Erratum of my article *On the Laplace and complex length spectra of locally symmetric spaces of negative curvature*, Math. Nachrichten **239-240** (1) (2002) 198-203. I thank Dorothee Schüth for making me aware of the mistake.

In fact, what is proved in the article is the weaker assertion that if two compact oriented locally symmetric manifolds of negative curvature are locally isometric, have the same complex length spectrum *and the same volume*, then they are strongly Laplace isospectral. (In the Proof of Proposition 3.1 I considered every element of the group Γ except the identity.)

I used a criterion developed by P. Bérard and D. De Turk-C. Gordon. Another possibility, which seems to allow one to exclude the equal volume hypothesis, is to use a Selberg's trace formula as in R. Gangolli [3]. A detailed justification would take me too long, but it seems that it works.

For compact surfaces of constant negative curvature it is well known that the length spectrum determines the Laplace spectrum and the volume of the manifold. I recall the arguments for that from Buser [2]

If the length spectrum is known to us, the function

$$F(t) = \sum_{0<\lambda_n\leq 1/4} e^{-\lambda_n t} - \sigma(t)\, e^{-t/4} \operatorname{area}(M) + \sum_{\lambda_n > 1/4} e^{-\lambda_n t}$$

is known to us also, where $\sigma(t) = O\left(t^{-3/2}\right)$ as $t \to \infty$. From $F(t)$ we first determine the small eigenvalues (if there are any) by taking the unique ω for which

$$\lim_{t\to\infty} e^{\omega t} F(t) =: m_\omega$$

is finite and positive. Then m_ω is the multiplicity of λ_1 and we remove $m_\omega e^{-\lambda_1 t}$ from F. In this way we proceed until the sum over all $\lambda_n \leq 1/4$ is removed. Then the area is determined by multiplication with $\sigma(t)^{-1} e^{t/4}$, and we continue as before.

It seems that proceeding analogously one can recover the Laplace spectrum and the volume from the complex length spectrum. Indeed, in the first displayed formula of page 416 of [3] we have for a compact locally symmetric space of negative curvature G/K the trace formula

$$L(t) = g_t(1) \operatorname{vol}(\Gamma\backslash G) + \sum_{i\geq 1} (4\pi t)^{-1/2} \varepsilon_i e^{-\left(\langle\rho,\rho\rangle t + \frac{1}{4}\ell_i^2/t\right)},$$

where $L(t)$, given in (4.7) by

$$L(t) = \sum_{\omega \in \mathcal{E}(G,1)} n_\Gamma(\omega, 1)\, e^{-(\langle \lambda_\omega, \lambda_\omega \rangle + \langle \rho, \rho \rangle)t},$$

contains the Laplace spectral information, g_t is the fundamental solution of the heat equation on G/K for spherical functions, ℓ_i are the lengths of primitive geodesics, and

$$\varepsilon_i = \ell_i \sum_{\{\gamma \in G_\Gamma | \ell(\gamma) = \ell_i\}} j(\gamma)^{-1}\, C(h(\gamma))$$

with $\gamma = \delta^{j(\gamma)}$, δ primitive. The number $C(h(\gamma))$ is computed in (3.2) and it seems to be determined by the complex length of γ, since it involves the length of γ and the angles $\Theta_\alpha(h_{\mathfrak{k}}(\gamma))$ of the holonomy along γ ($h_{\mathfrak{k}}(\gamma)$ is the rotational part of a conjugate of γ).

Thus, to proceed analogously as in [2], it would remain only to find what is $g_t(1)$ asymptotic to as $t \to \infty$. This can be found for instance in Theorem 5.9 of [1] for a family of homogeneous manifolds including the negatively curved symmetric spaces. One sees that

$$g_t(1) = O\left(t^{-3/2}\right) e^{-Q^2 t/4}$$

as $t \to \infty$, for some constant Q depending only on the isometry type of the space.

As one can see, by the inductive process as in Buser, I cannot recover separately the volume of the manifold from the complex length spectrum.

References.

[1] J.-P. Anker, E. Damek and C. Yacoub, *Spherical Analysis on harmonic AN groups*, Ann. S. N. S. Pisa **23** No. 4 (1996) 643–679.

[2] P. Buser, *Geometry and spectra of compact Riemann surfaces*, Birkhäuser, 1992.

[3] R. Gangolli, *The length spectra of some compact manifolds of negative curvature*, J. Diff. Geom. **12** No. 3 (1977) 403–425.