Vladimir I. Arnold (1937-2010): *facets of his mathematical thought*

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Framework for this talk:

- Different styles of research in history of mathematics: tensions between internal and external views.
- Dark zones between these views
- Methodological and conceptual troubles related to recent episodes (i.e., last decades).
- The “context of discovery-invention” and the relation between biographical episodes and the social environment.

The criteria for selecting the main topics:

- Some views on philosophy of math.
- Mathematical practices: just some aspects of the work of this mathematician.
- Soviet and Russian math communities. Historical and cultural scenario.
- Main focus on the “two cultures” in math: problem solving and theory building.

Why Arnold?

- A great mathematician
- A great teacher
- A prolific writer (many books and a lot of papers)
- An idiosyncratic style of thought. A representative of XX Century soviet and russian math
- His opinioins about the international community of mathematicians – (his observations about the priorities of publications)

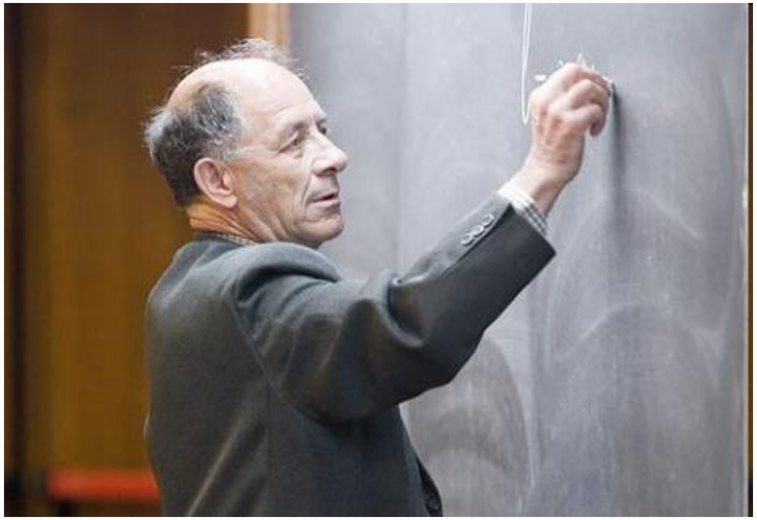
- His way of looking at the history of the discipline
- A view of math as a discipline very close to physics
- His particular view of the unity of math
- His provocative way of thinking about others mathematicians
- His interaction with some philosophers

What about all that here? Just a leaf of a tree into the forest.

An introduction to a line of research

Motivation: it is an interesting case-study of deep connections between math, history, teaching and philosophy.

- Around 1956, when he finished the solution to the 13th Hilbert problem



“I followed one line from the very beginning and there was essentially one problem I was working on all my life”

The mathematician:

- Dynamical systems – differential equations – celestial mechanics – classical mechanics – algebraic geometry – simplectic geometry – hidrodynamics – theory of singularities – caustics and wavefronts -
- 13 ° Hilbert problem - KAM – Arnold's cat – Arnold diffusion – trinities – A-D-E singularities
- Prizes: Lenin (1965) – Pierre et Marie Curie (1979) – Crafoord (1982) - Wolf (2001) – Shaw (2008)

  (No the Fields Medal. He was pre-selected in 1974)
- Vladarnolda

10031 Vladarnolda (*main belt asteroid*)

Discoverer L. G. Karachkina

Discovery date September 7, 1981

Discovery site Crimean Astrophysical Observatory

Orbital elements:

Eccentricity ***(e)*** 0.1992617

Semimajor axis ***(a)*** 2.5893518

AU **Perihelion** ***(q)*** 2.0733932 AU

Aphelion ***(Q)*** 3.1053104 AU

Orbital period ***(P)*** 4.17 a

Inclination ***(i)*** 12.92759°

Longitude of the ascending node ***(Ω)*** 255.60431°

Argument of Perihelion ***(ω)*** 47.21270°

Mean anomaly ***(M)*** 177.33203°

Mathematicians at the Mechmat:

- Teachers:

  Kolmogorov, Gelfand, Petrovskii, Pontriagin, P. Novikov, Markov, Gelfond, Lusternik, Khinchin, P.Alexandrov

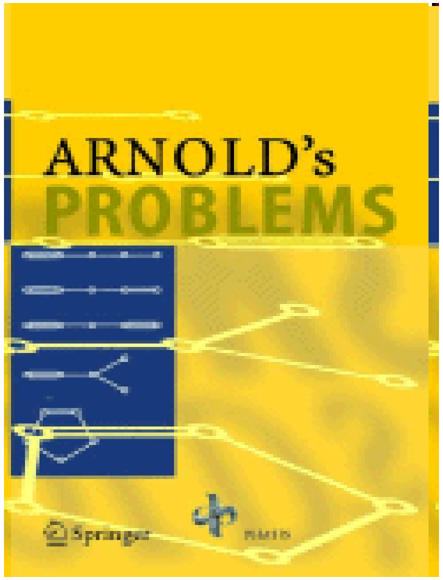
- Some students:

  Manin, Sinai, S.Novikov, Alexeev, Anosov, Kirillov, Arnold.

What is the mathematical activity for him?

- "When you are collecting mushrooms, you only see the mushroom itself. But if you are a mycologist, you know that the real mushroom is in the earth. There's an enormous thing down there, and you just see the fruit, the body that you eat. In mathematics, the upper part of the mushroom corresponds to theorems that you see. But you don't see the things which are below, namely *problems, conjectures, mistakes, ideas, and so on*."

Fig. 1. The mathematical mushroom



- (Springer – Verlag and PHASIS, 2005).
- Years 1956 – 2003 (772 problems with comments, in 639 pags.)

$$\lim_{x \to 0} \; [\sin(\tan x) - \tan(\sin x)]/[\arcsin(\arctan x) - \arctan(\arcsin x)$$

- Math as a problem solving activity

  *“You are never sure whether or not a problem is good unless you actually solve it”. (*M. Gromov)

- The art of asking good questions

  “To ask the right question is harder than to answer it”. (G. Cantor)

- Math as an adventure of ideas: concepts as a result of problem solving activity

- Mistakes as a source of new ideas and methods

  “Mistakes are an important and instructive part of math, perhaps as important a part as proofs”. (V. Arnold)

- The rule of multiplication was his first trouble with negative numbers. His father gave an explanation to him, but that experience influenced on his attitude against axiomatics.
- *“since that time I have disliked the axiomatic method with its nonmotivated definitions”.*
- That oriented him to attend to people far from the algebra, like Mandel’shtam, Tamm, Novikov, Feinberg, Leontovich, and Gurvich, who tried to explain the ideas and non trivial facts of different scientific disciplines.

An extreme classification:

- “All mathematics is divided into three parts: cryptography (paid by CIA, KGB and the like), Hydrodynamics (supported by manufacturers of atomic submarines) and celestial mechanics (financed by military and other institutions dealing with missiles, such as NASA).”
- “Cryptography has generated number theory, algebraic geometry over finite fields, algebra, combinatorics and computers.”
- “Hydrodynamics procreated complex analysis, partial differential equations, Lie groups and algebra theory, cohomology theory and scientific computing.”
- “Celestial mechanics is the origin of dynamical systems, linear algebra, topology, variational calculus and symplectic geometry.”

(Polymathematics:…)

“Mathematics is a part to physics. Physics is an experimental science, a part of natural sciences. Mathematics is the part of physics where experiments are cheap”.

“The Jacobi identity (which forces the heights of a triangle to cross at one point) is an experimental fact in the same way as that the Earth is round (that is, homeomorphic to a ball). But it can be discovered with less expense.”

“In the middle of the twentieth century it was attempted to divide physics and mathematics. The consequences turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science and, of course, in total ignorance of any other sciences.”

On teaching mathematics

(Palais de Découverte, Paris 1997)

“A teacher of math, who has not got to grips with at least some of the volumes of the courses by Landau and Lifshitz, will then become a relict like the one nowadays who does not know the difference between an open and a closed set.”

- *“The existence of mysterious relations between all these different domains is the most striking and delightful feature of mathematics (having no rational explanation)”.*

  “The experience of past centuries shows that the development of mathematics was due not to technical progress,… but rather to discoveries of unexpected interrelations between different domains…But the pernicious character of diverging modes of thought (to which the growing specialisation of mathematicians and the fragmentation of mathematics into small domains leads) becomes evident when one tries to understand the development of mathematics in the past with all its meandering”.

  (Polymathematics:…)

Newton's Principia - Lemma XXVIII:

There is no oval figure whose area, cut off by right lines at pleasure, can be universally found by means of equations of any number of finite terms and dimensions.

(Univ. Of California Press, Vol. I. Motte's Translation Revised by Cajori, pp. 110- 112).

- “In the *Principia* are two purely mathematical pages containing an astonishingly modern topological proof of a remarkable theorem on the transcendence of Abelian integrals.”

  “Hidden among research into celestial mechanics, this theorem of Newton has hardly been drawn to the attention of mathematicians. This is possibly because Newton’s topological arguments outstripped the level of the science of his time by two hundred years. Newton’s proof is essentially based on the investigation of a certain equivalent of the Riemann surfaces of algebraic curves, so it is incomprehensible both from the viewpoint of his contemporaries and also for those twentieth century mathematicians brought up on set theory and the theory of functions of a real variable who are afraid of multi-valued functions.”

- *(Huygens &…) (Chapter 5: Kepler's Second Law and the Topology of Abelian Integrals, p. 83)*
- Arnold V.I., Vasil'ev V.A.: Newton's Principia Read 300 Years Later. *Notices of the AMS* 36(9), pp. 1148- 1154, 1989.

- About self-intersecting closed curves that can be locally algebraically integrable, there are some letter between Huygens and Leibniz (1691).
- In Cajori's revised edition of Principia it is cited the work of H.Brougham and E.Routh: *Analytical view of Sir Isaac Newton's Principia,* London, 1855.

- “Newton’s forgotten proof of algebraic non-integrability of ovals was the first “impossibility proof” in the mathematics of the new era – the prototype of future proofs of insolubility of algebraic equations in radicals (Abel) and the insolubility of differential equations in elementary functions or in quadratures (Liouville), and not without reason did Newton compare it with the proof of the irrationality of square roots of integer numbers in the “Elements” of Euclid.”

- Chandrasekhar S.: *Newton's* Principia *for the Common Reader*, pp. 133- 139. Clarendon Press, 1995.
- He finishes the section with a quotation from Arnold:

  "Comparing today the texts of Newton with the comments of his successors, it is striking how Newton's original presentation is more modern, more understandable and richer in ideas than the translation due to commentators of his geometrical ideas into the formal language of the calculus of Leibniz".

- V. Alekseev: *Abel's Theorem in problems and solutions*. 1976. Springer 2004.

- A course on Abel theorem in 1964, asked by Kolmogorov, at a Moscow high school: it included the geometrical theory of complex numbers; topology of Riemannian surfaces; the fundamental group and monnodromy of branched coverings; and after them, normal divisors which are considered invariants, i.e., they do not depend on the choice of a “system of coordinates” of a subgroup of a group of representations; exact sequences; the description of groups of symmetries of regular polyhedra, including a dodecahedron and the five cubes inscribed into it by Kepler”.

- “Las cáusticas y los frentes de ondas en los sistemas de rayos se estudian desde hace mucho tiempo. No obstante, sólo recientemente se descubrió que las singularidades de los sistemas de rayos se rigen por la teoría de los grupos de reflexiones euclídeas y de los grupos de Weyl de álgebras simples de Lie. Esta relación, inesperada y algo enigmática, entre la óptica geométrica, el cálculo de variaciones y la teoría del control óptimo, por una parte, y la teoría de invariantes de los grupos de Lie y de las álgebras de Lie, la topología algebraica y la geometría diferencial, por otra parte, condujo a un considerable progreso en el desarrollo de la teoría de propagación de ondas.”

Singularidades de cáusticas y de frentes de ondas, 1995, 2000.

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