From Ether to Metric

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CONICET

Ether: the great problem of physics at the end of XIX century

Ether *by James Clerk Maxwell*

Ether, Encyclopædia Britannica Ninth Edition ***8****: 568 – 572.*

Contents

H. Hertz (1857-1894)

Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt (1894)

"All physicists agree that the problem of physics consists in tracing the phenomena of nature back to the simple laws of mechanics".

"Most physicist" (Die Meisten Physiker),

"Probably all physicist" (Wohl alle Physiker)

"The physicist" (Die Physiker).

Next to "agree" he added the word "completely" (vollständig),

E. Mach (1883)

"We think it is a prejudice to assume that mechanics must be considered the foundation of all other branches of physics, and that all physical processes must be explained mechanically."

WHY MECHANICS?

Weak sense: Mechanics was considered a perfect model for other disciplines

Strong sense: The latest objective of physics was to give a mechanical explanation of all natural phenomena.

*"... the fundamentals ideas of mechanics, together with the principles connecting them, represent the simple **image** which physics can produce of things in the sensible world and the proceses which occur in it."*

STRATEGY

To explain (physical) non mechanical phenomena through the movement of non observables mechanical systems.

This idea has his history:

- Newton: light rays = rays of particles of light in fast movement;
- Fresnel: Light = vibrations in a mechanical medium.

- Maxwell
- Boltzmann
- Helmholtz

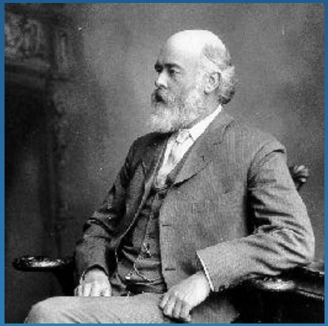
} Theory of Heat: movement of partícles

"The final aim of science is to reduce everything to mechanics" (Helmholtz-1869)

IN UNITED KINDOM:

W. Thomsom (1884)

“*I never satisfy myself until I can make a mechanical model of a thing. If I can make Aamechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electro-magnetism* ”



O. Lodge (1875) en el que estudia la ilustración, por medio de un modelo mecánico, de los fenómenos termo-eléctricos

Lodge propone modelos mecánicos para casi todos los fenómenos electromagnéticos:

Se destacan su modelo de cuerdas y botones de un circuito eléctrico, su modelo de rueda dentada del éter y su modelo hidrodinámico de las botellas de Leyden.

G. F. FitzGerald (1884) propone un modelo funcional, constituido por bandas y ruedas, para ilustrar el funcionamiento del éter electromagnético.

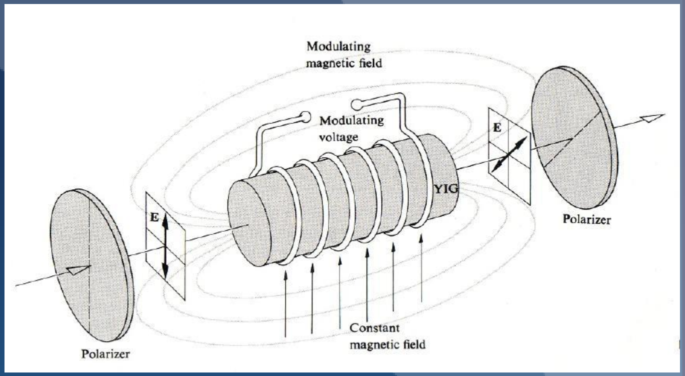
> X. *On the Structure of Mechanical Models illustrating some Properties of the Æther. By* Prof. GEORGE FRANCIS FITZ-GERALD, *F.R.S.**
>
> THE elements of which the model is constructed consist of pairs of wheels so geared together that when one of them rotates it causes the second to rotate in the same direction. The simplest way of effecting this is to connect them by a band, and this is sufficient for a one-dimensional model. Such a model may be constructed by fixing a number of wheels with their axes parallel and at right angles to a plane, and connecting each wheel with its neighbours by elastic bands. This represents a nonconducting region of the æther. A perfectly conducting region is one in which there are no bands, and a partially conducting region would be represented by the bands slipping more or less. A short description of how electrostatic, electrokinetic, and luminiferous phenomena are illustrated by such a one-dimensional model will be clearer than the corre-
>
> * Read March 28, 1885.

S. Poynting (1888) propuso un modelo mecánico con el propósito de ilustrar la carga residual de un dieléctrico.

Lodge
Heaviside
FitzGerald

Los Maxwellianos

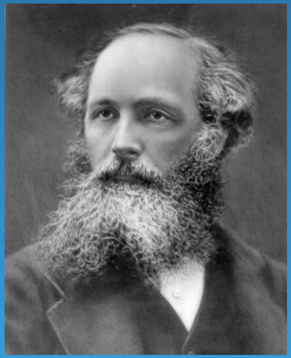
Efecto Faraday:



Rotación del plano de polarización de la luz polarizada en un campo magnético.

Thomson:

El campo magnético era debido a “vórtices moleculares” ➔ movimiento rotacional de alguna clase.

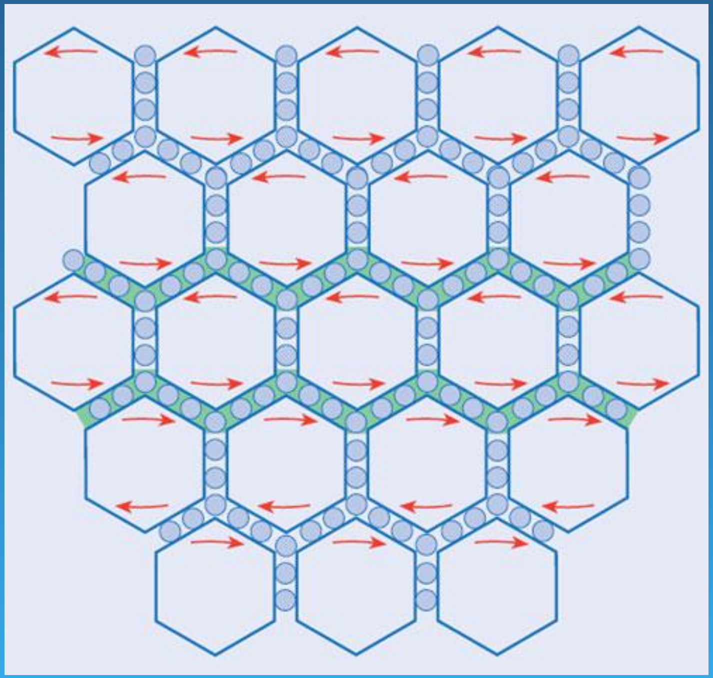


Maxwell cambia de idea sobre el campo de Faraday:

Tubos de sección variable por los que circula un fluído incompresible.

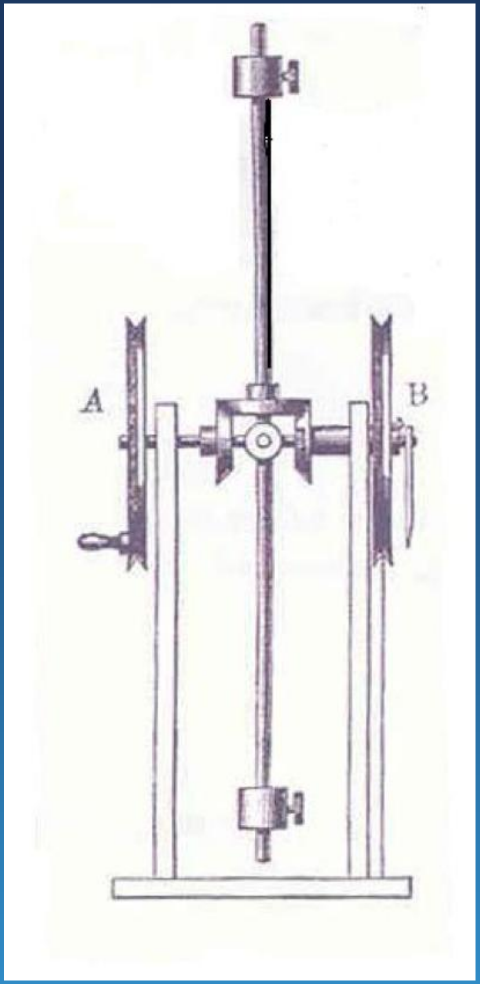
Vórtices

Creo tener una buena evidencia para la opinión de que algún fenómeno de rotación ocurre en el campo magnético; que esta rotación es realizada por un gran número de porciones pequeñas de materia, cada una rotando sobre sus propios ejes, siendo estos ejes paralelos a la dirección de la fuerza magnética, y que las rotaciones de esos vórtices dependen una de otras por medio de algún mecanismo que las conecta. El intento que he realizado para imaginar un modelo funcional de este mecanismo no debe tomarse más allá de lo que realmente es, una demostración de que podría imaginarse un mecanismo capaz de producir una conexión mecánicamente equivalente a la verdadera conexión de las partes del campo electromagnético” (1865)



El movimiento de las esferas representa una corriente eléctrica

Conocedor de aspectos mecánicos



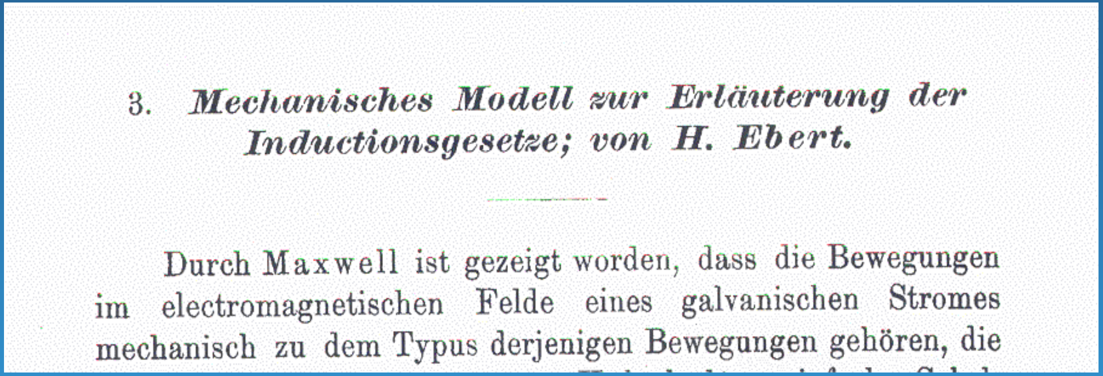
Esquema del modelo de Maxwell tomado del “*Treatise*”.

ETER

Maxwell, en una carta dirigida a L. Campbell en la Navidad de 1876, expresa:

"*He realizado un modelo mecánico de una bobina de inducción, en el cual las corrientes primaria y secundaria son representadas por el movimiento de las ruedas, y con el cual puedo representar todos los efectos de poner más o menos núcleo de hierro o más o menos resitencia y botellas de Leyden en uno u otro circuito*"

Volvamos a Europa Central…



3. ***Mechanisches Modell zur Erläuterung der Inductionsgesetze; von H. Ebert.***

Durch Maxwell ist gezeigt worden, dass die Bewegungen im electromagnetischen Felde eines galvanischen Stromes mechanisch zu dem Typus derjenigen Bewegungen gehören, die

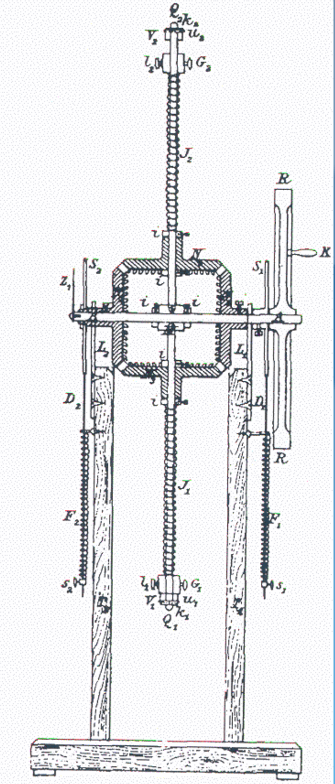
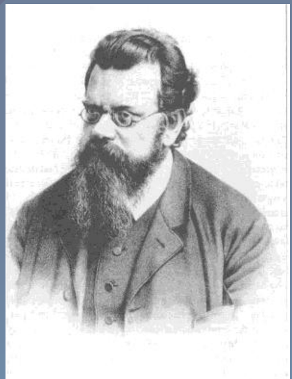
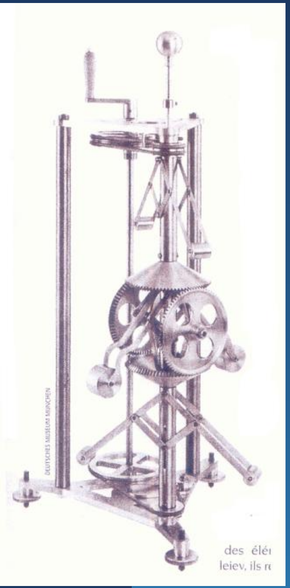


Fig. 2.



El “Bicykel” de Boltzmann



En 1890 Boltzmann se incorpora a la Universidad de Múnich.

•La dirección de la “colección de física-matemática”

•Ser miembro del directorio del seminario de física matemática

•Desarrollar un programa completo de física teórica.

Boltzmann agregó algunos aparatos relacionados con fenómenos electromagnéticos,

entre los cuales estaba su **“Bicykel”**


2009/06/12

Representación mecánica de dos circuitos eléctricos en interacción

La Sociedad Matemática Germana (1892)
publicó un catalogo de la colección

"Sobre los métodos de la Física Teórica"

"En matemáticas y geometría fue sin duda alguna el primer lugar en donde la evolución, desde los métodos puramente analíticos a los constructivos, y la ilustración o ejemplificación por medio de modelos, estuvo motivada por la economía de esfuerzos"

"Basta de esto, existe la necesidad de aprovechar lo más posible los medios que posee nuestra capacidad de percepción…

…de ello se sigue la necesidad de hacer intuitivos los resultados de los cálculos no solamente en nuestra imaginación, sino también de un modo más visible para nuestros ojos y palpable para nuestras manos…"

“Maxwell logró, a través de su modelo, ecuaciones cuyo poder singular y casi maravilloso describió tan entusiastamente el más capacitado de los científicos, llamado H. Hertz…
Quiero sólo añadir a las palabras de Hertz que las ***ecuaciones de Maxwell son mera consecuencia de sus modelos mecánicos*** *y que su elogio entusiasta corresponde, en un primer lugar, no al análisis de Maxwell, sino a su sagacidad para descubrir* ***analogías mecánicas.”***

Comentarios:

-Maxwell se sintió particularmente atraído por las analogías en la naturaleza; por la reaparición del **mismo plan** a través de toda la naturaleza:

-Las mismas leyes o ecuaciones diferenciales se aplican a la conducción del calor, a vórtices como a procesos electromagnéticos en el éter.

-En 1893 Boltzmann alerta sobre el riesgo que corre la física si se relaja de su **sello mecánico.**

CHAPTER XIV.

SECTION XXXI. MECHANISMS FOR ILLUSTRATING ELECTRO-MAGNETIC PHENOMENA; THEIR MECHANICS: CYCLES, AND LAGRANGE'S EQUATIONS OF MOTION FOR CYCLES. MONOCYCLES; A MONOCYCLIC MECHANISM.

In the derivation of the above well-known laws of electrodynamics we have followed essentially Hertz's development. Although this is undoubtedly correct, it fails to give us a clear insight into the real nature of the phenomena themselves. An exhaustive treatment of the theory of ponderable forces would naturally have to be deferred to the chapters on the electrodynamics of moving bodies and electro- and magneto-striction (see § 43). We shall, nevertheless, endeavour to show here how deep an insight can be obtained of the real nature of electro-magnetic phenomena from mechanical models. Such models or mechanisms must, however, be considered merely as means for facilitating our conceptive power and not as anything really existing; they are, indeed, to be regarded only as another feature of our concrete representation. As the mechanics of these mechanisms require a certain knowledge of the theory of cycles and their equations of motion, we shall begin with this important subject.

If a motion is imparted to a given system of bodies, not only the bodies themselves will change their position in space, but their state will, in general, also be altered.

Prácticamente copiado de Boltzmann

“El mecanismo de la corriente eléctrica es completamente diferente al del aparato; más aún es completamente desconocido para nosotros”

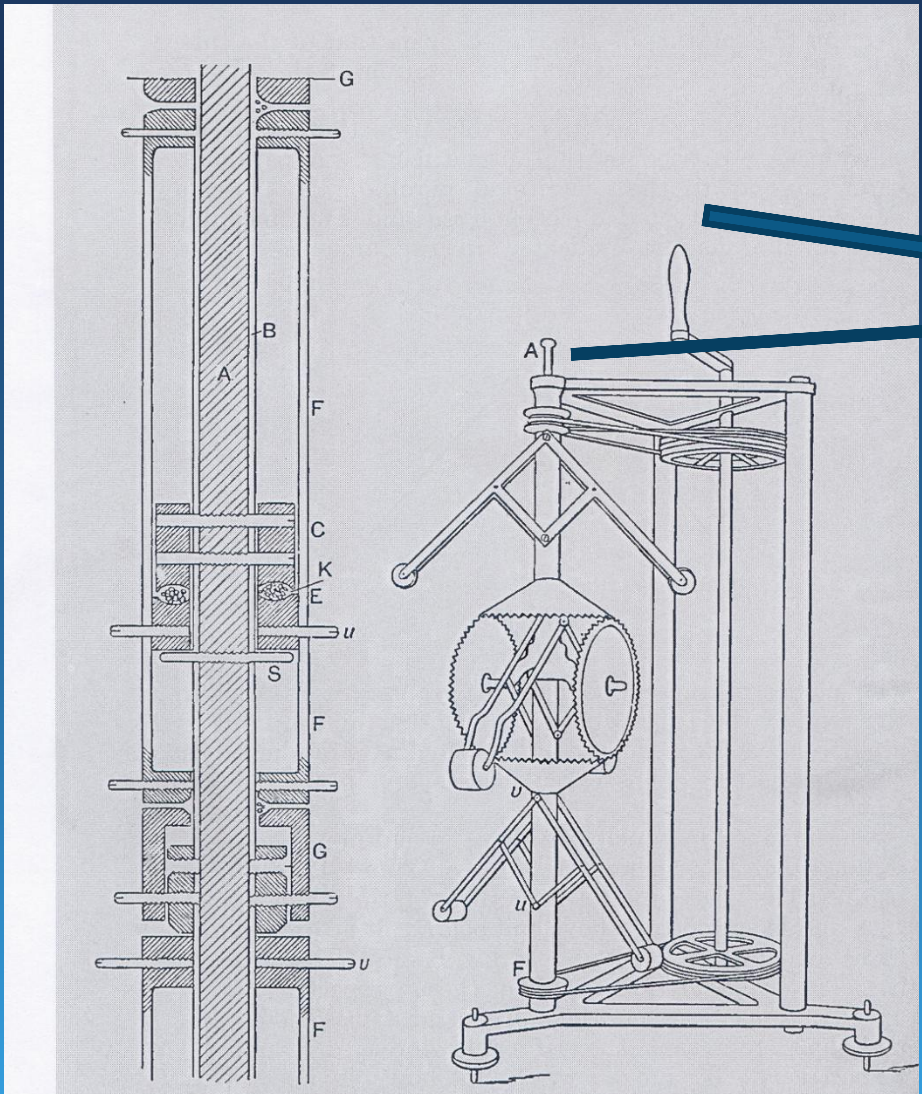


Fig. 41. Fig. 42.

Fuerza electromotriz inducida sobre el circuito primario y el secundario

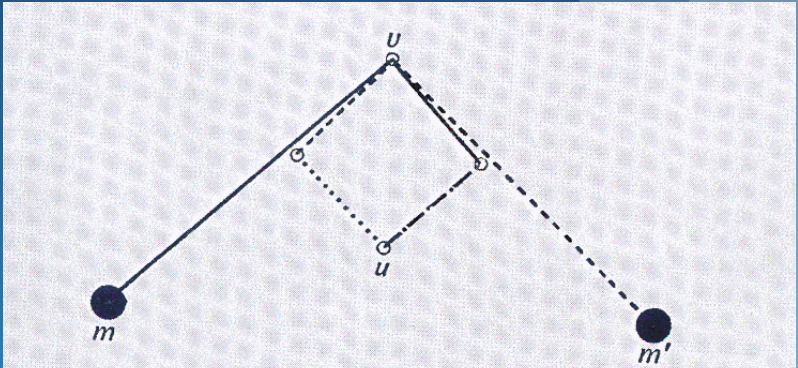


Fig. 43.

Variando la separación de las masas es posible modificar los coeficientes de autoinducción y de inducción mutua

Dos especímenes: uno en Munich, otro en Graz: ambos destruidos durante la II guerra mundial.

En el inventario del departamento de física de la Universidad de Graz (año 1914) se lee:

0 776: inexplicable apparatus with toothed gearing after Boltzmann

Una réplica del Bicykel de Boltzmann fue construido en 1985 por un maestro mecánico de Graz en ocasión de una exhibición dedicada a Boltzmann.

Sommerfeld (1944):

"This model, which even worked well, was made to Boltzmann's order; It was kept in my former department in Munich with all due reverence. However, it Was used less for electrodynamics than for mechanics, i.e. to make it understandable How the differential gear in cars, which is completely analogous to Boltzmann ́s Bicykel works".

"Boltzmann dedicó sus primeras clases a describir un sistema mecánico doblemente cíclico para ilustrar el efecto inductivo mutuo entre dos circuitos eléctricos"… "Para nosotros parecía mucho más complicado que la teoría de Maxwell que intentaba ilustrar"

Antecedentes y contexto del Bicykel

Atribuido a Lord Rayleigh (1890)

171.

ON HUYGENS'S GEARING IN ILLUSTRATION OF THE INDUCTION OF ELECTRIC CURRENTS*.

[*Philosophical Magazine*, xxx. pp. 30—32, 1890.]

As a mechanical model of the electric machinery at work in the induction of currents, Maxwell employed differential gearing; and an apparatus on this principle, designed by him, is in use at the Cavendish Laboratory. Wishing to show something similar in a recent course of lectures, and not having differential gearing at my disposal, I designed more than one combination of pulleys, the action of which should be analogous to that of electric currents. These eventually resolved themselves into Huygens's gearing, invented, I believe, in connexion with the winding of clocks. As this apparatus is easier to understand than differential gearing, and the parts of which it is composed are more likely to be useful for general purposes in a laboratory, I have thought that it might be worth while to give a description, accompanied by an explanation of the mode of action.

Two similar pulleys, *A*, *B*, turn upon a piece of round steel fixed horizontally †. Over these is hung an endless cord, and the two bights carry similar pendent pulleys, *C*, *D*, from which again hang weights, *E*, *F*. The weight of the cord being negligible, the system is devoid of potential energy; that is, it will balance, whatever may be the vertical distance between *C* and *D*.

Since either pulley *A*, *B* may turn independently of the other, the system is capable of two independent motions. If *A*, *B* turn in the same direction and with the same velocity, one of the pendent pulleys *C*, *D* rises, and the other falls. If, on the other hand, the motions of *A*, *B* are equal and opposite, the axes of the pendent pulleys and the attached weights remain at rest.

* Read before the Physical Society on May 16, 1890.

† Light wooden laths, variously coloured and revolving with the pulleys, render the movements evident at a distance.

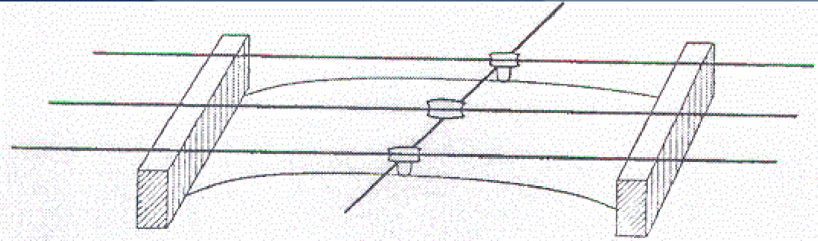
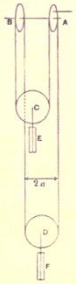


Fig. 38.

In the electrical analogy the rotatory velocity of *A* corresponds to a current in a primary circuit, that of *B* to a current in a secondary. If when all is at rest the rotation of *A* be suddenly started, by force applied at the handle or otherwise, the inertia of the masses, *E*, *F*, opposes their sudden movement, and the consequence is that the pulley *B* turns *backwards*, *i.e.*, in the opposite direction to the rotation imposed upon *A*. This is the current induced in a secondary circuit when an electromotive force begins to act in the primary. In like manner, if *A* having been for some time in uniform movement suddenly stops, *B* enters into motion in the direction of the former movement of *A*. This is the secondary current on the break of the current in the primary circuit.



It must be borne in mind that in the absence of friction there is nothing to correspond with electrical resistance, so that the conductors must be looked upon as perfect. The frictions which actually enter do not follow the same laws as electrical resistances, and only very imperfectly represent them. However, the frictions which oppose the rotations of *A* and *B* have a general effect of the right sort; but the rotations of *C* and *D*, corresponding to dielectric machinery, should be as free as possible.

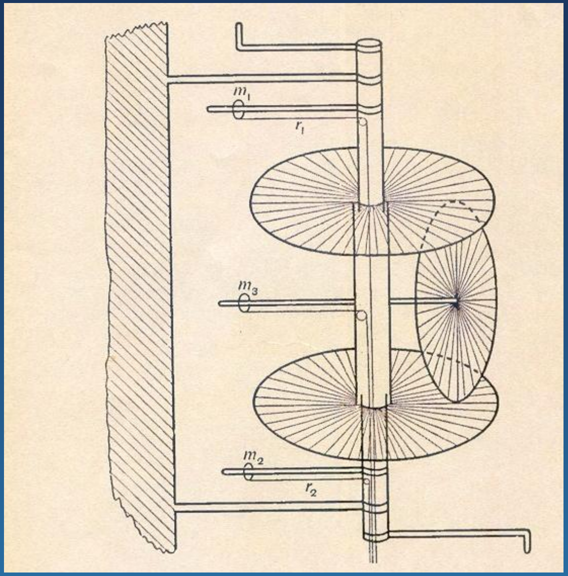
The effect of a condenser, to which the terminals of one of the circuits is joined, would be represented by a spiral spring (as in a watch) attached to the corresponding pulley, the stiffness of the spring being inversely as the capacity of the condenser. The absence of the spring, or (which comes to the same thing) the indefinite decrease of its stiffness, corresponds to infinite electrical capacity, or to a simply closed circuit.

The equations which express the mechanical properties of the system are readily found, and are precisely the same as those applicable in the electrical problem. Since the potential energy vanishes, everything turns upon the expression for the kinetic energy. If x and y denote the circumferential velocities, in the same direction, of the pulleys *A*, *B* where the cord is in contact with them, $\frac{1}{2}(x+y)$ is the vertical velocity of the pendent pulleys. Also $\frac{1}{2}(x-y)$ is the circumferential linear velocity of *C*, *D*, due to rotation, at the place where the cord engages. If the diameter be here $2a$, the angular velocity is $(x-y)/2a$. Thus, if M be the total mass of each pendent pulley and attachment, Mk^2 the moment of inertia of the revolving parts, the whole kinetic energy corresponding to each is

$$\tfrac{1}{2} M \left\{ \frac{(x+y)^2}{4} + \frac{k^2}{a^2} \frac{(x-y)^2}{4} \right\}.$$

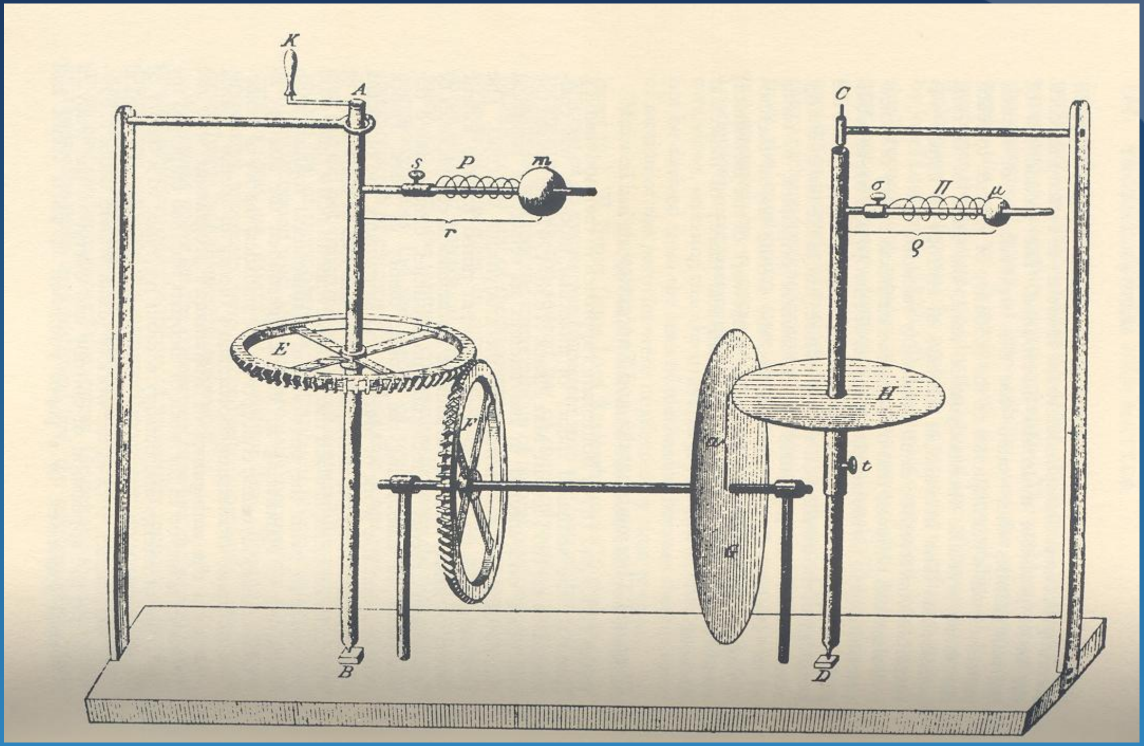
Bicykel ideal

Boltzmann adopta el formalismo Lagrangeano, al igual que Maxwell



$$\left.\begin{aligned}
T &= \left(\frac{m_1 r_1^2}{2} + \frac{m_3 r_3^2}{8}\right) l_1'^2 + \left(\frac{m_2 r_2^2}{2} + \frac{m_3 r_3^2}{8}\right) l_2'^2 + \frac{m_3 r_3^2}{4} l_1' l_2' \\
L_1 &= \frac{d}{dt}\left[\left(m_1 r_1^2 + \frac{m_3 r_3^2}{4}\right) l_1' + \frac{m_3 r_3^2}{4} l_2'\right] + W_1 \\
L_2 &= \frac{d}{dt}\left[\frac{m_3 r_3^2}{4} l_1' + \left(m_2 r_2^2 + \frac{m_3 r_3^2}{4}\right) l_2'\right] + W_2 \\
R_1 &= -m_1 r_1 l_1'^2, \quad R_2 = -m_2 r_2 l_2'^2, \quad R_3 = -\frac{m_3 r_3}{4}(l_1' + l_2')^2
\end{aligned}\right\};(14)$$

Antecedente para Boltzmann: Los monociclos de Helmholtz



Modelo mecánico de Boltzmann para ilustrar el segundo principio de la termodinámica (1884)

Helmholtz supone que los movimientos moleculares en un gas son análogos a los de una rueda en torno a su eje: Monociclos (una sola coordenada cíclica)

Hertz´s Mechanics

Three basic ingredients:

Space
Time
Mass

In his new image (BILD) of mechanics,

Hertz distinguishes
- •Physical content
- •Mathematical form

Physical content: No forces; No potential energy.

For Newton: Force
For Hamilton: Energy

For **Hertz**:

When a mechanical system seems to be acted on by forces, it is because it is rigidly connected to another system of hidden masses, whose fast cyclic motions have the same effect as forces in the traditional Newton-Lagrange <u>image</u> of mechanics.

Mathematical form:

Geometrical structure of configuration space:

"The geometry of systems of points"

Devoted to the description of the movement of a system of points, rather than single particles.

In the configuration space he introduce a Riemannian metric:

$$\frac{1}{3}\left(\sum_{i=1}^{3n} m_i\right) ds^2 = \sum_{i=1}^{3n} m_i dx_i^2$$

where: $\{m_{3j+1} = m_{3j+2} = m_{3j+3}\}$

is the mass of the point mass with Cartesian coordinates $\{x_{3j+1}, x_{3j+2}, x_{3j+3}\}$

The point masses can be bound together by constraints expressed by first order differential equations:

$$\sum_{i=1}^{3n} c_{ij} dx_i \qquad j = 1,2,\ldots k$$

Some of these constraints can be holonomic (a term invented by Hertz); that is, there exists a function f_j such that

$$f_j(x_1, x_2, \ldots, x_{3n}) = c_j = const.$$

If there exist k holonomic constraint, the number of independent coordinates is reduced to μ=3n-k.

In terms of generalized coordinates, the metric can be written

$$ds^2 = \sum_{i,j=1}^{\mu} \alpha_{ij} dq_i dq_j$$

Hertz allows also non-holonomic constraints.

Example: sphere moving on a plane.

In the Riemannianan manifold Hertz introduces several geometrical notions, specially the concept of angles.

From this notion he can introduce the notion of the straightest path.

The curvature c of the path followed by the system in the 3n dimensional space, is given by the expression:

$$mc^2 = \sum_{i=1}^{3n} m_i \left(\frac{d^2 x_i}{ds^2} \right)^2$$

For a free system, his single postulate is:

“the actual path is the one of least curvature, and it is traversed at uniform rate;

$$\delta(mc^2) = 0$$

Note: Gauss (1829) “On a new general principle of mechanics”

N particles under the action of forces $\vec{F}_k \; k = 1,\dots,N$

$$x_1, x_2, \dots, x_{3n} \; cartesian\,coordinates\,of\;the\;particles$$

$$X_1, X_2, \dots, X_{3n};\; cartesian\,components\,of\;the\;forces$$

$$m_i,\, i = 1,\dots,3n.\; m_1 = m_2 = m_3,\, etc.$$

Gauss introduces the quantity:

$$D = \sum_{k=1}^{3n} m_k \left(\frac{d^2 x_k}{dt^2} - \frac{X_k}{m_k} \right)^2$$

Without constraints

$$\frac{d^2 x_k}{dt^2} = \frac{X_k}{m_k}$$

Then the quantity D can be identified as the quadratic deviation from the movement given by the previous equation.

$$\delta D = 2\sum_{k=1}^{3n}\left(m_k \frac{d^2 x_k}{dt^2} - X_k - \sum_{i=1} \lambda_i \frac{\partial F_i}{\partial x_k} \right)\delta x_k = 0$$

Here Gauss leaves unaltered the instantaneous state of the system, the constraints and the force acting on it.

The End

MUCHAS GRACIAS!!