OPTIMIZATION OF COMPOSITE PLATES AND SHELLS USING A GENETIC ALGORITHM AND THE FINITE ELEMENT METHOD

Felipe Schaedler de Almeida[a], Armando Miguel Awruch[a]

[a] *Centro de Mecânica Aplicada e Computacional, Programa de Pós-Graduação em Engenharia Civil, Universidade Federal do Rio Grande do Sul, Av. Osvaldo Aranha, 99, 3° andar-90035-190, Porto Alegre, RS, Brasil, awruch@adufrgs.ufrgs, www.cpgec.ufrgs.br/cemacom*

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Abstract. An optimization technique, using a genetic algorithm, applied to plates and shells of laminate composite materials is presented in this work. Two examples are analyzed. In the first case weight and central deflection of a plate under a transverse pressure load are minimized, using as optimization variables thickness and the fiber angle of each layer (in this case a factor is introduced in the objective function in order to vary the relevance of each of the objectives to be minimized). In the second case, the stiffness maximization of a cylindrical shell, under a transverse pressure load, and with geometrically nonlinear behavior, is obtained using as optimization variable the fiber angle of each layer. Some aspects such as the number of structural analyses required for each case (where the finite element method is employed), as well as the algorithm reliability are also included, together with the results obtained for both examples.

1 INTRODUCTION

In the recent decades the use of composite laminate materials on structural applications has been growing, requiring great effort on development of analysis and design techniques. The large number of design variables and the complexity of the mechanical behavior are outstanding characteristics of composite material structures design. Such characteristics turn the project much more difficult and laborious than those involving conventional materials.

Optimization methods have been used in the sense of turn the composite material structural design a more systematic and well defined task (Gürdal et al., 1999). Moreover, less dependence with respect to designer sensitivity and maximum material performance can be achieved. Initially, the same method used to optimize conventional materials structures was adopted for composite material structures optimization. This mathematical method works with continuous variables and performs the search for the best solution through de design space based on gradient information. The effort on using such methods met with limited success, since composite laminate design is, in practice, restricted by the manufacturing process that limit the variables to few discrete values. Moreover, typically exist many configurations close to optimal laminate configuration (locally optima regions) that can prevent the gradient based methods to reach the global optimum.

As alternative to gradient based methods, many other techniques were tested, having the genetic algorithm (GA) stand out the others because it perfectly adjusts to the problem characteristics. GAs are probabilistic optimization methods that seek to mimic the biological reproduction and natural selection process through random, but structured, operations. The design variables are coded as genes and grouped together on chromosomes strings that represent an organism (possible solution on the design space), what allow GAs to manipulate discrete variables. Instead of working with just one search point in the design space, GA uses a population of designs that, by reproduction operations, evolve through successive generations. Many search points dispersed in the design space prevent the GA to get stuck in a local optimum area, and avoiding a premature convergence of the process. New possible designs (organisms) are generated by applying genetic operators on existing population organisms (mimicking the natural genetic mechanisms). The evolution of successive generations towards the optimization objectives is achieved by using concept of survival of the fittest (where fittest organisms have more chances to reproduce and continue in the next generation) what mimic the natural selection process. The organism fitness is obtained directly from an objective function, that uses simple structure information. No gradient evaluation is necessary to perform the search by GA.

Although GA has shown to be well adapted to composite laminate structural optimization problems, it requires a large number of analyses in each process, what is a fundamental drawback. In more complex problems, were numerical methods are necessary for the structural analysis, an excessive number of analyses can turn GA impracticable. Many techniques have been developed to minimize these problems. Essentially, the classical GA structure has been adapted to take advantage of composite laminates characteristics. The GA restructuring is done by a new variable codification and by the way new genetic operators act on the gene string.

Two examples of GA application on composite laminate structure optimization are presented in this work. The first one consists on a multiobjective optimization problem where the total weight and central deflection of a square plate under a transverse pressure load are minimized. The objective function formulation is based on a technique for multiobjective problems optimization, were the emphasis given to each objective can be adjusted, allowing the GA to obtain the pareto-optimal set. The second example shows the use of GA to optimize a cylindrical shell under pressure load, and with geometrically non-linear behavior. The optimization objective is to maximize the structure stiffness, using critical load and maximum central displacement as parameters to evaluate the feasible solutions. In both examples the whole design space was analyzed in order to prove that the GA is successful in finding global optimum at each case. The GA efficiency is evaluated by the number of analyses required for each optimization procedure and by the apparent reliability.

2 STRUCTURAL ANALYSIS

Real composite material structures optimization problems depends on a reliable structural analysis. Even for simple geometric configurations, the determination of the mechanical behavior is difficult in the case of composite materials. It happens because of the complex mechanisms like coupling between extension, bending and torsion deformations, depending on the stacking sequence. The available closed mathematical formulations introduce much simplification on the analysis or, in many cases, they are not able to predict the structural behavior, mainly for complex geometries. These make necessary to use numerical methods that can predict satisfactorily the structure response for a given design load.

There are many works on the field of numerical simulation of composite material structures using the finite element method (FEM). However, the expressive number of analyses usually required by GA limits the use of FEM as an analysis tool because of the high computational cost for the analysis of each individual design.

In this work, a triangular flat plate and shell element with 18 degrees of freedom called DKT (Discrete Kirchhoff Triangle) is used. This element was developed by Bathe and Ho (1981) for the non-linear analysis of isotropic plates and shells. Some modifications are introduced in the original formulation to allow the analysis of laminated composite structures. Additionally to the structure displacements, the analysis tool must be able to determinate the stress components at the composite layers in order to predict material failure. This is a very common constrain adopted in most of optimization problems, and it is used in this work too. The Tsai-Wu failure criterion (Daniel and Iashai, 1994) is used in the failure prediction, evaluated at both faces of each ply at each of the three numerical integration point of each finite element. In geometrically non-linear analysis the material failure is verified at each load step of the interactive solution method, which is stopped if material failure is detected. A safety factor against failure λ_f can be obtained using the Tsai-Wu failure function with the material strength parameter for traction, compression and shearing at each of the principal material axes.

3 GENETIC ALGORITHMS FOR COMPOSITE LAMINATES

This work uses a GA provided of many modifications with respect to the classical GA to adapt it to the specific case of composite structural optimization (Almeida, 2006). In the next sections special genetic operators and strategies are explained. None of the classical GA structure is presented, but the reader can find details about it in Goldberg (1989).

3.1 Composite laminate codification

In GA optimization process, the structure is considered as an organism with its characteristics defined in chromosomes, as occurs in natural organisms. Each stored information is seen as a gene that refers to one of the structure laminate plies. A computational representation of chromosomes is done by a string containing coded information of laminate properties. In this work each laminate is represented by a pair of chromosomes, as it was done by Soremekun (2001). In the first one, called “orientation chromosome”, information about fiber orientation of each laminate layer are stored. The second, called “material chromosome”, is used to point the layers material properties group (ply thickness, elastic and strength constants). So, a laminate layer is represented by a pair of genes, each in one of the two chromosomes but at the same relative position. The first pair is referred to the outermost layer, being the inner layers referred by the succeeding pairs. As only symmetric laminated are used in this work, just half of the laminate layers are coded in the gene strings, and so the total number of genes in a chromosome is proportional to half of the maximum admissible number of layers in a specific design.

Integer numbers are used to form two gene alphabets, one for each chromosome, that are used to code a composite laminate into two gene strings. The numbers on an alphabet represent the discrete possible values for the design variables. In the orientation gene alphabet, each number represents a predefined stack in a layer, which can contain more than one ply. The orientation genes define how many plies exists on a layer and how are the plies of these fiber oriented. Similarly to the orientation gene, material alphabet genes define the material properties group that can be assigned to each layer. Since in the examples included in this work no changes in the number of plies are possible, the laminate thickness variation can be achieved only by the change of individual ply thickness, using different discrete values that are coded in the material genes.

3.2 Genetic operators for composite laminate optimization

3.2.1 Crossover

Crossover is an essential GA operator, having the fundamental task to generate new organisms (child) in a reproduction process, combining genetic information taken from a pair of organisms (parents) selected from a pre-existing population. The parents selection is a probabilistic process, but greater chance of selection is given to fitter organisms. The created child will hopefully be better than his predecessors, since his genes were part of organisms with good fitness. The crossover operator used in this work is similar to classical crossover operator with few modifications. A crossover point is randomly determined and the gene strings of both material and orientation chromosomes are split at the same point in both parent (Soremekun, 2001). The left part of parent 1 and the right part of parent 2 are combined to form a child. The probability of application of crossover operators is set to 100% in all the examples presented, because it is considered a fundamental operator in GA.

3.2.2 Mutation

The classical GA operator acts over the chromosomal string changing a gene value. It is implemented to each gene, at a small probability, introducing a different value chosen from the gene alphabet. In spite of the randomity of this process, it is possible to incorporate to the mutation operator some knowledge about mechanical response of composite materials when one or more of its characteristics are altered. This may let to a less random process and guide the evolution towards optimization objectives. These modifications lead to new operators called orientation alteration, material alteration, replacing the classical mutation in the GA (Nagendra et al, 1996 and Soremekun, 2001).

Orientation alteration and material alteration operators are implemented very similarly to classical mutation. The differences are based in the fact that they are independently applied to orientation and material chromosomes, respectively. Different orientation and material operator probabilities (p_{oa} and p_{ma}) may be adopted, which is convenient, since orientation and material chromosome may converge at different velocities in an optimization process.

3.2.3 Permutation

The main characteristic of permutation operator is the ability to modify laminate stack sequence without changes of the total number of plies with fibers oriented on each permissible direction. This allows GA to change the bending behavior of the laminate without modifying its in-plane mechanical response. The permutation operator implemented in this work is equivalent to the gene-swap operator (Nagendra et al, 1996 and Soremekun, 2001), where two pairs of genes, are chosen randomly and have their position shifted in the chromosome, resulting on a new staking sequence. Such operation occurs at a given probability p_{per}, usually with a larger value than those corresponding mutation operators probabilities.

3.3 Selection Schemes

There are many ways to obtain the population of successive generations in a GA. In classical algorithms new generations are formed only by children created from an existing population. This process has many drawbacks since there is no warranty of improvement or maintenance of achieved evolution when all population is replaced. To solve this problem new selection schemes were created, being one of them the elitism scheme, which consists in transfer good organisms from old population to a new generation, preserving desirable genetic information. This papers deals with a multiple elitist scheme (Soremekun,1997).

In the multiple elitist scheme, both parent and child populations of size P are independently ranked from best to worst fitness. These two populations are then combined and ranked together, resulting in a combined population with $2P$ organisms. Then, best Ne individuals of the combined population are transferred to the new generation. The best individuals of child population that have not already been used are taken to fill the remainder of the new generation. The number of top elements (Ne) to be transferred to the new generation is a GA parameter to be adjusted at each application

4 NUMERICAL APPLICATIONS

In the following sections two examples of GA applied to optimize composite laminate structures are presented. To prove the success of the optimization procedure and to characterize the problem design space, all the possible laminate configurations are previously analyzed. Additionally, to obtain the algorithm reliability and computational cost, N optimizations with the GA are carried out for each example. The apparent reliability (R) is determined by taking the number of optimizations for which the GA finds at least one global optimum (No), divided by the total number of applications of the GA (N). It defines the chances of obtaining the global optimum in a single application of the GA. As the structural analysis employing the FEM is usually the most time consuming task in the optimization procedure, the GA cost is determined by

$$An = \frac{\sum_{i=1}^{N} X_g^i P}{N} \tag{1}$$

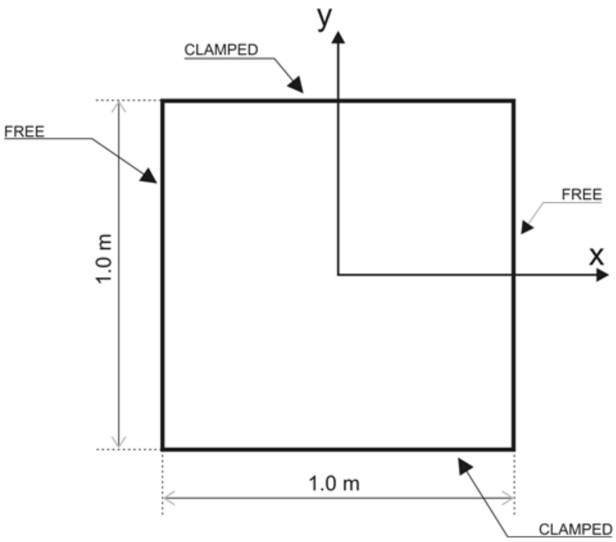
where X^i_g is the total number of generations analyzed in the *i-th* optimization procedure.

The criterion to stop the optimization process, used in both examples, is based in two parameters, the upper limit of the number of generations (N_{LG}) and the maximum number of generations with no improvement of the best design (N_{SD}). Once one of these limits is reached, the optimization process is stopped and the best laminate of the least generation is taken as the optimization result. N_{LG} and N_{SD} are defined in each optimization procedure, depending on the problem complexity.

4.1 Weight and deflection minimization of a composite laminated plate under transverse load.

This example deals with the optimization of a square plate of a composite laminate material and subjected to a uniform pressure load on its surface. The problem consists in a multiobjective optimization since weight and deflection are supposed to be minimized at the same time. Each one of these two objectives are opposite to the other because improvements in one of them leads to depreciation of the other. As a result of such characteristics this problem has a set of optimal solutions (pareto-optimal set) instead of a single solution as usually is found in simple optimization problems. The different points of the pareto-set are obtained by varying the emphasis given to each objective during an optimization process, which may be performed by introducing a weighting factor in the objective function. The material failure and a maximum value for thickness of contiguous plies with the same fiber orientation are taken as the optimization constraints.

The structure geometry, boundary conditions and the mechanical properties of the composite material are presented in Fig. 1. The elastic constants are the Young's modulus on fiber direction (*E1*) and transverse to fiber direction (*E2*), shear modulus (*G12*) and the Poisson's ratio (*v12*), respectively. Strength parameters for traction and compression for longitudinal and transversal directions are given by *F1t*, *F1c*, *F2t*, and *F2c* respectively. The remainder parameters are the shear strength (*F6*) and the specific weight (ρ). A finite element mesh with 288 elements and 169 nodes is used to represent the whole plate in the numerical analysis. The structure must support a design pressure load of 0.1 MPa with no material failure (Tsai-Wu failure function must be lower than 1.0 for the whole plate) and no more than 2mm of contiguous plies thickness with the same fiber orientation are allowed.



Composite Properties	
E1	181.0 GPa
E2	10.3 GPa
G12	7.17 GPa
v12	0.28
ρ	15.7 kN/m³
F1t	1500.0 MPa
F1c	1500.0 MPa
F2t	40.0 MPa
F2c	246.0 MPa
F6	68.0 MPa

Figure 1: Structure geometry, boundary conditions and composite properties

The laminate must be symmetric with 8 layers, being represented in the GA by a pair of chromosomes (with 4 genes each chromossome). The optimization variables are the fiber orientation and the thickness of each layer, defined by code values given in Table 1. The total number of genes in each chromosome combined with the number of possible values of each variable leads to a total number of 65536 possible laminates.

Orientation gene alphabet		Material gene alphabet	
code	Orientation angle	code	Ply thickness
1	1 ply at 0°	1	0.75mm
2	1 ply at -45°	2	1.00mm
3	1 ply at +45°	3	1.50mm
4	1 ply at 90°	4	2.00mm

Table 1: Gene alphabet and possible variable values

A graphical representation of the distribution of weight and central displacement for all the feasible designs of the problem is shown in Fig. 2. The points A to P in this figure are the pareto-optimal designs, which must be obtained by the GA, according to the emphasis given to each of the objectives. Details of the pareto-optimal set are presented in Tab. 2.

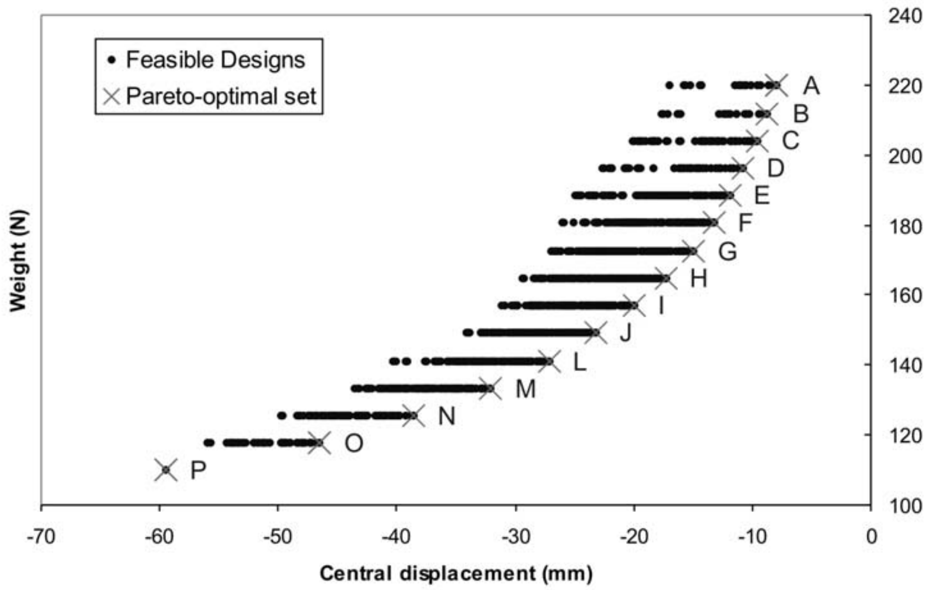


Figure 2: Weight and Central displacement of feasible designs

To perform the search througth the design space, the GA must attribute a fitness value to each of the organisms created in the optimization process. In this example the fitness must consider both objectives, weight and deflection reduction, weighted by the factor α, and include some penalization for unfeasible designs that violate constrains of material failure or the limit of contiguous plies tickness with the same fiber orientation. The fitness evaluation is done by Eq. 2a, where W^* and D^* are the total weight and the central deflection nomalized by their maximum and minimum values. The lower and higher weigth limits can be easily obtained taking all the plies thickness equal to 0.75 mm or equal to 2.00 mm, respectively. The minimum and maximum values of displacement to be used for the normalization are obtained by the results of optimizations performed with values of α equal to 0.0 and equal to 1.0, respectively. When α is equal to 0.0 only the displacement is reduced, given a design that have the lowest displacement. Taking α equal to 1.0 the GA obtains the lightest structure. This structure may have a high displacement, which is used as the maximum value in the normalization. Using Eq. (2b) nulls values of W^* and D^* are avoided.

$$
\begin{cases}
FIT = \dfrac{1}{Tv\left(\alpha W^{*} + \left(1-\alpha\right)D^{*}\right)} \quad , \quad if \;\; FF \le 1 \\[2ex]
FIT = \dfrac{1}{FF \cdot Tv\left(\alpha W^{*} + \left(1-\alpha\right)D^{*}\right)} \quad , \quad if \;\; FF > 1
\end{cases} \quad , \qquad (2a)
$$

where $W^{*} = \dfrac{W - W_{\min}}{W_{\max} - W_{\min}} + 1$, and $D^{*} = \dfrac{D - D_{\min}}{D_{\max} - D_{\min}} + 1$ (2b)

The penalization to unfeasible designs is done by the introduction of the parameters *FF* and *Tv* in the equation that determines the fittness. The first parameter represents the maximum value of the failure funcion evaluated in the structure. The parameter *Tv* is refered to the violation of the limit of contiguous plies thickness with the same fiber orientation. It is taken as 1.0 plus the exceding value that violates the limit. As an example, if the thickness of each one of two comtiguous plies with the same fiber orientation is equal to 1.5 mm, the exceding value violationg the limit that must be added to the parameter *Tv* is 1.0 mm, since this work adopts a limit of 2.0 mm. Then *Tv* = 1.0 + (1.5 x 2 - 2.0) = 2.0.

Optimal design	Laminate	Weight (N)	Deflection (mm)	α
A	$\left[90^{2,0},\ +45^{2,0},\ 90^{2,0},\ -45^{1,0}\right]_S$	219.7	-7.9	0.0 – 0.20
B	$\left[90^{2,0},\ -45^{2,0},\ 90^{2,0},\ +45^{0,75}\right]_S$	211.9	-8.8	-
C	$\left[90^{2,0},\ -45^{1,75},\ 90^{2,0},\ +45^{1,0}\right]_S$	204,0	-9,6	0,25
D	$\left[90^{2,0},\ -45^{1,75},\ 90^{2,0},\ +45^{0,75}\right]_S$	196,2	-10,8	-
E	$\left[90^{2,0},\ -45^{1,0},\ 90^{2,0},\ +45^{1,0}\right]_S$	188,4	-11,8	0,30
F	$\left[90^{2,0},\ -45^{0,75},\ 90^{2,0},\ +45^{1,0}\right]_S$	180,5	-13,2	0,35
G	$\left[90^{2,0},\ -45^{0,75},\ 90^{2,0},\ +45^{0,75}\right]_S$	172,7	-15,0	0,40
H	$\left[90^{2,0},\ -45^{0,75},\ 90^{1,75},\ +45^{1,0}\right]_S$	164,8	-17,3	0,45
I	$\left[90^{2,0},\ -45^{0,75},\ 90^{1,75},\ +45^{0,75}\right]_S$	157,0	-19,9	0,50
J	$\left[90^{2,0},\ -45^{0,75},\ 90^{1,0},\ +45^{1,0}\right]_S$	149,1	-23,3	0,55
L	$\left[90^{2,0},\ -45^{0,75},\ 90^{1,0},\ +45^{0,75}\right]_S$	141,3	-27,2	0,60
M	$\left[90^{2,0},\ -45^{0,75},\ 90^{0,75},\ -45^{0,75}\right]_S$	133,4	-32,1	0,65
N	$\left[90_2^{1,0},\ -45^{1,0},\ 90^{1,0}\right]_S$	125,6	-38,5	0,70
O	$\left[90_2^{1,0},\ +45^{0,75},\ -45^{1,0}\right]_S$	117,7	-46,6	0,75 - 0,85
P	$\left[90_2^{1,0},\ 0^{0,75},\ +45^{0,75}\right]_S$	109,9	-59,5	0,90 – 1,0

Table 2: Pareto-optimal designs

In the optimization process, the GA is applied 50 times for each *α*, which is taken varying from 0.0 to 1.0 with increments equal to 0.05. The GA is used with a population size *P*=50 and the elitist scheme parameter *Ne*=5. The genetic operators are used with the following probabilities p_{ao}=3%, p_{am}=2%, and p_{per}=80%. The criterion parameters to stop the process are

N_{LG}=200 and N_{SD}=50. All these vairables are defined in sections 3.2 and 3.3.

Good reliability value were obtained for the optimization with most of the α values, as can be seen in Table 3, where the column σ represents the standard deviation of the apparent reliability R. Even for those optimizations with lower values of the reliability, their values are not so far from a desirable value, suggesting that some modifications in the GA parameters could lead to better reliability levels. The average number of analyses required in each optimization process varied form 3682 to 4679, which represents 5.89% to 7.14% of the size of the complete design space. These values are low if compared with the total number of possibilities (65536), but a high computational effort is required to perform all these analyses.

α	Optimal designs	R	σ	An	α	Optimal designs	R	σ	An
0.00	A	90%	4,2%	3906	0.55	J	88%	4,6%	4273
0.05	A	96%	2,8%	4196	0.60	L	94%	3,4%	4183
0.10	A	98%	2,0%	4022	0.65	M	94%	3,4%	4234
0.15	A	86%	4,9%	4223	0.70	N	62%	6,9%	3920
0.20	A	94%	3,4%	4356	0.75	O	86%	4,9%	4342
0.25	C	74%	6,2%	4679	0.80	O	78%	5,9%	3905
0.30	E	98%	2,0%	4154	0.85	O	80%	5,7%	4186
0.35	F	90%	4,2%	4221	0.90	P	76%	6,0%	4195
0.40	G	98%	2,0%	3862	0.95	P	80%	5,7%	4180
0.45	H	96%	2,8%	4219	1.00	P	92%	3,8%	4222
0.50	I	96%	2,8%	4099					

Table 3: GA optimization results.

As can be seen in the Table 2 and Table 3, the GA is successful in finding most of pareto-optimal designs, but the points B and D are not obtained. In fact this points are one of the solutions of the optimization, but they are located out of a convex curve defined by the other optimal points. This fact avoids the GA to find the points B and D, since the fitness is a convex combination of the objectives. The Fig 3a and 3b show the difference of the fitness of the points B and D with respect to their neighbor points in a range of α where the optimal solution changes from A to C and from C to E, respectively (see Table 3). The figures show that the fitness of the points B and D are never greater than those of their neighbor points at same time and so they can not be obtained by the GA, no matter the value of the weighting parameter.

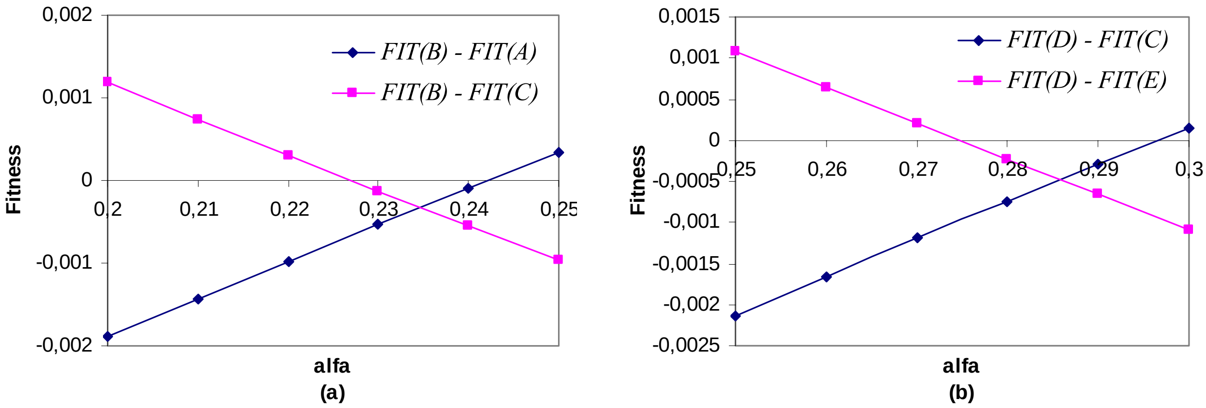
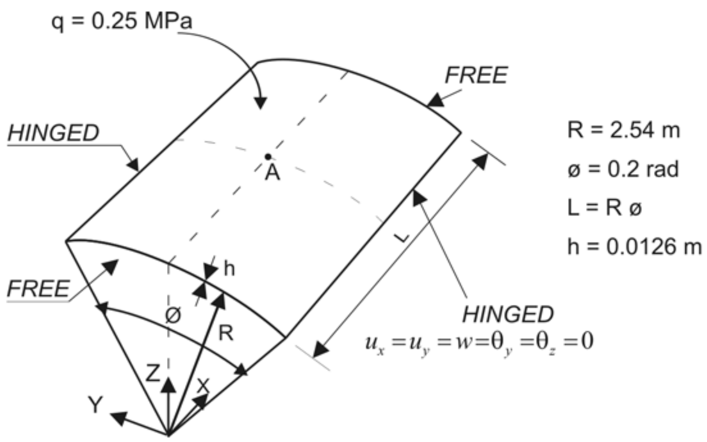


Figure 3: Difference of fitness of points B and D to their neighbor points.
(a) When FIT(B) > FIT(A), then FIT(C) > FIT(B)
(b) When FIT(D) > FIT(C), then FIT(E) > FIT(D)

4.2 Stiffness maximization of a composite laminated shell with geometrically non-linear behavior

The cylindrical shell under uniform pressure load is shown in Fig .4. In the optimization process, the finite element analysis is carried out taken into account geometrically non-linear effects. It was considered that in the composite laminate material only the fiber angles may assume different discrete values, while all the other parameters remain fixed. Material failure and the number of contiguous plies with the same fiber orientation are considered as design constrains. This last constrain is imposed in order to avoid the failure of the composite material due to matrix rupture.



Glass-epoxy properties	
E1	39.0 GPa
E2	8.6 GPa
G12	3.8 GPa
ν12	0.28
ρ	20.6 kN/m³
F1t	1080.0 MPa
F1c	620.0 MPa
F2t	39.0 MPa
F2c	128.0 MPa
F6	89.0 MPa

Figure 4: Cylindrical shell under a uniform pressure load

A finite element mesh for the whole domain having 800 elements and 441 nodes is adopted and the Generalized Displacement Control Method (Yang and Shieh, 1990) is used to solve the non-linear problem. In Fig. 4 are also included material properties (elastic constants, specific weight and strength parameters) corresponding to glass-epoxy. It is considered that the composite material is formed by 14 layers having a fixed total thickness h=12.6 mm. Each layer is formed by 2 plies that may have fiber orientations such as $0°_2$, ±45° and $90°_2$. Contiguous plies with same fiber orientation are limited to 4 plies. An orientation chromosome with 7 genes is used. Codes 1, 2 and 3 are attributed, respectively, to the laminate sequence $0°_2$, ±45° and $90°_2$.

To evaluate the shell stiffness two parameters, obtained form the structural analysis, are used. The first one is the critical load level (NC_{crit}) determined when the curve pressure x displacement at the central point A reaches the first peak. The second parameter is the maximum value of the displacement of the central point A ($U_{máx}$) at the end of the load increment or when material failure is observed. Two variables are used in order to consider situations where constrains are satisfied or they are violated. The first variable is the maximum load level acting on the structure without material failure ($NC_{máx}$) and the second variable is an integer number (V_{nlc}) indicating how many times the constrain referred to contiguous plies with the same fiber orientation have been violated. In Fig. 5 values of NC_{crit} and $U_{máx}$ corresponding to situations where the two constrains are not violated (feasible designs) are shown.

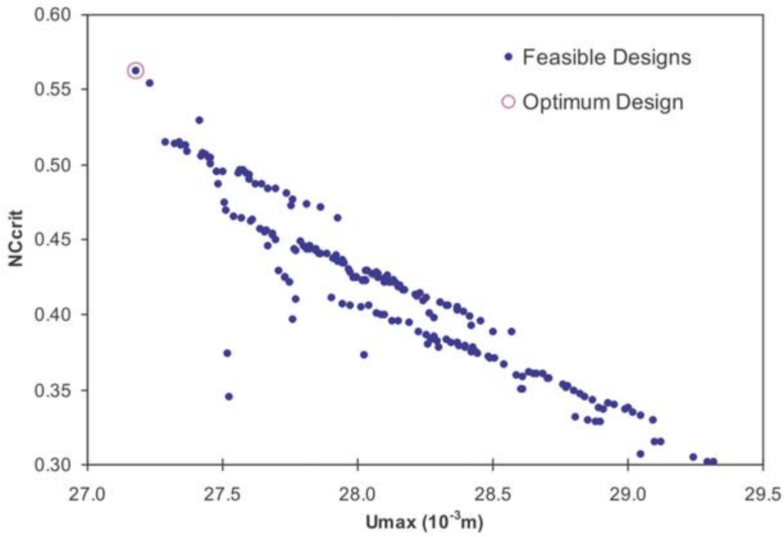
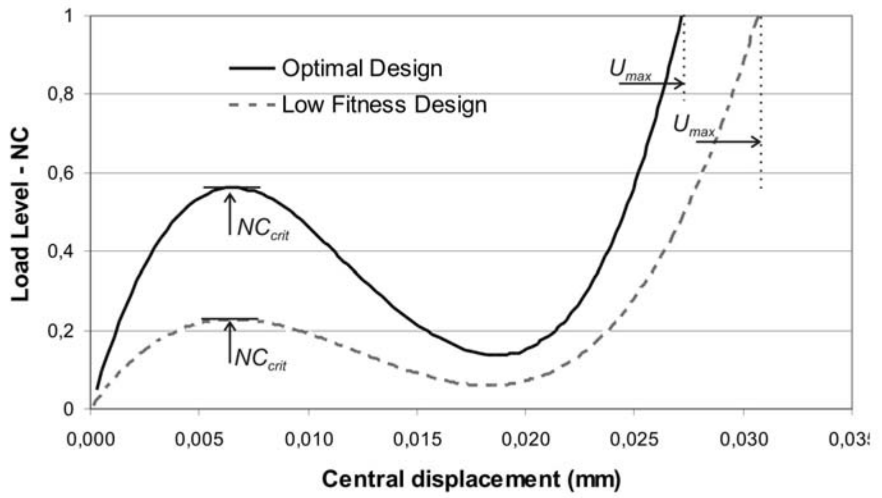


Figure 5: Critical load level x maximum central displacement of feasible designs

The objective here is to maximize the structure stiffness, obtaining the maximum value of NC_{crit} associated to the minimum value of $U_{máx}$. The structure fitness is given in the GA by the following function:

$$FIT = \left(\frac{NC_{crit} \cdot NC_{\max}^2}{U_{\max} \cdot (V_{nlc} + 1)} \right) \tag{3}$$

In Eq. 3, $NC^2_{máx}$ is used to penalize structural failures before the application of the total load, whereas (V_{nlc} + 1) is employed to penalize configurations where the maximum number of plies with the same fiber orientation is greater than 4. The optimal solution, contained in Fig. 5, is defined by the stacking sequence $[(90_4, \ \pm 45)_2, \ 90_2]_S$ and the following value of the parameters: NC_{crit}=0.563, U_{max}=27.2 x 10^{-3} m and FIT=20.698. The curve load x displacement at point A for the optimal solution, indicated in Fig. 5, is presented in Fig. 6. The same curve for a configuration of the structure with a low fitness is also shown in Fig. 6. In this case the stacking sequence is $[\pm 45_3, \ 0_4 \ \ \pm 45_2]_S$, NC_{crit}=0.228, U_{max}=30.7 x 10^{-3} m and FIT=7.427.



Load level x central displacement for optimal and low fitness design

In order to evaluate the parameter N_{LG} (limit for the number of generations) and P (number of individuals in the population) with respect to the performance of the GA, 5 values of each of parameter are adopted and 50 optimizations are carried out for each of the 25 combinations of the two parameters. All these combinations of N_{LG} and P are presented in Table 4, where Ne and N_{SD}, associated with N_{LG} and P had also been included. The probability of fiber orientation alterations is taken in this case as being p_{oa}=4%, while for the probability of permutation p_{per}=80% is adopted. The probabilities p_{am}=0.0, since the thickness of layers remains fixed and only glass-epoxy is used.

In Fig. 7, some results of the optimization are presented. The main goal of this case is reached because the design with the maximum stiffness is obtained for different sets of GA parameters. Additionally, the research about the effects of the values of N_{LG} and P is shown in Fig. 7, where the variation of R and An with respect to these parameters is well defined. The best values for the GA parameters depend on the required reliability of the process, since the minimum average number of structural analyses demanded by the optimization procedure is proportional to R.

Comb	*P (Ne)*	N_{LG} (N_{SD})	*Comb*	*P (Ne)*	N_{LG} (N_{SD})	*Comb*	*P (Ne)*	N_{LG} (N_{SD})
1	50 (8)	300 (100)	10	6 (1)	180 (60)	19	10 (1)	60 (20)
2	30 (5)	300 (100)	11	50 (8)	108 (36)	20	6 (1)	60 (20)
3	18 (3)	300 (100)	12	30 (5)	108 (36)	21	50 (8)	33 (11)
4	10 (1)	300 (100)	13	18 (3)	108 (36)	22	30 (5)	33 (11)
5	6 (1)	300 (100)	14	10 (1)	108 (36)	23	18 (3)	33 (11)
6	50 (8)	180 (60)	15	6 (1)	108 (36)	24	10 (1)	33 (11)
7	30 (5)	180 (60)	16	50 (8)	60 (20)	25	6 (1)	33 (11)
8	18 (3)	180 (60)	17	30 (5)	60 (20)			
9	10 (1)	180 (60)	18	18 (3)	60 (20)			

Table 4: Study of some parameters used in the GA.

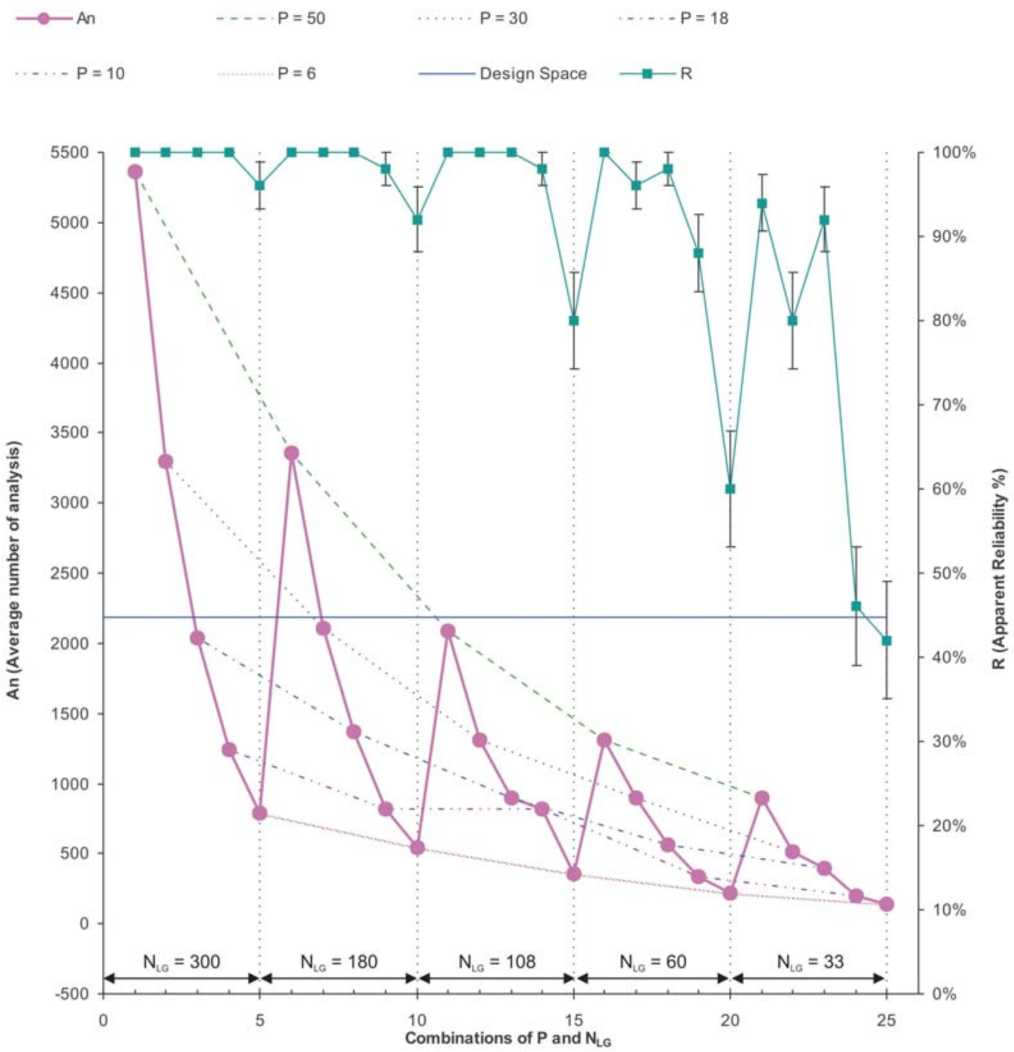


Figure 7: Average value of the number of structural analyses per optimization process and apparent reliability for each combination of the parameters of the GA

Table 5 shows the combination with the lower computational cost for each value of R above 90%. In this table, J is the relationship between An and total number of solutions in the design space. As it can be observed in Fig. 6 and Table 5, 18 individuals is the ideal size for the population, appearing three times with the lower values of An and the best values of R. It is also observed that in combination number 18, with P=18, the apparent reliability is R=98%, the number of structural analyses is not very high (567) and, for this case, N_{LG}=60.

R	*Comb*	*P*	N_{LG}	*An*	*J*
92%	23	18	33	394	18%
94%	21	50	33	896	41%
96%	5	6	300	793	36%
98%	18	18	60	567	26%
100%	13	18	108	893	41%

Table 5: The best combinations according to increasing values of the apparent reliability R.

5 CONCLUSIONS

The GA was applied successfully to optimize composite laminate structures. In the first example the objective function containing the optimization objectives normalized with respec to their limit values was able to find 13 of the 15 pareto-optimal points. An additional research was performed to show that the objective function is unable to find the two remaining points, even for smaller intervals of the weighting factor, because they are located out of the convex curve that is described by the other 13 points. For most of the values of α the GA presented good reliability and the average number of analyses required in a optimization process was found to be about 6% to 7% of the complete design space. In the second example a cylindrical shell with geometrically non-linear behavior was analyzed. In this problem the influence of different values of P and N_{LG}, taking R and An as parameters, was studied. Using 25 combinations of P and N_{LG} the tendency of R and An was analyzed. Finally, intervals of P and N_{LG} where the GA is more efficient were determined.

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REFERENCES

F. S. Almeida. *Laminated composite material structures optimization with genetic algorithms*. (In Portuguese) M. Sc. Thesis – PPGEC/UFRGS, Porto Alegre, Rio Grande do Sul – Brazil. 2006.

K. J. Bathe and L. Ho. A Simple and Effective Element for Analysis of General Shell Structures. *Computer & Structures*, vol. 13, p. 673-681, 1981.

I. M. Daniel and O.Ishai. *Engineering Mechanics of Composite Materials*. Oxford University Press, 1994.

D. E. Goldberg. *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, Reading, MA. 1989.

Z. Gürdal, R. T. Haftka and P. Hajela. *Design and Optimization of Laminated Composite Materials*. Wiley & Sons, 1999.

S. Nagendra, D. Jestin, Z. Gürdal, R. T. Haftka and L. T. Watson. Improved Genetic Algorithm for the Design of Stiffened Composite Panels. *Computer & Structures*, vol. 58, no. 3, p. 543-555, 1996.

G. A. E. Soremekun, Z. Gürdal, R. T. Haftka and L. T. Watson. Composite laminate design optimization by genetic algorithm with generalized elitist selection. *Computer & Structures*, vol. 79, p. 131-143, 2001.

G. A. E. Soremekun, *Genetic Algorithms for Composite Laminate Design and Optimization*. Dissertation (Master of Science in Engineering Mechanics) – Polytechnic Institute and State University, Blacksbourg, Virginia – USA. 157 p. 1997.

Y. Yang, M. Shieh, Solution Method for Nonlinear Problems with Multiple Critical Points. *AIAA JOURNAL*, vol. 28, no. 12, p. 2110-2116, 1990.