On the group-cograded multiplier Hopf algberas

Shuanhong Wang

Department of Mathematics,
Southeast University, Nanjing, China

September 18, 2009/Cordoba

Contents

1 1. Definition and Examples

2 2. Characterization and Properties

3 3. Drinfeld double

4 4. Quasitriangular Structure

5 References

Definition and Examples

Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

Definition and Examples
Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

Definition and Examples

Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

- (i) $A = \bigoplus_{p \in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

Definition and Examples

Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

- (i) $A = \bigoplus_{p \in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

Definition and Examples

Definition

Definition Let G be a group and let (A,Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p\in G}$ of A so that

- (i) $A=\bigoplus_{p\in G}A_p$ with $A_pA_q=0$ whenever $p,q\in G$ and $p\neq q$;
- (ii) $\Delta(A_{pq})(1\otimes A_q)=A_p\otimes A_q$ and $(A_p\otimes 1)\Delta(A_{pq})=A_p\otimes A_q$ for all $p,q\in G$. Then we call (A,Δ) a G-cograded multiplier Hopf algebra.

Definition and Examples
Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p\in G}$ of A so that

- (i) $A = \bigoplus_{p\in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;
- (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

Definition and Examples

Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

• (i) $A = \bigoplus_{p \in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

• (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

If (A, Δ) is a multiplier Hopf $*$-algebra, we will assume that all these subalgebras are $*$-subalgebras and call it a G-cograded multiplier Hopf $*$-algebra.

Definition and Examples
Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p \in G}$ of A so that

• (i) $A = \bigoplus_{p \in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

• (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

If (A, Δ) is a multiplier Hopf $*$-algebra, we will assume that all these subalgebras are $*$-subalgebras and call it a G-cograded multiplier Hopf $*$-algebra.

Definition and Examples

Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p\in G}$ of A so that

• (i) $A = \bigoplus_{p\in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

• (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

If (A, Δ) is a multiplier Hopf $*$-algebra, we will assume that all these subalgebras are $*$-subalgebras and call it a G-cograded multiplier Hopf $*$-algebra.

Definition and Examples
Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p\in G}$ of A so that

• (i) $A = \bigoplus_{p\in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

• (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

If (A, Δ) is a multiplier Hopf $*$-algebra, we will assume that all these subalgebras are $*$-subalgebras and call it a G-cograded multiplier Hopf $*$-algebra.

Definition and Examples
Definition

Definition Let G be a group and let (A, Δ) be a multiplier Hopf algebra.

Assume that there is a family of (non-trivial) subalgebras $(A_p)_{p\in G}$ of A so that

• (i) $A = \bigoplus_{p\in G} A_p$ with $A_p A_q = 0$ whenever $p, q \in G$ and $p \neq q$;

• (ii) $\Delta(A_{pq})(1 \otimes A_q) = A_p \otimes A_q$ and $(A_p \otimes 1)\Delta(A_{pq}) = A_p \otimes A_q$ for all $p, q \in G$. Then we call (A, Δ) a G-cograded multiplier Hopf algebra.

If (A, Δ) is a multiplier Hopf $*$-algebra, we will assume that all these subalgebras are $*$-subalgebras and call it a G-cograded multiplier Hopf $*$-algebra.

Definition and Examples
Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Definition and Examples
Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Definition and Examples

Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Consider now the direct sum B of these algebras. It is an algebra, without identity, except when G is a finite group, but the product is non-degenerate. The maps $\Delta_{p,q}$ can be used to define a coproduct Δ on B and the conditions imposed on these maps give that (A, Δ) is a multiplier Hopf algebra. It is G-cograded mha.

Definition and Examples
Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Consider now the direct sum B of these algebras. It is an algebra, without identity, except when G is a finite group, but the product is non-degenerate. The maps $\Delta_{p,q}$ can be used to define a coproduct Δ on B and the conditions imposed on these maps give that (A, Δ) is a multiplier Hopf algebra. It is G-cograded mha.

Definition and Examples
Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Consider now the direct sum B of these algebras. It is an algebra, without identity, except when G is a finite group, but the product is non-degenerate. The maps $\Delta_{p,q}$ can be used to define a coproduct Δ on B and the conditions imposed on these maps give that (A, Δ) is a multiplier Hopf algebra. It is G-cograded mha.

Definition and Examples

Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Consider now the direct sum B of these algebras. It is an algebra, without identity, except when G is a finite group, but the product is non-degenerate. The maps $\Delta_{p,q}$ can be used to define a coproduct Δ on B and the conditions imposed on these maps give that (A, Δ) is a multiplier Hopf algebra. It is G-cograded mha.

Definition and Examples
Examples

Examples (1) (Motivating example) Let G be a group and assume that $(A_p)_{p\in G}$ is a family of algebras with identity.

We call $A = (A_p)_{p\in G}$ a Hopf G-coalgebra in the sense of Turaev if, for each pair $p, q \in G$, there is given a unital homomorphism $\Delta_{p,q} : A_{pq} \longrightarrow A_p \otimes A_q$ satisfying certain properties.

Consider now the direct sum B of these algebras. It is an algebra, without identity, except when G is a finite group, but the product is non-degenerate. The maps $\Delta_{p,q}$ can be used to define a coproduct Δ on B and the conditions imposed on these maps give that (A, Δ) is a multiplier Hopf algebra. It is G-cograded mha.

Definition and Examples
Examples

Examples (2) (Trivial example, but it is important.) Let G be a group. Let A be $K(G)$, the algebra of complex functions with finite support on G (with pointwise sum and product). The product in G gives rise to a coproduct Δ_G on $K(G)$ defined by $(\Delta_G(f))(p,q) = f(pq)$ where $f \in K(G)$ and $p,q \in G$.

Notice that Δ_G maps $K(G)$ into the multiplier algebra $M(K(G)\otimes K(G))$ where first $K(G)\otimes K(G)$ is identified with $K(G\times G)$ and then $M(K(G)\otimes K(G))$ is identified with the algebra of all complex functions on $G\times G$. With this coproduct, $K(G)$ is a multiplier Hopf algebra. It is trivially G-cograded. In fact, it is a G-cograded multiplier Hopf $*$-algebra if we take the complex conjugate of a function as the involution.

Definition and Examples
Examples

Examples (2) (Trivial example, but it is important.) Let G be a group. Let A be $K(G)$, the algebra of complex functions with finite support on G (with pointwise sum and product). The product in G gives rise to a coproduct Δ_G on $K(G)$ defined by $(\Delta_G(f))(p,q) = f(pq)$ where $f \in K(G)$ and $p, q \in G$.

Notice that Δ_G maps $K(G)$ into the multiplier algebra $M(K(G)\otimes K(G))$ where first $K(G)\otimes K(G)$ is identified with $K(G\times G)$ and then $M(K(G)\otimes K(G))$ is identified with the algebra of all complex functions on $G\times G$. With this coproduct, $K(G)$ is a multiplier Hopf algebra. It is trivially G-cograded. In fact, it is a G-cograded multiplier Hopf $*$-algebra if we take the complex conjugate of a function as the involution.

Definition and Examples
Examples

Examples (2) (Trivial example, but it is important.) Let G be a group. Let A be $K(G)$, the algebra of complex functions with finite support on G (with pointwise sum and product). The product in G gives rise to a coproduct Δ_G on $K(G)$ defined by $(\Delta_G(f))(p,q) = f(pq)$ where $f \in K(G)$ and $p,q \in G$.

Notice that Δ_G maps $K(G)$ into the multiplier algebra $M(K(G) \otimes K(G))$ where first $K(G) \otimes K(G)$ is identified with $K(G \times G)$ and then $M(K(G) \otimes K(G))$ is identified with the algebra of all complex functions on $G \times G$. With this coproduct, $K(G)$ is a multiplier Hopf algebra. It is trivially G-cograded. In fact, it is a G-cograded multiplier Hopf $*$-algebra if we take the complex conjugate of a function as the involution.

Definition and Examples
Examples

Examples (2) (Trivial example, but it is important.) Let G be a group. Let A be $K(G)$, the algebra of complex functions with finite support on G (with pointwise sum and product). The product in G gives rise to a coproduct Δ_G on $K(G)$ defined by $(\Delta_G(f))(p,q) = f(pq)$ where $f \in K(G)$ and $p,q \in G$.

Notice that Δ_G maps $K(G)$ into the multiplier algebra $M(K(G)\otimes K(G))$ where first $K(G) \otimes K(G)$ is identified with $K(G \times G)$ and then $M(K(G)\otimes K(G))$ is identified with the algebra of all complex functions on $G \times G$. With this coproduct, $K(G)$ is a multiplier Hopf algebra. It is trivially G-cograded. In fact, it is a G-cograded multiplier Hopf $*$-algebra if we take the complex conjugate of a function as the involution.

Definition and Examples
Examples

Examples (2) (Trivial example, but it is important.) Let G be a group. Let A be $K(G)$, the algebra of complex functions with finite support on G (with pointwise sum and product). The product in G gives rise to a coproduct Δ_G on $K(G)$ defined by $(\Delta_G(f))(p,q) = f(pq)$ where $f \in K(G)$ and $p,q \in G$.

Notice that Δ_G maps $K(G)$ into the multiplier algebra $M(K(G)\otimes K(G))$ where first $K(G) \otimes K(G)$ is identified with $K(G \times G)$ and then $M(K(G)\otimes K(G))$ is identified with the algebra of all complex functions on $G \times G$. With this coproduct, $K(G)$ is a multiplier Hopf algebra. It is trivially G-cograded. In fact, it is a G-cograded multiplier Hopf $*$-algebra if we take the complex conjugate of a function as the involution.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Definition and Examples

Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Now, we consider again the algebra A, defined as $K(G) \otimes A_0$ with the tensor product algebra structure.

Definition and Examples

Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Now, we consider again the algebra A, defined as $K(G) \otimes A_0$ with the tensor product algebra structure.

Definition and Examples

Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Now, we consider again the algebra A, defined as $K(G) \otimes A_0$ with the tensor product algebra structure.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Now, we consider again the algebra A, defined as $K(G) \otimes A_0$ with the tensor product algebra structure.

Definition and Examples
Examples

Examples (3) Let (A_0, Δ_0) be a multiplier Hopf algebra. Let G be a group and assume that G acts on the multiplier Hopf algebra A_0 in the following sense.

For each $p \in G$, we have an automorphism α_p of the algebra A_0 such that also $\Delta_0(\alpha_p(a)) = (\alpha_p \otimes \alpha_p)\Delta_0(a)$ for all $a \in A_0$ and all $p \in G$.

We also assume that $\alpha_e = \iota$ and $\alpha_p(\alpha_q(a)) = \alpha_{pq}(a)$ for all $a \in A_0$ and $p, q \in G$.

Now, we consider again the algebra A, defined as $K(G) \otimes A_0$ with the tensor product algebra structure.

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p,q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

The antipode S on A is given by

$$S(a)(p) = S_0(\alpha_p(a(p^{-1})))$$

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

The antipode S on A is given by

$$S(a)(p) = S_0(\alpha_p(a(p^{-1})))$$

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p,q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

The antipode S on A is given by

$$S(a)(p) = S_0(\alpha_p(a(p^{-1})))$$

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

The antipode S on A is given by

$$S(a)(p) = S_0(\alpha_p(a(p^{-1})))$$

Definition and Examples
Examples

We will consider elements in A as functions on G with finite support and values in A_0.

This algebra is made into a multiplier Hopf algebra by defining the coproduct Δ by $(\Delta(a))(p,q) = (\alpha_q \otimes \iota)(\Delta_0(a(pq)))$, for $a \in A$ and $p, q \in G$.

It is straightforward to show that this is a multiplier Hopf algebra. The counit ε on A is given by $\varepsilon(a) = \varepsilon_0(a(e))$ where ε_0 is the counit on A_0.

The antipode S on A is given by

$$S(a)(p) = S_0(\alpha_p(a(p^{-1})))$$

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.

Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$. Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.

Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.

Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.

Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.

Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

Definition and Examples

Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

It is not hard to find more concrete examples using the above construction.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

It is not hard to find more concrete examples using the above construction.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

It is not hard to find more concrete examples using the above construction.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.

Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.

Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

It is not hard to find more concrete examples using the above construction.

Definition and Examples
Examples

If the action is trivial, we get the tensor product coalgebra structure on $K(G) \otimes A_0$.
Now, we have the imbedding $\gamma : K(G) \longrightarrow M(A)$ given by $\gamma(f) = f \otimes 1$ where 1 stands for the identity in $M(A_0)$.
Of course, this is a central embedding and it is trivial to verify that it is compatible with the comultiplications. So, we have a G-cograded multiplier Hopf algebra. All the components are the same, namely A_0.

However, not all the maps $\Delta_{p,q}$ are equal. We have $\Delta_{p,q} = (\alpha_q \otimes \iota)\Delta_0$ for all $p, q \in G$.

It is not hard to find more concrete examples using the above construction.

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying

(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying

(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying

(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;

(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying

(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;

(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma\otimes\gamma)(\Delta(f))$ for all $f \in K(G)$.

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.

In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.

In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma\otimes\gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....

$\Leftarrow$) Define the subalgebra A_p by $\gamma(f_p)A$ where $p \in G$ and f_p is the function on G satisfying $f_p(p) = 1$ and $f_p(q) = 0$ when $q \neq p$. Because γ maps into the center, A_p will be a subalgebra of A. It is non-trivial for each p because γ is assumed to be injective....

Characterization and Properties
Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.
Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....
$\Leftarrow$) Define the subalgebra A_p by $\gamma(f_p)A$ where $p \in G$ and f_p is the function on G satisfying $f_p(p) = 1$ and $f_p(q) = 0$ when $q \neq p$. Because γ maps into the center, A_p will be a subalgebra of A. It is non-trivial for each p because γ is assumed to be injective.....

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A,Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma\otimes\gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.

Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....
$\Leftarrow$) Define the subalgebra A_p by $\gamma(f_p)A$ where $p \in G$ and f_p is the function on G satisfying $f_p(p) = 1$ and $f_p(q) = 0$ when $q \neq p$. Because γ maps into the center, A_p will be a subalgebra of A. It is non-trivial for each p because γ is assumed to be injective.....

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma \otimes \gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.
Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....
$\Leftarrow$) Define the subalgebra A_p by $\gamma(f_p)A$ where $p \in G$ and f_p is the function on G satisfying $f_p(p) = 1$ and $f_p(q) = 0$ when $q \neq p$. Because γ maps into the center, A_p will be a subalgebra of A. It is non-trivial for each p because γ is assumed to be injective.....

Characterization and Properties

Characterization

Theorem 2.1 A multiplier Hopf algebra (A, Δ) is G-cograded if and only if there is a non-degenerate algebra embedding $\gamma : K(G) \longrightarrow M(A)$ satisfying
(i) $\gamma(K(G))$ belongs to the center $Z(M(A))$ of $M(A)$;
(ii) γ is also a coalgebra map, that is, $\Delta(\gamma(f)) = (\gamma\otimes\gamma)(\Delta(f))$ for all $f \in K(G)$.
In the case of a G-cograded multiplier Hopf $*$-algebra, we have that γ is a $*$-map.
Proof. $\Rightarrow$) Define a map $\gamma : K(G) \longrightarrow M(A)$ by $(\gamma(f))(p) = f(p)1_p$ for any $p \in G$ where 1_p is the identity in $M(A_p)$.....
$\Leftarrow$) Define the subalgebra A_p by $\gamma(f_p)A$ where $p \in G$ and f_p is the function on G satisfying $f_p(p) = 1$ and $f_p(q) = 0$ when $q \neq p$. Because γ maps into the center, A_p will be a subalgebra of A. It is non-trivial for each p because γ is assumed to be injective.....

Characterization and Properties

Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

Characterization and Properties

Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

Characterization and Properties
Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

For all $p \in G$, $S : A_p \longrightarrow A_{p^{-1}}$ satisfies $m(S_{p^{-1}} \otimes \iota)(\Delta_{p^{-1},p}(a)(1 \otimes b)) = \varepsilon_e(a)b$, $m(\iota \otimes S_{p^{-1}})((b \otimes 1)\Delta_{p,p^{-1}}(a)) = \varepsilon_e(a)b$, for all $p \in A_e$ and $b \in A_p$.

Characterization and Properties

Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

For all $p \in G$, $S : A_p \longrightarrow A_{p^{-1}}$ satisfies $m(S_{p^{-1}} \otimes \iota)(\Delta_{p^{-1},p}(a)(1 \otimes b)) = \varepsilon_e(a)b$, $m(\iota \otimes S_{p^{-1}})((b \otimes 1)\Delta_{p,p^{-1}}(a)) = \varepsilon_e(a)b$, for all $p \in A_e$ and $b \in A_p$.

Characterization and Properties
Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

For all $p \in G$, $S : A_p \longrightarrow A_{p^{-1}}$ satisfies $m(S_{p^{-1}} \otimes \iota)(\Delta_{p^{-1},p}(a)(1 \otimes b)) = \varepsilon_e(a)b$, $m(\iota \otimes S_{p^{-1}})((b \otimes 1)\Delta_{p,p^{-1}}(a)) = \varepsilon_e(a)b$, for all $p \in A_e$ and $b \in A_p$.

Characterization and Properties
Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

For all $p \in G$, $S : A_p \longrightarrow A_{p^{-1}}$ satisfies $m(S_{p^{-1}} \otimes \iota)(\Delta_{p^{-1},p}(a)(1 \otimes b)) = \varepsilon_e(a)b$, $m(\iota \otimes S_{p^{-1}})((b \otimes 1)\Delta_{p,p^{-1}}(a)) = \varepsilon_e(a)b$, for all $p \in A_e$ and $b \in A_p$.

Characterization and Properties

Properties

Proposition 2.2 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Then $\varepsilon(A_p) = 0$ when $p \neq e$ and $S(A_p) \subseteq M(A_{p^{-1}})$ for all p. If the multiplier Hopf algebra is regular, we have $S(A_p) = A_{p^{-1}}$.

Moreover, $\varepsilon_e : A_e \longrightarrow \mathbb{C}$ satisfies, for all $p \in G$, $(\iota \otimes \varepsilon_e)((a \otimes 1)\Delta_{p,e}(b)) = ab$, $(\varepsilon_e \otimes \iota)(\Delta_{e,p}(a)(1 \otimes b)) = ab$, whenever $a, b \in A_p$.

For all $p \in G$, $S : A_p \longrightarrow A_{p^{-1}}$ satisfies $m(S_{p^{-1}} \otimes \iota)(\Delta_{p^{-1},p}(a)(1 \otimes b)) = \varepsilon_e(a)b$, $m(\iota \otimes S_{p^{-1}})((b \otimes 1)\Delta_{p,p^{-1}}(a)) = \varepsilon_e(a)b$, for all $p \in A_e$ and $b \in A_p$.

Characterization and Properties

Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a) 1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Characterization and Properties

Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a)1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Characterization and Properties
Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a) 1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Also $\Delta_{p,q}(\delta_{pq})(b \otimes c) = \delta_p b \otimes \delta_q c$ for all $a \in A_{pq}$, $b \in A_p$, $c \in A_q$ and $p, q \in G$.

Characterization and Properties

Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a)1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Also $\Delta_{p,q}(\delta_{pq})(b \otimes c) = \delta_p b \otimes \delta_q c$ for all $a \in A_{pq}$, $b \in A_p$, $c \in A_q$ and $p, q \in G$.

Characterization and Properties
Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p \in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a)1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Also $\Delta_{p,q}(\delta_{pq})(b \otimes c) = \delta_p b \otimes \delta_q c$ for all $a \in A_{pq}, b \in A_p, c \in A_q$ and $p, q \in G$.

Characterization and Properties
Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a)1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Also $\Delta_{p,q}(\delta_{pq})(b \otimes c) = \delta_p b \otimes \delta_q c$ for all $a \in A_{pq}, b \in A_p, c \in A_q$ and $p, q \in G$.

Characterization and Properties
Integrals

Proposition 2.3 Let A be a G-cograded multiplier Hopf algebra with decomposition $A = \bigoplus_{p\in G} A_p$. Let φ be a left integral on A. Then all the components φ_p are faithful. We have $(\iota \otimes \varphi_q)(\Delta_{p,q}(a)) = \varphi_{pq}(a)1_p$ for all $a \in A_{pq}$ and all $p, q \in G$.

Moreover, for the components (δ_p) of the modular element δ, we have $(\varphi_p \otimes \iota)\Delta_{p,q}(a) = \varphi_{pq}(a)\delta_q$ when $a \in A_{pq}$ and $p, q \in G$.

Also $\Delta_{p,q}(\delta_{pq})(b \otimes c) = \delta_p b \otimes \delta_q c$ for all $a \in A_{pq}, b \in A_p, c \in A_q$ and $p, q \in G$.

Characterization and Properties
Integrals

Theorem 2.4 Let (A, Δ) be a regular G-cograded multiplier Hopf algebra with finite dimensional unital components. For any $f \in A'_e$, define a linear functional φ_f on A by

$$\varphi_f(a) = \sum_{p,i} (f_{p,i} f)(a S^2(e_{p,i})).$$

Then φ_f is left invariant on (A, Δ).

Where, for each $p \in G$, $(e_{p,i})_i$ is a basis in A_p and $(f_{p,i})_i$ a dual basis in the dual space A'_p.

Characterization and Properties
Integrals

Theorem 2.4 Let (A,Δ) be a regular G-cograded multiplier Hopf algebra with finite dimensional unital components. For any $f \in A'_e$, define a linear functional φ_f on A by

$$\varphi_f(a) = \sum_{p,i} (f_{p,i} f)(aS^2(e_{p,i})).$$

Then φ_f is left invariant on (A,Δ).

Where, for each $p \in G$, $(e_{p,i})_i$ is a basis in A_p and $(f_{p,i})_i$ a dual basis in the dual space A'_p.

Characterization and Properties

Integrals

Theorem 2.4 Let (A, Δ) be a regular G-cograded multiplier Hopf algebra with finite dimensional unital components. For any $f \in A'_e$, define a linear functional φ_f on A by

$$\varphi_f(a) = \sum_{p,i} (f_{p,i} f)(aS^2(e_{p,i})).$$

Then φ_f is left invariant on (A, Δ).

Where, for each $p \in G$, $(e_{p,i})_i$ is a basis in A_p and $(f_{p,i})_i$ a dual basis in the dual space A'_p.

Characterization and Properties

Integrals

Theorem 2.4 Let (A, Δ) be a regular G-cograded multiplier Hopf algebra with finite dimensional unital components. For any $f \in A'_e$, define a linear functional φ_f on A by

$$\varphi_f(a) = \sum_{p,i} (f_{p,i} f)(aS^2(e_{p,i})).$$

Then φ_f is left invariant on (A, Δ).

Where, for each $p \in G$, $(e_{p,i})_i$ is a basis in A_p and $(f_{p,i})_i$ a dual basis in the dual space A'_p.

Characterization and Properties
Integrals

Theorem 2.4 Let (A, Δ) be a regular G-cograded multiplier Hopf algebra with finite dimensional unital components. For any $f \in A'_e$, define a linear functional φ_f on A by

$$\varphi_f(a) = \sum_{p,i} (f_{p,i} f)(aS^2(e_{p,i})).$$

Then φ_f is left invariant on (A, Δ).

Where, for each $p \in G$, $(e_{p,i})_i$ is a basis in A_p and $(f_{p,i})_i$ a dual basis in the dual space A'_p.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p\in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

Drinfeld double

Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

Drinfeld double

Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

In the $*$-case, we assume that π_p is a $*$-automorphism for all $p \in G$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

In the $*$-case, we assume that π_p is a $*$-automorphism for all $p \in G$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

In the $*$-case, we assume that π_p is a $*$-automorphism for all $p \in G$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

In the $*$-case, we assume that π_p is a $*$-automorphism for all $p \in G$.

Drinfeld double
Definition

Definition 3.1 Let B be a G-cograded multiplier Hopf ($*$-)algebra. So B has the form $B = \bigoplus_{p \in G} B_p$.

Let $Aut(B)$ denote the group of algebra automorphisms on B. By an action of the group G on B, we mean a group homomorphism $\pi : G \longrightarrow Aut(B)$. Further, we require that for all $p \in G$,

(1) π_p respects the comultiplication on B in the sense that $\Delta(\pi_p(b)) = (\pi_p \otimes \pi_p)(\Delta(b))$ for all $b \in B$.

We call this action admissible if there is an action ρ of G on itself so that: (2) $\pi_p(B_q) = B_{\rho_p(q)}$, (3) $\pi_{\rho_p(q)} = \pi_{pqp^{-1}}$ for all $p, q \in G$.

In the $*$-case, we assume that π_p is a $*$-automorphism for all $p \in G$.

Drinfeld double
Deformation

Take B and π as above. For $b \in B$, we define the multiplier $\widetilde{\Delta}(b)$ in $M(B \otimes B)$ by the following formulas.

Take $b' \in B_q$, then we define

$$\widetilde{\Delta}(b)(1 \otimes b') = (\pi_{q^{-1}} \otimes \iota)(\Delta(b)(1 \otimes b'));$$

$$(1 \otimes b')\widetilde{\Delta}(b) = (\pi_{q^{-1}} \otimes \iota)((1 \otimes b')\Delta(b))$$

in $B \otimes B$. By the associativity of the product in $B \otimes B$, we have that $\widetilde{\Delta}(b)$ is a multiplier in $M(B \otimes B)$.

Drinfeld double

Deformation

Take B and π as above. For $b \in B$, we define the multiplier $\widetilde{\Delta}(b)$ in $M(B \otimes B)$ by the following formulas.

Take $b' \in B_q$, then we define

$$\widetilde{\Delta}(b)(1 \otimes b') = (\pi_{q^{-1}} \otimes \iota)(\Delta(b)(1 \otimes b');$$
$$(1 \otimes b')\widetilde{\Delta}(b) = (\pi_{q^{-1}} \otimes \iota)((1 \otimes b')\Delta(b))$$

in $B \otimes B$. By the associativity of the product in $B \otimes B$, we have that $\widetilde{\Delta}(b)$ is a multiplier in $M(B \otimes B)$.

Drinfeld double
Deformation

Take B and π as above. For $b \in B$, we define the multiplier $\widetilde{\Delta}(b)$ in $M(B \otimes B)$ by the following formulas.

Take $b' \in B_q$, then we define

$$\widetilde{\Delta}(b)(1 \otimes b') = (\pi_{q^{-1}} \otimes \iota)(\Delta(b)(1 \otimes b'));$$
$$(1 \otimes b')\widetilde{\Delta}(b) = (\pi_{q^{-1}} \otimes \iota)((1 \otimes b')\Delta(b))$$

in $B \otimes B$. By the associativity of the product in $B \otimes B$, we have that $\widetilde{\Delta}(b)$ is a multiplier in $M(B \otimes B)$.

Drinfeld double
Deformation

Take B and π as above. For $b \in B$, we define the multiplier $\widetilde{\Delta}(b)$ in $M(B \otimes B)$ by the following formulas.

Take $b' \in B_q$, then we define

$$\widetilde{\Delta}(b)(1 \otimes b') = (\pi_{q^{-1}} \otimes \iota)(\Delta(b)(1 \otimes b');$$
$$(1 \otimes b')\widetilde{\Delta}(b) = (\pi_{q^{-1}} \otimes \iota)((1 \otimes b')\Delta(b))$$

in $B \otimes B$. By the associativity of the product in $B \otimes B$, we have that $\widetilde{\Delta}(b)$ is a multiplier in $M(B \otimes B)$.

Drinfeld double
Deformation

Take B and π as above. For $b \in B$, we define the multiplier $\widetilde{\Delta}(b)$ in $M(B \otimes B)$ by the following formulas.

Take $b' \in B_q$, then we define

$$\widetilde{\Delta}(b)(1 \otimes b') = (\pi_{q^{-1}} \otimes \iota)(\Delta(b)(1 \otimes b');$$
$$(1 \otimes b')\widetilde{\Delta}(b) = (\pi_{q^{-1}} \otimes \iota)((1 \otimes b')\Delta(b))$$

in $B \otimes B$. By the associativity of the product in $B \otimes B$, we have that $\widetilde{\Delta}(b)$ is a multiplier in $M(B \otimes B)$.

Drinfeld double
Deformation

Proposition 3.2 Take the notations as above. For all $b \in B_p$ and $b' \in B_q$, we have

$$(b' \otimes 1)\widetilde{\Delta}(b) = \sum b'(\pi_{qp^{-1}}(b_{(1)}) \otimes b_{(2)});$$
$$\widetilde{\Delta}(b)(b' \otimes 1) = \sum (\pi_{qp^{-1}}(b_{(1)})b' \otimes b_{(2)})$$

in $B \otimes B$. The map $\widetilde{\Delta} : B \longrightarrow M(B \otimes B)$ is a non-degenerate homomorphism and coassociative.

Drinfeld double

Deformation

Proposition 3.2 Take the notations as above. For all $b \in B_p$ and $b' \in B_q$, we have

$$(b' \otimes 1)\widetilde{\Delta}(b) = \sum b'(\pi_{qp^{-1}}(b_{(1)}) \otimes b_{(2)});$$

$$\widetilde{\Delta}(b)(b' \otimes 1) = \sum (\pi_{qp^{-1}}(b_{(1)})b' \otimes b_{(2)})$$

in $B \otimes B$. The map $\widetilde{\Delta} : B \longrightarrow M(B \otimes B)$ is a non-degenerate homomorphism and coassociative.

Drinfeld double
Deformation

Proposition 3.2 Take the notations as above. For all $b \in B_p$ and $b' \in B_q$, we have

$$(b' \otimes 1)\widetilde{\Delta}(b) = \sum b'(\pi_{qp^{-1}}(b_{(1)}) \otimes b_{(2)});$$
$$\widetilde{\Delta}(b)(b' \otimes 1) = \sum (\pi_{qp^{-1}}(b_{(1)})b' \otimes b_{(2)})$$

in $B \otimes B$. The map $\widetilde{\Delta} : B \longrightarrow M(B \otimes B)$ is a non-degenerate homomorphism and coassociative.

Drinfeld double
Deformation

Proposition 3.2 Take the notations as above. For all $b \in B_p$ and $b' \in B_q$, we have

$$(b' \otimes 1)\widetilde{\Delta}(b) = \sum b'(\pi_{qp^{-1}}(b_{(1)}) \otimes b_{(2)});$$
$$\widetilde{\Delta}(b)(b' \otimes 1) = \sum(\pi_{qp^{-1}}(b_{(1)})b' \otimes b_{(2)})$$

in $B \otimes B$. The map $\widetilde{\Delta} : B \longrightarrow M(B \otimes B)$ is a non-degenerate homomorphism and coassociative.

Drinfeld double
Deformation

Proposition 3.2 Take the notations as above. For all $b \in B_p$ and $b' \in B_q$, we have

$$(b' \otimes 1)\widetilde{\Delta}(b) = \sum b'(\pi_{qp^{-1}}(b_{(1)}) \otimes b_{(2)});$$
$$\widetilde{\Delta}(b)(b' \otimes 1) = \sum (\pi_{qp^{-1}}(b_{(1)})b' \otimes b_{(2)})$$

in $B \otimes B$. The map $\widetilde{\Delta} : B \longrightarrow M(B \otimes B)$ is a non-degenerate homomorphism and coassociative.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\tilde{\Delta}$ be defined as above. Then we have the following.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$. Assume that π is an admissible action of G on B. Let $\tilde{\Delta}$ be defined as above. Then we have the following.

Drinfeld double

Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p\in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

(2) If B is a G-cograded multiplier Hopf $*$-algebra, then $(B, \widetilde{\Delta})$ is again a multiplier Hopf $*$-algebra.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

(2) If B is a G-cograded multiplier Hopf $*$-algebra, then $(B, \widetilde{\Delta})$ is again a multiplier Hopf $*$-algebra.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

(2) If B is a G-cograded multiplier Hopf $*$-algebra, then $(B, \widetilde{\Delta})$ is again a multiplier Hopf $*$-algebra.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p\in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

(2) If B is a G-cograded multiplier Hopf $*$-algebra, then $(B, \widetilde{\Delta})$ is again a multiplier Hopf $*$-algebra.

Drinfeld double
Deformation

Theorem 3.3 Let B be a regular G-cograded multiplier Hopf algebra. So, as an algebra, B has the form $B = \bigoplus_{p \in G} B_p$.

Assume that π is an admissible action of G on B. Let $\widetilde{\Delta}$ be defined as above. Then we have the following.

(1) $(B, \widetilde{\Delta})$ is a regular multiplier Hopf algebra. The counit $\widetilde{\varepsilon}$ is the original counit ε. The antipode $\widetilde{S}$ is given by the formula $\widetilde{S}(b) = \pi_{p^{-1}}(S(b))$ for $b \in B_p$.

(2) If B is a G-cograded multiplier Hopf $*$-algebra, then $(B, \widetilde{\Delta})$ is again a multiplier Hopf $*$-algebra.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

Drinfeld double

Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B,\widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B,\widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B,\widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B,\widetilde{\Delta})$.

Remark that we will use $\widehat{B}$ for the deformed multiplier Hopf algebra $(B,\widetilde{\Delta})$.

Drinfeld double

Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Remark that we will use $\widetilde{B}$ for the deformed multiplier Hopf algebra $(B, \widetilde{\Delta})$.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Remark that we will use $\widetilde{B}$ for the deformed multiplier Hopf algebra $(B, \widetilde{\Delta})$.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Remark that we will use $\widetilde{B}$ for the deformed multiplier Hopf algebra $(B, \widetilde{\Delta})$.

Drinfeld double
Deformation

(3) If φ is a left integral on B, then φ is also a left integral on $(B, \widetilde{\Delta})$.

However, if ψ is a right integral, it is in general not right invariant on $(B, \widetilde{\Delta})$. It has to be modified.

For $b \in B_p$, define $\widetilde{\psi}(b) = \psi_B(\pi_{p^{-1}}(b))$. Then $\widetilde{\psi}$ is a right integral.

In the $*$-case, we have that a positive left integral on B is again a positive left integral on $(B, \widetilde{\Delta})$. A positive right integral ψ on B gives rise to a positive right integral $\widetilde{\psi}$ on $(B, \widetilde{\Delta})$.

Remark that we will use $\widetilde{B}$ for the deformed multiplier Hopf algebra $(B, \widetilde{\Delta})$.

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$;

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B \rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p \in G}$ of A such that

(1) $A = \bigoplus_{p \in G} A_p$ and $A_p A_q \subseteq A_{pq}$,

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$.

Drinfeld double

Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$.

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_p A_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Drinfeld double

Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Proof. Define a subspace A_p in A by $A_p = 1_p \blacktriangleright A$ for any $p \in G$, where 1_p denotes the unit in $M(B_p)$.....

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Proof. Define a subspace A_p in A by $A_p = 1_p \blacktriangleright A$ for any $p \in G$, where 1_p denotes the unit in $M(B_p)$.....

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Proof. Define a subspace A_p in A by $A_p = 1_p \blacktriangleright A$ for any $p \in G$, where 1_p denotes the unit in $M(B_p)$.....

Drinfeld double

Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Proof. Define a subspace A_p in A by $A_p = 1_p \blacktriangleright A$ for any $p \in G$, where 1_p denotes the unit in $M(B_p)$.....

Drinfeld double
Pairing

Proposition 3.4 Let $\langle A, B\rangle$ be a pairing of two regular multiplier Hopf algebras. Suppose that B is G-cograded. Then there exist subspaces $\{A_p\}_{p\in G}$ of A such that

(1) $A = \bigoplus_{p\in G} A_p$ and $A_pA_q \subseteq A_{pq}$,

(2) $\langle A_p, B_q\rangle = 0$ whenever $p \neq q$,

(3) $\langle \Delta(A_p), B_q \otimes B_r\rangle = 0$ if $q \neq p$ or $r \neq p$, where $p, q, r \in G$.

Proof. Define a subspace A_p in A by $A_p = 1_p \blacktriangleright A$ for any $p \in G$, where 1_p denotes the unit in $M(B_p)$.....

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B \rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^\pi = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

Drinfeld double

Pairing

Theorem 3.4 Let $\langle A, B\rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^\pi = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B \rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^{\pi} = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

- the product: $(a \bowtie b)(a' \bowtie b') = (m_A \otimes m_B)(\iota_A \otimes R \otimes \iota_B)(a \otimes b \otimes a' \otimes b')$ where $R(b \otimes a') = \sum (\pi_{p^{-1}}(b_{(1)}) \blacktriangleright a \blacktriangleleft S^{-1}(b_{(3)})) \otimes b_{(2)}$ for all $a' \in A$ and $b \in B_p$,

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B\rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^{\pi} = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

- the product: $(a \bowtie b)(a' \bowtie b') = (m_A \otimes m_B)(\iota_A \otimes R \otimes \iota_B)(a \otimes b \otimes a' \otimes b')$ where $R(b \otimes a') = \sum(\pi_{p^{-1}}(b_{(1)}) \blacktriangleright a \blacktriangleleft S^{-1}(b_{(3)})) \otimes b_{(2)}$ for all $a' \in A$ and $b \in B_p$,

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B\rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^{\pi} = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

• the product: $(a \bowtie b)(a' \bowtie b') = (m_A \otimes m_B)(\iota_A \otimes R \otimes \iota_B)(a \otimes b \otimes a' \otimes b')$ where $R(b \otimes a') = \sum(\pi_{p^{-1}}(b_{(1)}) \blacktriangleright a \blacktriangleleft S^{-1}(b_{(3)})) \otimes b_{(2)}$ for all $a' \in A$ and $b \in B_p$,

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B\rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^{\pi} = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

• the product: $(a \bowtie b)(a' \bowtie b') = (m_A \otimes m_B)(\iota_A \otimes R \otimes \iota_B)(a \otimes b \otimes a' \otimes b')$ where $R(b \otimes a') = \sum(\pi_{p^{-1}}(b_{(1)}) \blacktriangleright a \blacktriangleleft S^{-1}(b_{(3)})) \otimes b_{(2)}$ for all $a' \in A$ and $b \in B_p$,

Drinfeld double
Pairing

Theorem 3.4 Let $\langle A, B\rangle$ be a pair of multiplier Hopf algebras and assume that B is a (regular) G-cograded multiplier Hopf algebra. Let π be an admissible action of G on B.

(1) The space $D^{\pi} = A^{cop} \otimes \widetilde{B}$ becomes a (regular) multiplier Hopf algebra, called the Drinfel'd double, with the multiplication, the comultiplication, the counit and the antipode, depending on the pairing as well as on the action π, defined in the following way:

• the product: $(a \bowtie b)(a' \bowtie b') = (m_A \otimes m_B)(\iota_A \otimes R \otimes \iota_B)(a \otimes b \otimes a' \otimes b')$ where $R(b \otimes a') = \sum(\pi_{p^{-1}}(b_{(1)}) \blacktriangleright a \blacktriangleleft S^{-1}(b_{(3)})) \otimes b_{(2)}$ for all $a' \in A$ and $b \in B_p$,

Drinfeld double

Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^{\pi} \otimes D^{\pi})$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$.

Drinfeld double

Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^{\pi} \otimes D^{\pi})$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$.

Drinfeld double

Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^\pi \otimes D^\pi)$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^{\pi} \otimes D^{\pi})$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^{\pi} \otimes D^{\pi})$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

(2) If moreover A is a multiplier Hopf $*$-algebra, B a G-cograded multiplier Hopf $*$-algebra and $\langle A, B\rangle$ a $*$-pairing, then D^{π} is again a multiplier Hopf $*$-algebra with the $*$-operation given by $(a \bowtie b)^* = R(b^* \otimes a^*)$ for all $a \in A$ and $b \in B$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^\pi \otimes D^\pi)$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

(2) If moreover A is a multiplier Hopf $*$-algebra, B a G-cograded multiplier Hopf $*$-algebra and $\langle A, B\rangle$ a $*$-pairing, then D^π is again a multiplier Hopf $*$-algebra with the $*$-operation given by $(a \bowtie b)^* = R(b^* \otimes a^*)$ for all $a \in A$ and $b \in B$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^\pi \otimes D^\pi)$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

(2) If moreover A is a multiplier Hopf $*$-algebra, B a G-cograded multiplier Hopf $*$-algebra and $\langle A, B\rangle$ a $*$-pairing, then D^π is again a multiplier Hopf $*$-algebra with the $*$-operation given by $(a \bowtie b)^* = R(b^* \otimes a^*)$ for all $a \in A$ and $b \in B$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^\pi \otimes D^\pi)$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

(2) If moreover A is a multiplier Hopf $*$-algebra, B a G-cograded multiplier Hopf $*$-algebra and $\langle A, B\rangle$ a $*$-pairing, then D^π is again a multiplier Hopf $*$-algebra with the $*$-operation given by $(a \bowtie b)^* = R(b^* \otimes a^*)$ for all $a \in A$ and $b \in B$.

Drinfeld double
Pairing

- $\overline{\Delta}(a \bowtie b) = \Delta^{cop}(a)\widetilde{\Delta}(b)$ for all $a \in A$ and $b \in B$ where $\Delta^{cop}(a)$ and $\widetilde{\Delta}(b)$ are considered as multipliers in $M(D^\pi \otimes D^\pi)$,
- $\overline{\varepsilon}(a \bowtie b) = \varepsilon(a)\varepsilon(b)$ for all $a \in A$ and $b \in B$,
- $\overline{S}(a \bowtie b) = R(\pi_{p^{-1}}(S(b)) \otimes S^{-1}(a))$ for all $a \in A$ and $b \in B_p$.

(2) If moreover A is a multiplier Hopf $*$-algebra, B a G-cograded multiplier Hopf $*$-algebra and $\langle A, B\rangle$ a $*$-pairing, then D^π is again a multiplier Hopf $*$-algebra with the $*$-operation given by $(a \bowtie b)^* = R(b^* \otimes a^*)$ for all $a \in A$ and $b \in B$.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called ”topologically” quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called ”topologically” quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Another way to avoid topological difficulties is to consider the formal dual point of view via coquasitriangular Hopf algebras.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Another way to avoid topological difficulties is to consider the formal dual point of view via coquasitriangular Hopf algebras.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Another way to avoid topological difficulties is to consider the formal dual point of view via coquasitriangular Hopf algebras.

Quasitriangular Structure
Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called "topologically" quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Another way to avoid topological difficulties is to consider the formal dual point of view via coquasitriangular Hopf algebras.

Quasitriangular Structure

Motivation

The motivating example for quasitriangular Hopf algebras is given by the Hopf algebra $H = U_q(g)$, for g a finite-dimensional semisimple Lie algebra over $k = \mathbb{C}$.

However, H is not quasitriangular in the strict sense of the definition. The R-matrix lies in a completion of $H \otimes H$ rather than in $H \otimes H$ itself. The Hopf algebra $H = U_q(g)$ is called ”topologically” quasitriangular.

The explicit construction of the universal R-matrix is complicated. There are some ways to evade this problem.

One can study the finite-dimensional modules to obtain solutions of the Yang-Baxter equation.

Another way to avoid topological difficulties is to consider the formal dual point of view via coquasitriangular Hopf algebras.

Quasitriangular Structure
Definition

An approach with multiplier Hopf algebras gives an alternative way to construct a generalized R-matrix in purely algebraic setting.

Definition. A regular multiplier Hopf algebra (A, Δ) is called quasitriangular if there exists an invertible multiplier R in $M(A \otimes A)$ such that the conditions of the usual quasitriangular Hopf algberas hold.

Quasitriangular Structure
Definition

An approach with multiplier Hopf algebras gives an alternative way to construct a generalized R-matrix in purely algebraic setting.

Definition. A regular multiplier Hopf algebra (A, Δ) is called quasitriangular if there exists an invertible multiplier R in $M(A \otimes A)$ such that the conditions of the usual quasitriangular Hopf algberas hold.

Quasitriangular Structure
Definition

An approach with multiplier Hopf algebras gives an alternative way to construct a generalized R-matrix in purely algebraic setting.

Definition. A regular multiplier Hopf algebra (A, Δ) is called quasitriangular if there exists an invertible multiplier R in $M(A \otimes A)$ such that the conditions of the usual quasitriangular Hopf algberas hold.

Quasitriangular Structure
Definition

An approach with multiplier Hopf algebras gives an alternative way to construct a generalized R-matrix in purely algebraic setting.

Definition. A regular multiplier Hopf algebra (A,Δ) is called quasitriangular if there exists an invertible multiplier R in $M(A\otimes A)$ such that the conditions of the usual quasitriangular Hopf algberas hold.

Quasitriangular Structure
Definition

An approach with multiplier Hopf algebras gives an alternative way to construct a generalized R-matrix in purely algebraic setting.

Definition. A regular multiplier Hopf algebra (A, Δ) is called quasitriangular if there exists an invertible multiplier R in $M(A \otimes A)$ such that the conditions of the usual quasitriangular Hopf algberas hold.

Quasitriangular Structure
Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

Quasitriangular Structure

Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

Quasitriangular Structure
Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

We, furthermore, suppose that $R(a \otimes 1), (a \otimes 1)R \in A \otimes M(A)$ for all $a \in A$. We call R the generalized matrix for A.

Quasitriangular Structure
Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

We, furthermore, suppose that $R(a \otimes 1), (a \otimes 1)R \in A \otimes M(A)$ for all $a \in A$. We call R the generalized matrix for A.

We denote $R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}$ in $A \otimes M(A)$ and $(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}$.

Quasitriangular Structure

Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A\otimes A)$ so that for all $a\in A$ we have $(a\otimes 1)R$ and $R(a\otimes 1)$ are elements in $A\otimes M(A)$.

We use the following notation in $A\otimes M(A)$.

$$(a\otimes 1)R=\sum aR^{(1)}\otimes R^{(2)},\quad R(a\otimes 1)=\sum R^{(1)}a\otimes R^{(2)}.$$

We, furthermore, suppose that $R(a\otimes 1),(a\otimes 1)R\in A\otimes M(A)$ for all $a\in A$. We call R the generalized matrix for A.

We denote $R(a\otimes 1)=\sum R^{(1)}a\otimes R^{(2)}$ in $A\otimes M(A)$ and $(a\otimes 1)R=\sum aR^{(1)}\otimes R^{(2)}$.

Quasitriangular Structure
Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

We, furthermore, suppose that $R(a \otimes 1), (a \otimes 1)R \in A \otimes M(A)$ for all $a \in A$. We call R the generalized matrix for A.

We denote $R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}$ in $A \otimes M(A)$ and $(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}$.

Quasitriangular Structure

Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

We, furthermore, suppose that $R(a \otimes 1), (a \otimes 1)R \in A \otimes M(A)$ for all $a \in A$. We call R the generalized matrix for A.

We denote $R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}$ in $A \otimes M(A)$ and $(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}$.

Quasitriangular Structure

Multipliers

Let A be any regular multiplier Hopf algebra. Let R denote a multiplier in $M(A \otimes A)$ so that for all $a \in A$ we have $(a \otimes 1)R$ and $R(a \otimes 1)$ are elements in $A \otimes M(A)$.

We use the following notation in $A \otimes M(A)$.

$$(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}, \quad R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}.$$

We, furthermore, suppose that $R(a \otimes 1), (a \otimes 1)R \in A \otimes M(A)$ for all $a \in A$. We call R the generalized matrix for A.

We denote $R(a \otimes 1) = \sum R^{(1)}a \otimes R^{(2)}$ in $A \otimes M(A)$ and $(a \otimes 1)R = \sum aR^{(1)} \otimes R^{(2)}$.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

We will give a condition on R in order that u is a two-sided multiplier in $M(A)$ as follows.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

We will give a condition on R in order that u is a two-sided multiplier in $M(A)$ as follows.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

We will give a condition on R in order that u is a two-sided multiplier in $M(A)$ as follows.

Quasitriangular Structure

Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

We will give a condition on R in order that u is a two-sided multiplier in $M(A)$ as follows.

Quasitriangular Structure
Multipliers

Definition 4.1 Take R as above. We define the left multiplier u as follows.

Take $a \in A$. We set

$$ua = \sum S(R^{(2)})R^{(1)}a \quad \text{in } A.$$

It is easy to see that u is a left multiplier of A. Notice that we use the extension to $M(A)$ of the anti-isomorphism S on A.

We will give a condition on R in order that u is a two-sided multiplier in $M(A)$ as follows.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

(2) If R is invertible in $M(A \otimes A)$, then u is invertible in $M(A)$. Then S^2 is an inner automorphism. If $a \in A$, then we have $S^2(a) = uau^{-1} = S(u)^{-1}aS(u)$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

(2) If R is invertible in $M(A \otimes A)$, then u is invertible in $M(A)$. Then S^2 is an inner automorphism. If $a \in A$, then we have $S^2(a) = uau^{-1} = S(u)^{-1}aS(u)$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

(2) If R is invertible in $M(A \otimes A)$, then u is invertible in $M(A)$. Then S^2 is an inner automorphism. If $a \in A$, then we have $S^2(a) = uau^{-1} = S(u)^{-1}aS(u)$.

Quasitriangular Structure

Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

(2) If R is invertible in $M(A \otimes A)$, then u is invertible in $M(A)$. Then S^2 is an inner automorphism. If $a \in A$, then we have $S^2(a) = uau^{-1} = S(u)^{-1}aS(u)$.

Quasitriangular Structure
Multipliers

Proposition 4.2 Take $R \in M(A \otimes A)$ as above. Suppose that for all $a \in A$, $R\Delta(a) = R^{cop}(a)R$ in $M(A \otimes A)$. Then we have

(1) u is a multiplier in $M(A)$ so that $ua = S^2(a)u$ for all $a \in A$.

Remark: the right multiplier with u is given by the formula $S^2(a)u = ua$ for all $a \in A$.

(2) If R is invertible in $M(A \otimes A)$, then u is invertible in $M(A)$. Then S^2 is an inner automorphism. If $a \in A$, then we have $S^2(a) = uau^{-1} = S(u)^{-1}aS(u)$.

Quasitriangular Structure

Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

Quasitriangular Structure

Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$,

Quasitriangular Structure

Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a\otimes 1) = (\iota\otimes S^{-1})(R(a\otimes 1))$, $(a\otimes 1)R^{-1} = (\iota\otimes S^{-1})((a\otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

(4) u is invertible in $M(A)$ and u^{-1} is given as $u^{-1}a = \sum S^{-2}(R^{(2)}) R^{(1)} a = \sum S^{-1}(R^{(2)}) S(R^{(1)}) a = \sum R^{(2)} S^2(R^{(1)}) a$.

Quasitriangular Structure

Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

(4) u is invertible in $M(A)$ and u^{-1} is given as $u^{-1}a = \sum S^{-2}(R^{(2)}) R^{(1)}a = \sum S^{-1}(R^{(2)})S(R^{(1)})a = \sum R^{(2)}S^{2}(R^{(1)})a$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

(4) u is invertible in $M(A)$ and u^{-1} is given as $u^{-1}a = \sum S^{-2}(R^{(2)}) R^{(1)}a = \sum S^{-1}(R^{(2)})S(R^{(1)})a = \sum R^{(2)}S^{2}(R^{(1)})a$.

Quasitriangular Structure

Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a \otimes 1) = (\iota \otimes S^{-1})(R(a \otimes 1))$, $(a \otimes 1)R^{-1} = (\iota \otimes S^{-1})((a \otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

(4) u is invertible in $M(A)$ and u^{-1} is given as $u^{-1}a = \sum S^{-2}(R^{(2)}) R^{(1)}a = \sum S^{-1}(R^{(2)})S(R^{(1)})a = \sum R^{(2)}S^2(R^{(1)})a$.

Quasitriangular Structure
Multipliers

The following results are known for usual quasitriangular Hopf algebras.

Lemma 4.3. Let R be a generalized R-matrix for A. Then we have for all $a \in A$:

(1) $(\varepsilon \otimes \iota)(R) = 1 = (\iota \otimes \varepsilon)(R)$ in $M(A)$.

(2) $R^{-1}(a\otimes 1) = (\iota\otimes S^{-1})(R(a\otimes 1))$, $(a\otimes 1)R^{-1} = (\iota\otimes S^{-1})((a\otimes 1)R)$, $R^{-1}(a \otimes 1) = (S \otimes \iota)((S^{-1}(a) \otimes 1)R)$, $(a \otimes 1)R^{-1} = (S \otimes \iota)(R(S^{-1}(a) \otimes 1))$.

(3) $(S \otimes S)(R) = R$ in $M(A \otimes A)$.

(4) u is invertible in $M(A)$ and u^{-1} is given as $u^{-1}a = \sum S^{-2}(R^{(2)}) R^{(1)}a = \sum S^{-1}(R^{(2)})S(R^{(1)})a = \sum R^{(2)}S^2(R^{(1)})a$.

Quasitriangular Structure
Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\hat{A}}$) denote the modular element in $M(A)$ (respectively $M(\hat{A})$).

Quasitriangular Structure
Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\hat{A}}$) denote the modular element in $M(A)$ (respectively $M(\hat{A})$).

Quasitriangular Structure

Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\widehat{A}}$) denote the modular element in $M(A)$ (respectively $M(\widehat{A})$).

Then we have $uS(u)^{-1} = \delta_A^{-1}((\iota \otimes \langle \delta_{\widehat{A}}^{-1}, \cdot \rangle)(R))$ in $M(A)$.

Quasitriangular Structure

Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\widehat{A}}$) denote the modular element in $M(A)$ (respectively $M(\widehat{A})$).

Then we have $uS(u)^{-1} = \delta_A^{-1}((\iota \otimes \langle \delta_{\widehat{A}}^{-1}, \cdot \rangle)(R))$ in $M(A)$.

Quasitriangular Structure
Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\widehat{A}}$) denote the modular element in $M(A)$ (respectively $M(\widehat{A})$).

Then we have $uS(u)^{-1} = \delta_A^{-1}((\iota \otimes \langle \delta_{\widehat{A}}^{-1}, \cdot \rangle)(R))$ in $M(A)$.

Quasitriangular Structure
Antipode

Proposition 4.4 Let (A, R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\widehat{A}}$) denote the modular element in $M(A)$ (respectively $M(\widehat{A})$).

Then we have $uS(u)^{-1} = \delta_A^{-1}((\iota \otimes \langle \delta_{\widehat{A}}^{-1}, \cdot \rangle)(R))$ in $M(A)$.

Quasitriangular Structure
Antipode

Proposition 4.4 Let (A,R) be a quasitriangular multiplier Hopf algebra. Then for all $a \in A$ we have

$$S^4(a) = gag^{-1},$$

where $g = uS(u)^{-1}$ is a grouplike multiplier in $M(A)$.

Proposition 4.5 Let A be a discrete quasitriangular multiplier Hopf algebra. Let δ_A (respectively $\delta_{\widehat{A}}$) denote the modular element in $M(A)$ (respectively $M(\widehat{A})$).

Then we have $uS(u)^{-1} = \delta_A^{-1}((\iota \otimes \langle \delta_{\widehat{A}}^{-1}, \ \cdot \rangle)(R))$ in $M(A)$.

Quasitriangular Structure
Example

Example will be found in Section 2 of the paper: L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

In the group-cograded case, also see the above paper!

Quasitriangular Structure
Example

Example will be found in Section 2 of the paper: L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

In the group-cograded case, also see the above paper!

Quasitriangular Structure
Example

Example will be found in Section 2 of the paper: L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

In the group-cograded case, also see the above paper!

Quasitriangular Structure

Example

Example will be found in Section 2 of the paper: L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

In the group-cograded case, also see the above paper!

Quasitriangular Structure

Example

Example will be found in Section 2 of the paper: L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

In the group-cograded case, also see the above paper!

References

1 A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf (∗-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.

2 L. Delvaux, A. Van Daele, The Drinfel' d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.

References

1 A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf (∗-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.

2 L. Delvaux, A. Van Daele, The Drinfel’ d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.

References

1 A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf ($*$-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.

2 L. Delvaux, A. Van Daele, The Drinfel' d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.

3 L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

References

1 A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf ($*$-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.

2 L. Delvaux, A. Van Daele, The Drinfel’ d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.

3 L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

References

1 A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf ($*$-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.

2 L. Delvaux, A. Van Daele, The Drinfel' d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.

3 L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

References

1. A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf ($*$-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.
2. L. Delvaux, A. Van Daele, The Drinfel' d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.
3. L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

References

1. A. T. Abd El-hafez, L. Delvaux, A. Van Daele, Group-cograded multiplier Hopf ($*$-)algebras. Algebras and Representation Theory 10(1)(2007), 77 95.
2. L. Delvaux, A. Van Daele, The Drinfel' d double for group-cograded multiplier Hopf algebras. Algebras and Representation Theory 10(3)(2007), 197 221.
3. L. Delvaux, A. Van Daele, S. Wang, Quasitriangular (G-cograded) multiplier Hopf algebras. J. Algebra 289(2005), 484 514.

Thank you!