

REDES NEURONALES

2021

Clase 6 Parte 2

Facultad de Matemática, Astronomía, Física y Computación
Universidad Nacional de Córdoba

Jueves 2 de septiembre 2021

<http://www.famaf.unc.edu.ar/~ftamarit/redes2021>

<https://www.famaf.unc.edu.ar/course/view.php?id=798>

Bifurcaciones Pitchfork

Supercríticas

Recordemos que cerca de la bifurcación hacemos un desarrollo en serie de Taylor en las dos variables x y r :

$$\begin{aligned}\dot{x} &= f(x, r) \\ &= \cancel{f(x^*, r_c)} + (x - x^*) \frac{\partial f}{\partial x} \Big|_{(x^*, r_c)} + (r - r_c) \frac{\partial f}{\partial r} \Big|_{(x^*, r_c)} + \frac{1}{2} (x - x^*)^2 \frac{\partial^2 f}{\partial x^2} \Big|_{(x^*, r_c)} + \dots\end{aligned}$$

Si los términos que sobreviven son los siguientes

$$\dot{x} = r x - x^3$$

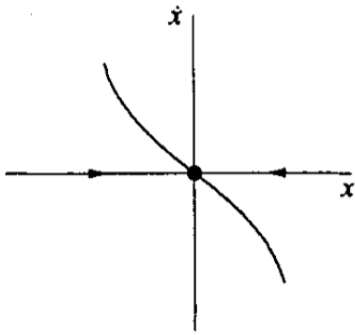
invarianza $x \rightarrow -x$

$$(-\dot{x}) = -(\dot{x}) = -\dot{x} = r(-x) - (-x)^3$$

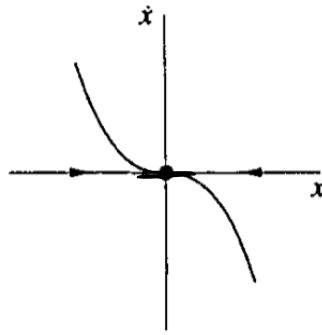
$$-\dot{x} = -r x + x^3$$

$$\dot{x} = r x - x^3$$

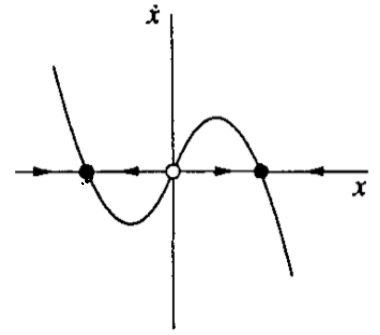
volvemos a lo mismo



(a) $r < 0$



(b) $r = 0$



(c) $r > 0$

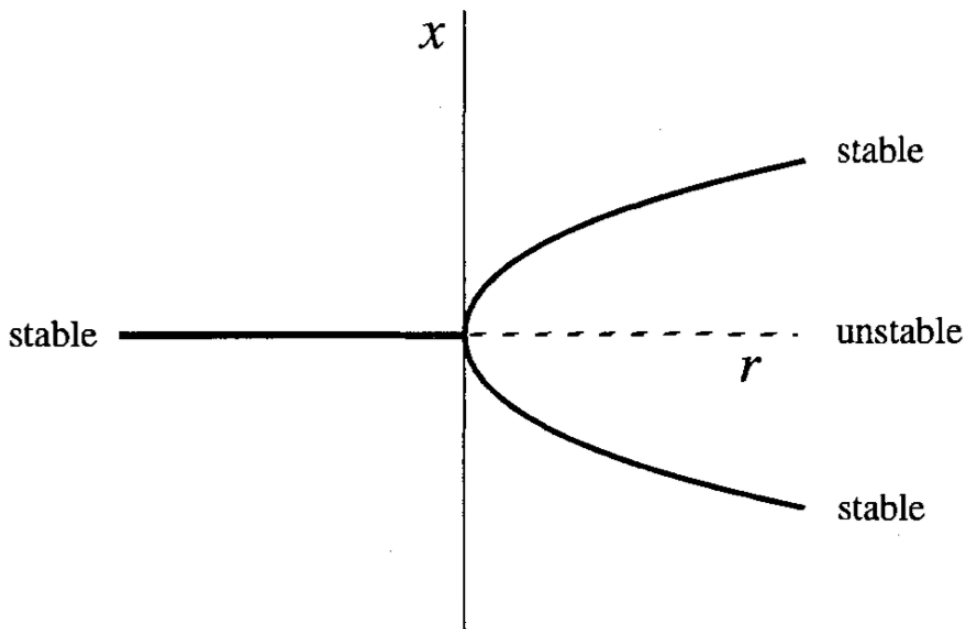
↑
critical
slowing
down

→
ya es una perturbación
no decae en forma
exponencial sino
ley de potencia

Para $r > 0$, tenemos 2 raíces:

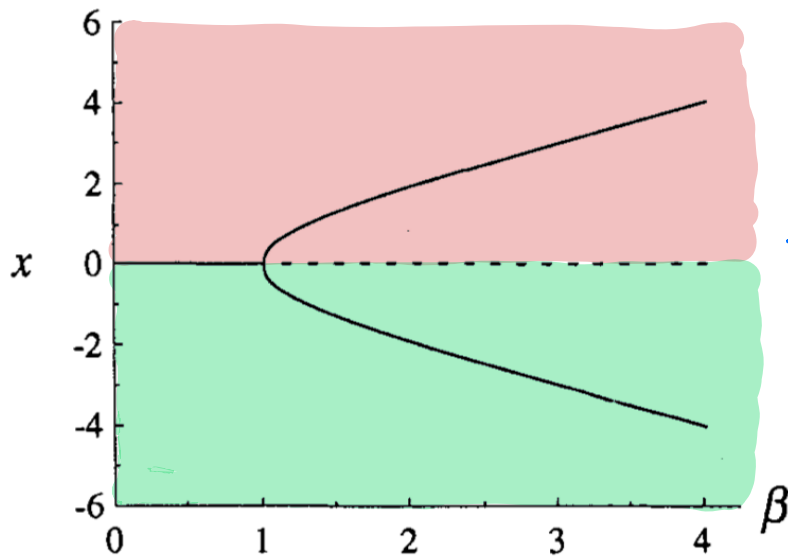
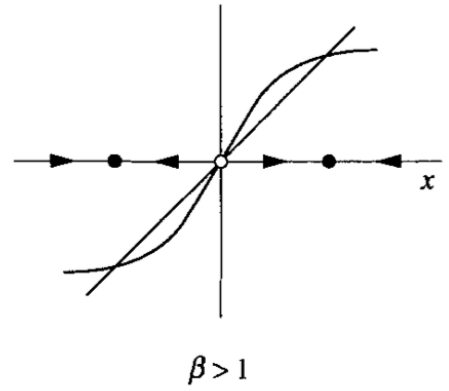
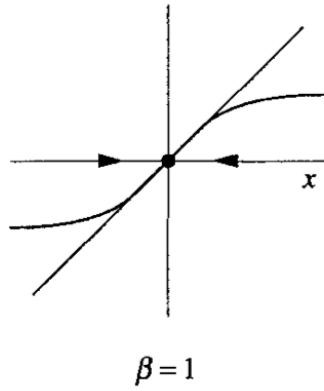
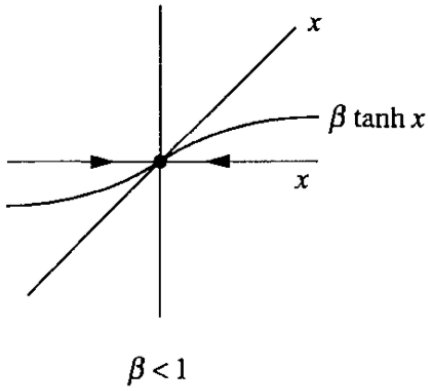
$$x^*_+ = \sqrt{r}$$

$$x^*_- = -\sqrt{r}$$



EJEMPLO

$$\dot{x} = -x + \beta \tanh(x) = f(x)$$



Mostramos la forma normal de la bifurcación pitchfork supercrítica

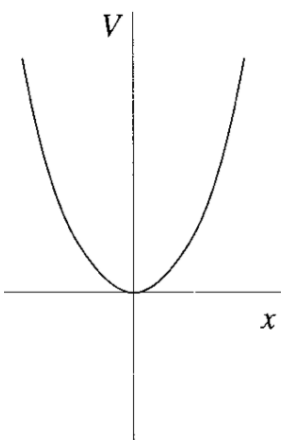
$$\dot{x} = rx - x^3$$

Podemos definir un potencial

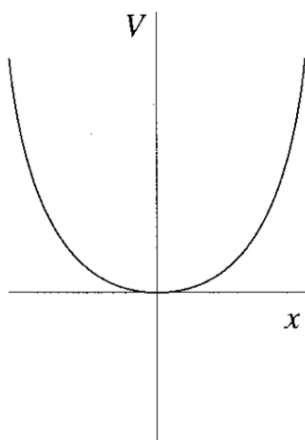
$$V(x) = -\frac{rx^2}{2} + \frac{1}{4}x^4$$

$$\rightarrow \frac{dV(x)}{dx} = rx - x^3$$

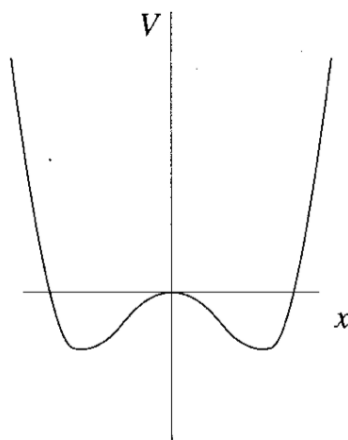
$$f(x) = -\frac{dV}{dx} = rx - x^3$$



$r < 0$



$r = 0$

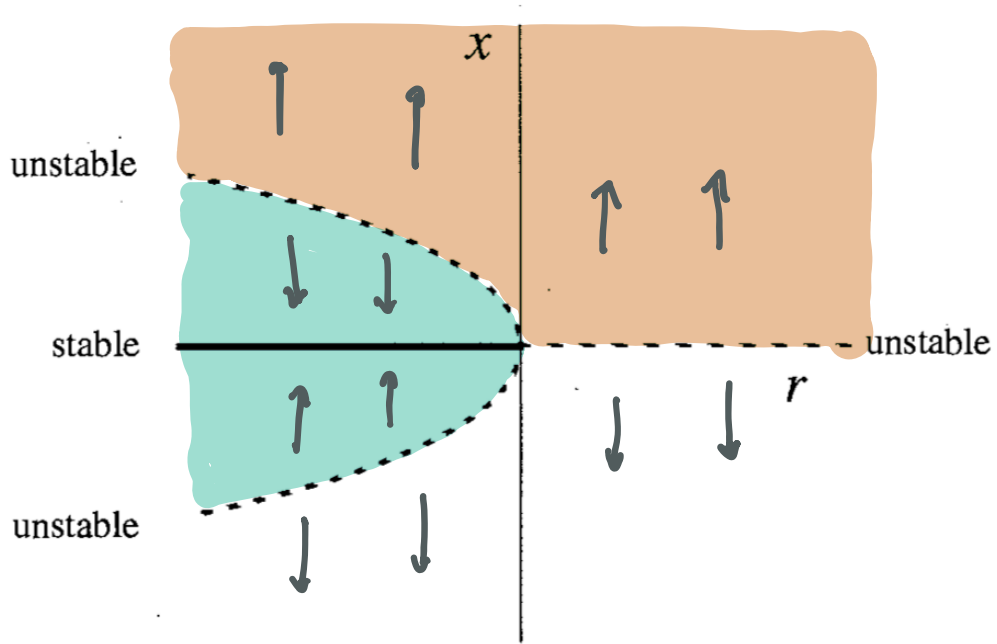


$r > 0$

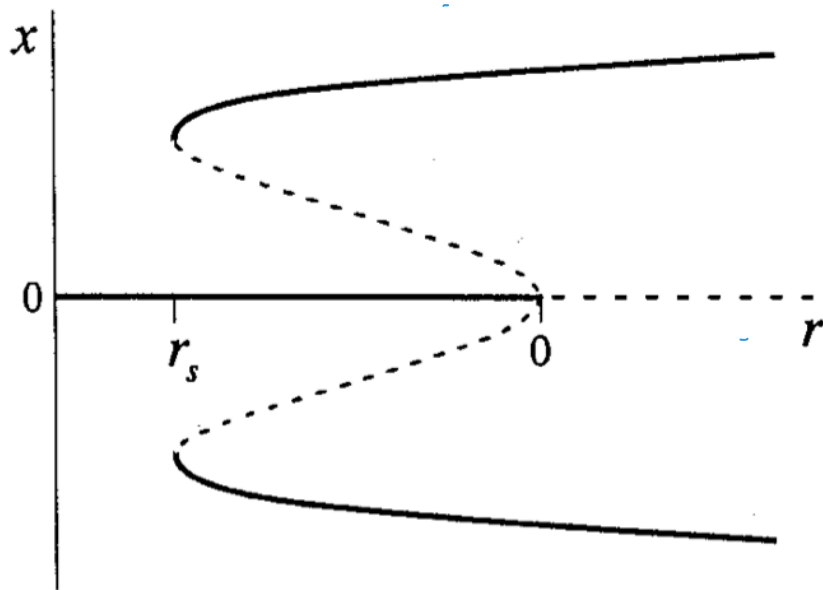
Subcríticas

$$\dot{X} = \Gamma X + X^3$$

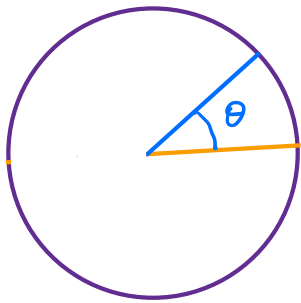
signo que desestabiliza



$$\dot{x} = rx + x^3 - x^5$$



FLUJO EN EL CÍRCULO



$$\dot{\theta} = f(\theta)$$

$\theta(t)$: ángulo

$\dot{\theta}(t)$: velocidad angular

De esta forma podemos tener un sistema 1D oscilante, aunque es claramente tiempo uniforme esto.

$$\dot{\theta} = \omega$$

$$\theta(t) = \omega t + \theta_0 \quad \theta(t=0) = \theta_0$$

NO INVOLUCRA AMPLITUDES