021

Clase 10 Parte 3

Facultad de Matemática, Astronomía, Física y Computación
Universidad Nacional de Córdoba

Jueves 16 de septiembre 2021

http://www.famaf.unc.edu.ar/~ftamarit/redes2021

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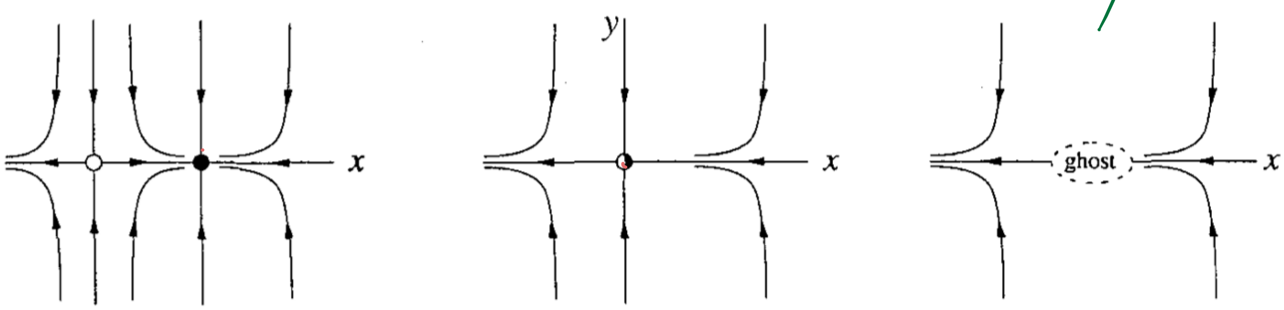
BIFURCACIONES REVISITADAS EN 2D

SADDLE NODE

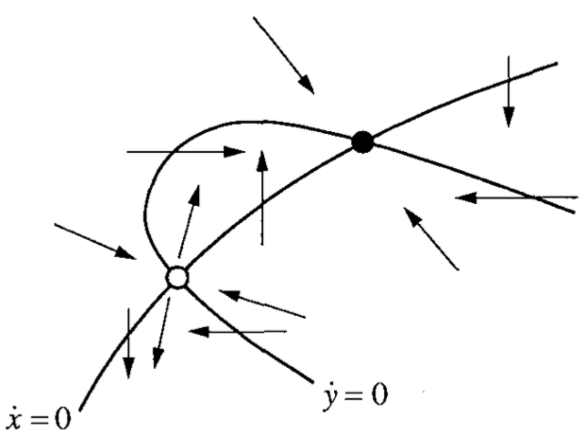
$$\left.\begin{aligned} \dot{x} &= \mu - x^2 \\ \dot{y} &= -y \end{aligned}\right\} \text{forma normal}$$

Punto fijo $(x^*, y^*) = (\sqrt{\mu}, 0)$

$\tau \propto (\mu - \mu_c)^{-1/2}$



$\mu > 0$ $\mu = 0$ $\mu < 0$



$$\dot{x} = -ax + y$$

$$\dot{y} = \frac{x^2}{1+x^2} - by$$

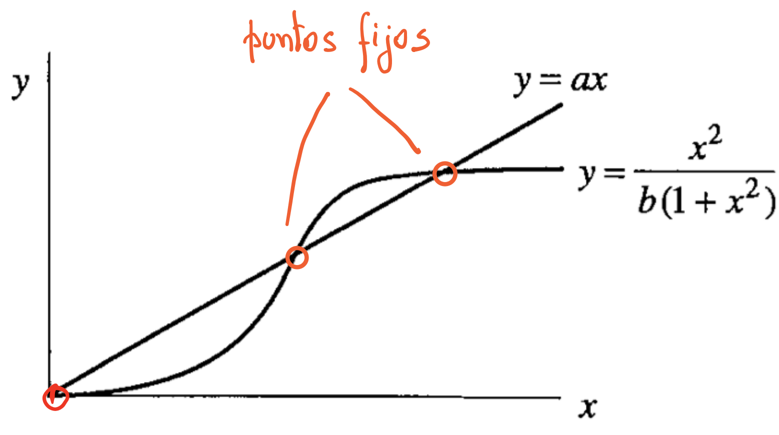
x: concentración de proteína

y: concentración de RNA

Los nullclines son

$$y = ax$$

$$y = \frac{x^2}{b(1+x^2)}$$



Estas curvas se intersectan cuando

$$ax = \frac{x^2}{b(1+x^2)}$$

$x^* = 0$ e $y^* = 0$

La otra solución es

$$ab(1+x^2) = x$$

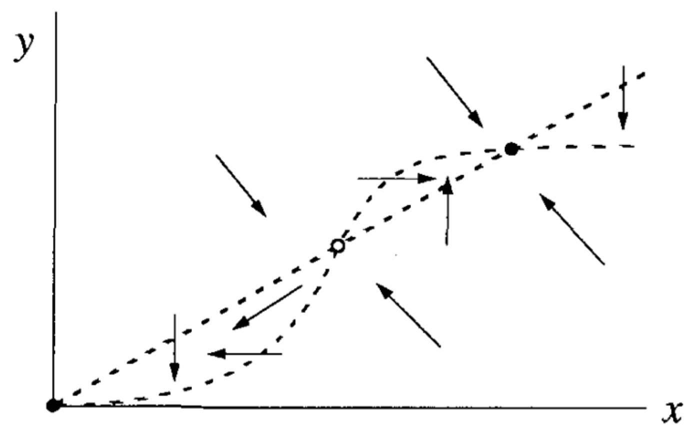
$$x^*_\pm = \frac{1 \pm \sqrt{1-4a^2b^2}}{2ab} \,,$$

si $1-4a^2b^2 > 0$ o $2ab < 1$. Las dos curvas colapsan cuando

$$a_c = \frac{1}{2b}$$

$$x^* = 1$$

$$x^* = \frac{1 \pm \sqrt{1-4\,b^2/4b^2}}{2b/2b} = \frac{1 \pm \sqrt{1-1}}{1} = 1$$



$$A = \begin{pmatrix} -a & 1 \\ \frac{2x}{(1+x^2)^2} & -b \end{pmatrix}$$

$$\tau = -(a+b) < 0$$

$$(x^*, y^*) = (0,0) \Rightarrow \Delta = ab > 0 \qquad \text{nodo estable}$$

$$\tau^2 - 4\Delta = (a-b)^2 > 0$$

Para los otros dos puntos:

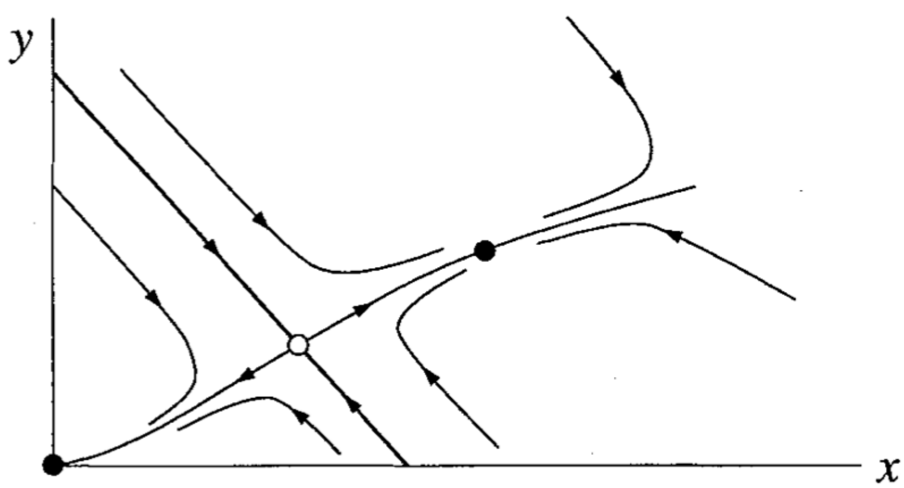
$$\Delta = ab - \frac{2x^*}{(1+(x^*)^2)^2} = ab\left[\frac{(x^*)^2-1}{1+(x^*)^2}\right].$$

Para el punto con $0 < x^* < 1$ (intermedio)

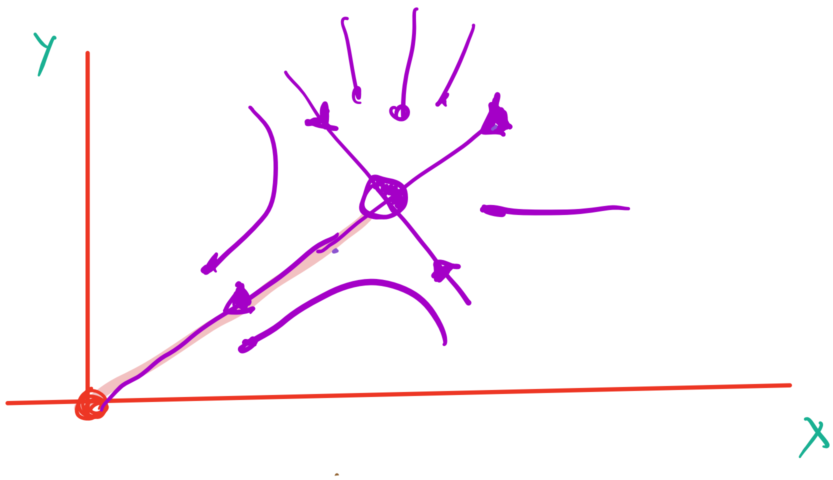
$\Delta < 0$ y el punto fijo es un saddle node.

Para el punto con $x^* > 1$ (derecho)

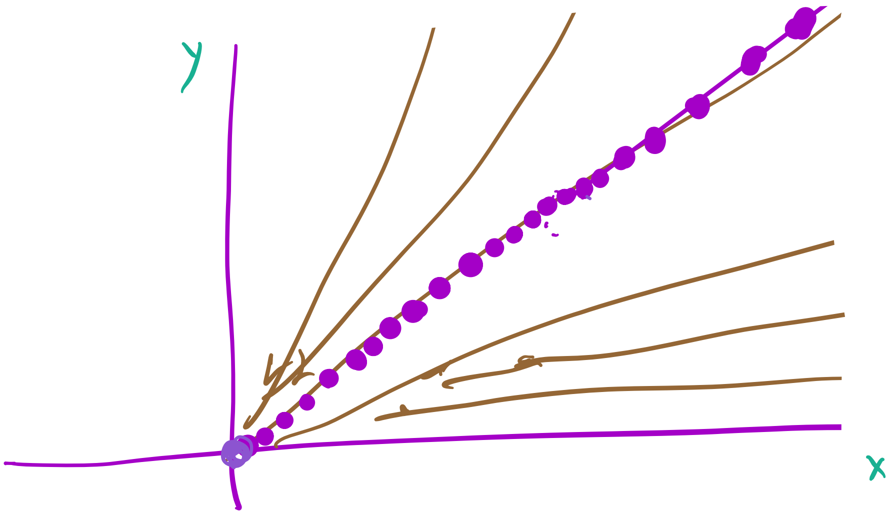
$\Delta > 0$ y el punto fijo es estable.



$\mu < \mu_c$



$\mu = \mu_c$



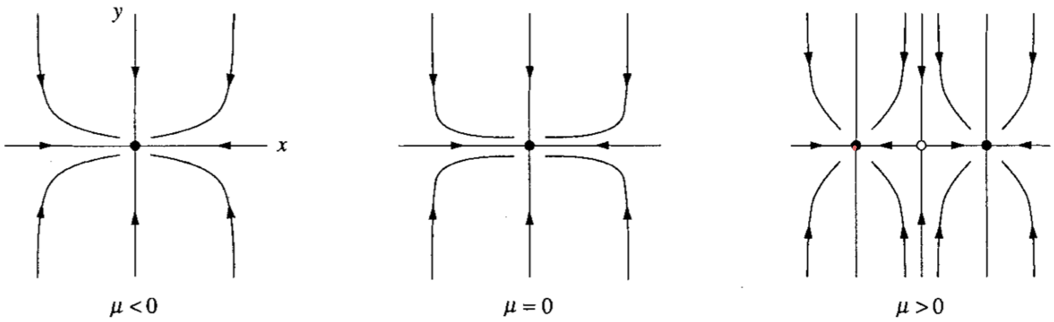
$\mu > \mu_c$

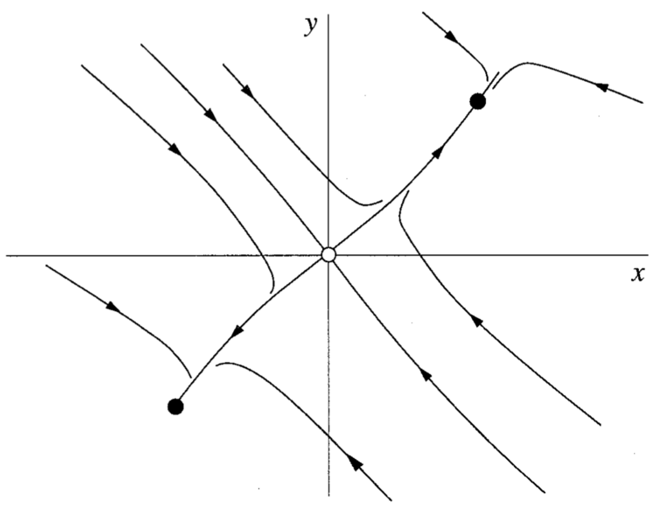
BIFURCACIÓN TRANSCRÍTICA

$$\dot{x} = \mu x - x^2, \qquad \dot{y} = -y$$

BIFURCACIÓN PITCHFORK SUPERCRÍTICA

$$\dot{x} = \mu x - x^3, \qquad \dot{y} = -y$$





Ejemplo

$$\dot{x} = \mu x + y + \operatorname{sen}(x)$$

$$\dot{y} = x - y$$

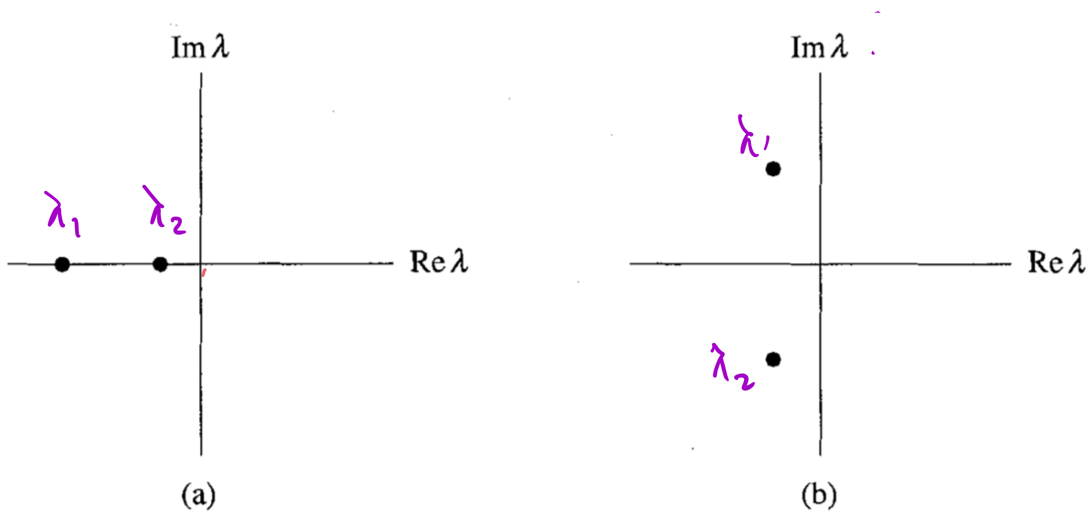
BIFURCACIÓN PITCHFORK SUBCRÍTICA

$$\dot{x} = \mu x + x^3 \,, \qquad \dot{y} = -y$$

BIFURCACIÓN DE HOPF (NUEVO en 2D)

Supongamos que tenemos un punto fijo estable

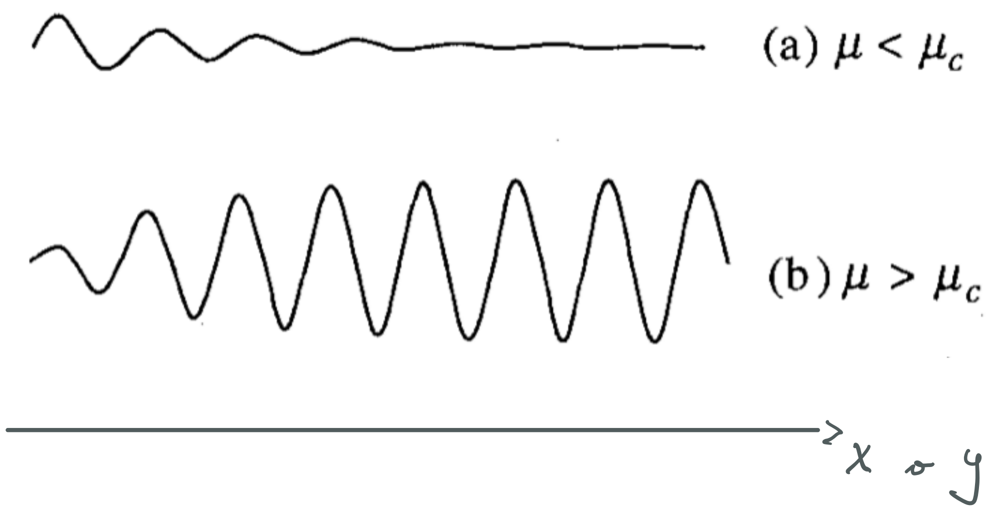
$$\mathrm{Re}(\lambda_1) \leq \mathrm{Re}(\lambda_2) < 0$$

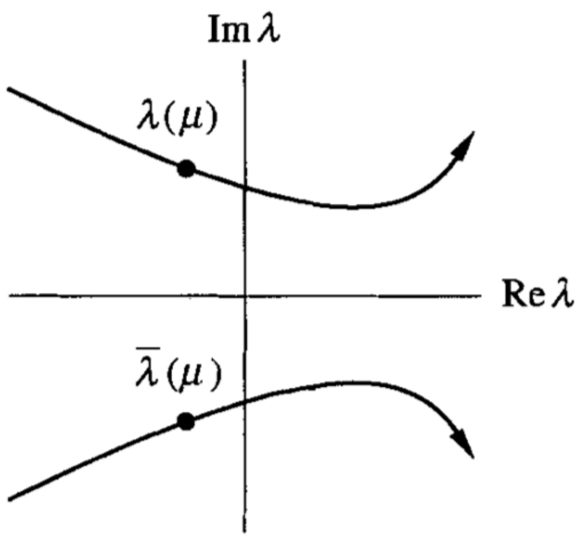


nodos estables   espirales estables

Bifurcación supercrítica de Hopf

Tenemos originalmente un espiral estable





EL parámetro μ controla el pasaje de espiral estable a espiral inestable y un ciclo límite

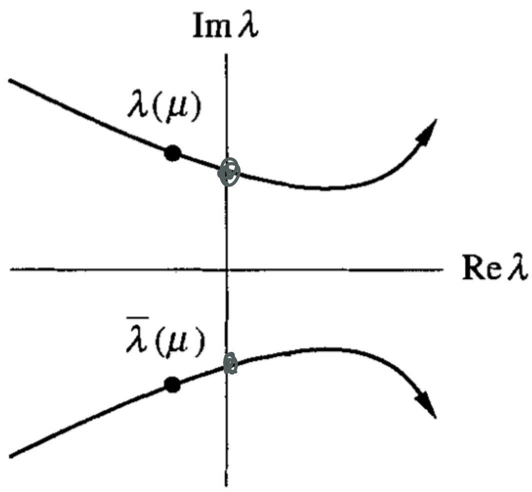
$$
\begin{cases}
\dot{r} = \mu r - r^3 \\
\dot{\theta} = \omega + b r^2 .
\end{cases}
$$

$$
x = r\cos\theta, \; y = r\sin\theta.
$$

$$
\begin{cases}
\begin{aligned}
\dot{x} &= \dot{r}\cos\theta - r\dot{\theta}\sin\theta \\
&= (\mu r - r^3)\cos\theta - r(\omega + b r^2)\sin\theta \\
&= \left(\mu - [x^2 + y^2]\right)x \;-\; \left(\omega + b[x^2 + y^2]\right)y \\
&= \mu x - \omega y + \text{cubic terms}
\end{aligned} \\
\\
\dot{y} = \omega x + \mu y + \text{cubic terms.}
\end{cases}
$$

$$
A = \begin{pmatrix} \mu & -\omega \\ \omega & \mu \end{pmatrix},
$$

$$\lambda = \mu \pm i\omega.$$



Bifurcación de Hopf subcrítica

$$\dot{r} = \mu r + r^3 - r^5$$
$$\dot{\theta} = \omega + br^2.$$