

# Diferencias divididas

## Polinomio de newton

$$P_n(x) = \sum_{i=0}^n f[x_0, \dots, x_i] Q_i(x).$$

$$P_n(x) = f[x_0] + f[x_0, x_1] Q_1(x) + f[x_0, \dots, x_2] Q_2(x) + \\ + f[x_0, \dots, x_3] Q_3(x) + \dots + f[x_0, \dots, x_n] Q_n(x).$$

## Propiedad recursiva

$$f[x_0, \dots, x_k] = \frac{f[x_1, \dots, x_k] - f[x_0, \dots, x_{k-1}]}{x_0 - x_k}$$

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## Propiedad recursiva

$$f[x_0, \dots, x_k] = \frac{f[x_1, \dots, x_k] - f[x_0, \dots, x_{k-1}]}{x_0 - x_k}$$

$$x_0 \quad f[x_0]$$

$$x_1 \quad f[x_1]$$

$$x_2 \quad f[x_2]$$

$$x_3 \quad f[x_3]$$

$$x_4 \quad f[x_4]$$

$$x_5 \quad f[x_5]$$

orden 0

$$x_0 \quad f[x_0]$$

$$f[x_0, x_1] = \frac{f[x_0] - f[x_1]}{x_0 - x_1}$$

$$x_1 \quad f[x_1]$$

$$f[x_1, x_2] = \frac{f[x_1] - f[x_2]}{x_1 - x_2}$$

$$x_2 \quad f[x_2]$$

$$f[x_2, x_3] = \frac{f[x_2] - f[x_3]}{x_2 - x_3}$$

$$x_3 \quad f[x_3]$$

$$f[x_3, x_4] = \frac{f[x_4] - f[x_3]}{x_4 - x_3}$$

$$x_4 \quad f[x_4]$$

$$f[x_4, x_5] = \frac{f[x_4] - f[x_5]}{x_4 - x_5}$$

$$x_5 \quad f[x_5]$$

orden 0

orden 1

$$x_0 \quad f[x_0]$$

$$f[x_0, x_1]$$

$$x_1 \quad f[x_1]$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$f[x_1, x_2]$$

$$x_2 \quad f[x_2]$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$

$$f[x_2, x_3]$$

$$x_3 \quad f[x_3]$$

$$f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}$$

$$f[x_3, x_4]$$

$$x_4 \quad f[x_4]$$

$$f[x_3, x_4, x_5] = \frac{f[x_4, x_5] - f[x_3, x_4]}{x_5 - x_3}$$

$$f[x_4, x_5]$$

$$x_5 \quad f[x_5]$$

orden 0

orden 1

orden 2

$x_0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1$	$f[x_1]$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] = \dots$
$x_2$	$f[x_2]$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4] = \dots$
$x_3$	$f[x_3]$		$f[x_2, x_3, x_4]$	
		$f[x_3, x_4]$		$f[x_2, x_3, x_4, x_5] = \dots$
$x_4$	$f[x_4]$		$f[x_3, x_4, x_5]$	
		$f[x_4, x_5]$		
$x_5$	$f[x_5]$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$			
$x_0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1$	$f[x_1]$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] = \dots$
$x_2$	$f[x_2]$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4] = \dots$
$x_3$	$f[x_3]$		$f[x_2, x_3, x_4]$	
		$f[x_3, x_4]$		$f[x_2, x_3, x_4, x_5] = \dots$
$x_4$	$f[x_4]$		$f[x_3, x_4, x_5]$	
		$f[x_4, x_5]$		
$x_5$	$f[x_5]$			
	orden 0	orden 1	orden 2	orden 3



$x(:)$	$A(:,1)$	$A(:,2)$		
$x_0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1$	$f[x_1]$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] = \dots$
$x_2$	$f[x_2]$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4] = \dots$
$x_3$	$f[x_3]$		$f[x_2, x_3, x_4]$	
		$f[x_3, x_4]$		$f[x_2, x_3, x_4, x_5] = \dots$
$x_4$	$f[x_4]$		$f[x_3, x_4, x_5]$	
		$f[x_4, x_5]$		
$x_5$	$f[x_5]$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	
$x_0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1$	$f[x_1]$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] = \dots$
$x_2$	$f[x_2]$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4] = \dots$
$x_3$	$f[x_3]$		$f[x_2, x_3, x_4]$	
		$f[x_3, x_4]$		$f[x_2, x_3, x_4, x_5] = \dots$
$x_4$	$f[x_4]$		$f[x_3, x_4, x_5]$	
		$f[x_4, x_5]$		
$x_5$	$f[x_5]$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	$A(:,4)$
$x_0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1$	$f[x_1]$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3] = \dots$
$x_2$	$f[x_2]$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4] = \dots$
$x_3$	$f[x_3]$		$f[x_2, x_3, x_4]$	
		$f[x_3, x_4]$		$f[x_2, x_3, x_4, x_5] = \dots$
$x_4$	$f[x_4]$		$f[x_3, x_4, x_5]$	
		$f[x_4, x_5]$		
$x_5$	$f[x_5]$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	$A(:,4)$
$x(1)$	$A(1,1)$	$f[x_0, x_1]$		
$x(2)$	$A(2,1)$	$f[x_1, x_2]$	$f[x_0, x_1, x_2]$	$f[x_0, x_1, x_2, x_3] = \dots$
$x(3)$	$A(3,1)$	$f[x_2, x_3]$	$f[x_1, x_2, x_3]$	$f[x_1, x_2, x_3, x_4] = \dots$
$x(4)$	$A(4,1)$	$f[x_3, x_4]$	$f[x_2, x_3, x_4]$	$f[x_2, x_3, x_4, x_5] = \dots$
$x(5)$	$A(5,1)$	$f[x_4, x_5]$	$f[x_3, x_4, x_5]$	
$x(6)$	$A(6,1)$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	$A(:,4)$
$x(1)$	$A(1,1)$	$A(1,2) = \frac{A(1,1) - A(2,1)}{x(1) - x(2)}$		
$x(2)$	$A(2,1)$	$A(2,2) = \frac{A(2,1) - A(3,1)}{x(2) - x(3)}$	$f[x_0, x_1, x_2]$	$f[x_0, x_1, x_2, x_3] = \dots$
$x(3)$	$A(3,1)$	$A(3,2) = \frac{A(3,1) - A(4,1)}{x(3) - x(4)}$	$f[x_1, x_2, x_3]$	$f[x_1, x_2, x_3, x_4] = \dots$
$x(4)$	$A(4,1)$	$A(4,2) = \frac{A(4,1) - A(5,1)}{x(4) - x(5)}$	$f[x_2, x_3, x_4]$	$f[x_2, x_3, x_4, x_5] = \dots$
$x(5)$	$A(5,1)$	$A(5,2) = \frac{A(5,1) - A(6,1)}{x(5) - x(6)}$	$f[x_3, x_4, x_5]$	
$x(6)$	$A(6,1)$			
	orden 0	orden 1	orden 2	orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	$A(:,4)$
$x(1)$	$A(1,1)$	$A(1,2) = \frac{A(1,1) - A(2,1)}{x(1) - x(2)}$	$A(1,3) = \frac{A(1,2) - A(2,2)}{x(1) - x(3)}$	$f[x_0, x_1, x_2, x_3]$
$x(2)$	$A(2,1)$	$A(2,2) = \frac{A(2,1) - A(3,1)}{x(2) - x(3)}$	$A(2,3) = \frac{A(2,2) - A(3,2)}{x(2) - x(4)}$	$f[x_1, x_2, x_3, x_4]$
$x(3)$	$A(3,1)$	$A(3,2) = \frac{A(3,1) - A(4,1)}{x(3) - x(4)}$	$A(3,3) = \frac{A(3,2) - A(4,2)}{x(3) - x(5)}$	$f[x_2, x_3, x_4, x_5]$
$x(4)$	$A(4,1)$	$A(4,2) = \frac{A(4,1) - A(5,1)}{x(4) - x(5)}$	$A(4,3) = \frac{A(4,2) - A(5,2)}{x(4) - x(6)}$	
$x(5)$	$A(5,1)$	$A(5,2) = \frac{A(5,1) - A(6,1)}{x(5) - x(6)}$		
$x(6)$	$A(6,1)$			

orden 0

orden 1

orden 2

orden 3

$x(:)$	$A(:,1)$	$A(:,2)$	$A(:,3)$	$A(:,4)$
$x(1)$	$A(1,1)$	$A(1,2) = \frac{A(1,1) - A(2,1)}{x(1) - x(2)}$	$A(1,3) = \frac{A(1,2) - A(2,2)}{x(1) - x(3)}$	$A(1,4) = \dots$
$x(2)$	$A(2,1)$	$A(2,2) = \frac{A(2,1) - A(3,1)}{x(2) - x(3)}$	$A(2,3) = \frac{A(2,2) - A(3,2)}{x(2) - x(4)}$	$A(2,4) = \dots$
$x(3)$	$A(3,1)$	$A(3,2) = \frac{A(3,1) - A(4,1)}{x(3) - x(4)}$	$A(3,3) = \frac{A(3,2) - A(4,2)}{x(3) - x(5)}$	$A(3,4) = \dots$
$x(4)$	$A(4,1)$	$A(4,2) = \frac{A(4,1) - A(5,1)}{x(4) - x(5)}$	$A(4,3) = \frac{A(4,2) - A(5,2)}{x(4) - x(6)}$	
$x(5)$	$A(5,1)$	$A(5,2) = \frac{A(5,1) - A(6,1)}{x(5) - x(6)}$		
$x(6)$	$A(6,1)$			

orden 0

orden 1

orden 2

orden 3

$$A(k,j) = \frac{A(k,j-1) - A(k+1,j-1)}{x(k) - x(k+j-1)}$$

```
function w=diferencias(x,y)
n=columns(x);
A=zeros(n,n);
A(:,1)=y';
w=zeros(1,n);
w(1)=y(1);
for j=2:n
    for k=1:n-j+1
        A(k,j)=(A(k,j-1)-A(k+1,j-1))/(x(k)-x(k+j-1));
    end
    w(j)=A(1,j);
end
endfunction
```



$$A(k,j) = \frac{A(k,j-1) - A(k+1,j-1)}{x(k) - x(k+j-1)}$$

```
function w=diferencias(x,y)
n=columns(x);
A=zeros(n,n);
A(:,1)=y';
w=zeros(1,n);
w(1)=y(1);
for j=2:n
    for k=1:n-j+1
        A(k,j)=(A(k,j-1)-A(k+1,j-1))/(x(k)-x(k+j-1));
    end
    w(j)=A(1,j);
end
endfunction
```

```
function pz=evalnewton(c,x,z)
k=columns(c);
pz=zeros(size(z));
for j=1:k
    wj=c(j)*ones(size(z));
    for i=1:j-1
        wj=wj.*(z-x(i));
    end
    pz=pz+wj;
end
end
```

# Multiplicación encajada - Método de Horner

$$p(x) = \sum_{i=0}^n a_i x^i.$$

$$\begin{aligned}
 p(x) &= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n \\
 &= a_0 + x(a_1 + a_2x + a_3x^2 + \cdots + a_nx^{n-1}) \\
 &= a_0 + x(a_1 + x(a_2 + a_3x + \cdots + a_nx^{n-2})) \\
 &= a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + a_nx^{n-3}))) \\
 &\vdots \\
 &= a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))
 \end{aligned}$$

# Multiplicación encajada - Método de Horner

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$$\begin{aligned}
 p(x) &= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n \\
 &= a_0 + x(a_1 + a_2x + a_3x^2 + \cdots + a_nx^{n-1}) \\
 &= a_0 + x(a_1 + x(a_2 + a_3x + \cdots + a_nx^{n-2})) \\
 &= a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + a_nx^{n-3}))) \\
 &\quad \vdots \\
 &= a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))
 \end{aligned}$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x$$

$$p(x) =$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x$$

$$p(x) =$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(m-1) + x))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(m-1) + xa(m)))))$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(m-1) + xa(m))))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(m-1) + xw(m))))))$$



$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(m-1) + xw(m))))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + xw(m-1))))$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + xw(5))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + xw(4)))$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + xw(4)))$$

$$p(x) = a(1) + x(a(2) + xw(3))$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + x(a(2) + xw(3))$$

$$p(x) = a(1) + xw(2)$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + xw(2)$$

$$p(x) = w(1)$$

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n))))$$

$$p(x) = a(1) + x(a(2) + x(a(3) + x(a(4) + \cdots + x(a(n) + xa(n+1)))))$$

$$p(x) = a(1) + xw(2)$$

$$p(x) = w(1)$$

$$w(k) = a(k) + xw(k+1)$$

```
function pz=horner(a,z)
m=columns(a);
pz=0*z;
for j=m:-1:1
    pz=a(j)+pz.*z;
end
endfunction
```

# Multiplicación encajada para el método Newton

$$P_n(x) = \sum_{i=0}^n C_i Q_i(x).$$

$$Q_i(x) = Q_{i-1}(x)(x - x_i)$$

$$P_n(x) = c_0 + (x - x_1)(c_1 + (x - x_2)(c_2 + (x - x_3)(c_3 + \dots)))$$



# Multiplicación encajada para el método Newton

$$P_n(x) = \sum_{i=0}^n C_i Q_i(x).$$

$$Q_i(x) = Q_{i-1}(x)(x - x_i)$$

$$P_n(x) = c_0 + (x - x_1) (c_1 + (x - x_2) (c_2 + (x - x_3) (c_3 + \dots)))$$

# Multiplicación encajada para el método Newton

$$P_n(x) = \sum_{i=0}^n C_i Q_i(x).$$

$$Q_i(x) = Q_{i-1}(x)(x - x_i)$$

$$P_n(x) = c_0 + (x - x_1) (c_1 + (x - x_2) (c_2 + (x - x_3) (c_3 + \cdots)))$$

## newtonh.m

```
function pz=horner(c,x,z)
m=columns(c);
pz=0*z;
for j=m:-1:1
    pz=c(j)+pz.*(z-x(j));
end
endfunction
```

# Errores y tiempos de ejecución en Newton

Interpolarse la función  $f(x) = x^k$  en el intervalo  $[0,001, 1]$  usando los puntos  $[0,001 (1 : k)/k]$ . Calcular el valor de  $P_k(1)$ . Graficar los valores obtenidos para  $k = 1 : 200$ . Analizar los resultados obtenidos.

Interpolarse la función  $f(x) = x^k$  en el intervalo  $[0,001, 1]$  usando los puntos  $[0,001 (1 : k)/k]$ . Calcular el valor de  $P_k(0)$ . Graficar los valores obtenidos para  $k = 1 : 200$ . Analizar los resultados obtenidos.

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