Facultad de Matemática, Astronomía y Física, U.N.C.

Métodos Matemáticos de la Física I

Guía N° 6 (2015)

Problema 1: Probar las siguientes identidades.

a) $\displaystyle\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$

b) $\displaystyle\int_0^\infty \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$

c) $\displaystyle\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$

d) $\displaystyle\int_0^\infty \frac{x^2\,dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$

e) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+4\mathrm{sen}(\theta)} = \frac{2\pi}{3}$

f) $\displaystyle\int_0^{2\pi} \frac{d\theta}{1+a\cos(\theta)} = \frac{2\pi}{\sqrt{1-a^2}}, \quad -1 < a < 1.$

g) $\displaystyle\int_0^{\pi} (\mathrm{sen}(\theta))^{2n} d\theta = \frac{(2n)!\pi}{2^{2n}(n!)^2}$

h) $\displaystyle\int_{-\pi}^{\pi} \frac{dt}{1+\mathrm{sen}(t)^2} = \sqrt{2}\,\pi$

i) $\displaystyle\int_0^\infty \frac{\log(x)\,dx}{1+x^2} = 0$

j) $\displaystyle\int_0^\infty \frac{\log(x)\,dx}{(1+x^2)^2} = -\frac{\pi}{4}$

k) $\displaystyle\int_0^\infty \frac{\cos(x)\,dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{(a^2-b^2)}\left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a}\right) \quad (a > b > 0)$

Problema 2: Calcular las integrales

a) $\displaystyle\int_0^\infty \frac{\sin(x)}{x}\,dx,$ *b)* $\displaystyle\int_{-\infty}^\infty \frac{dx}{x^4-\pi^4},$ *c)* $\displaystyle\int_0^\infty \frac{\cos(x)-1}{x^2}dx$