Evaluation of the Main Atomic Number,
Absorption and Fluorescence Correction Models in
Quantitative Microanalysis

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The principal $\mathcal{Z}$ absorption correction models have been tested for a set of 471 experimental values from heavy element binary specimens ($Z > 12$) and another set of 116 determinations for oxides, both compiled by Love et al. New fluorescence correction models have been developed, based on a Gaussian distribution of ionizations. The atomic number, absorption and fluorescence correction models corresponding to this distribution are the best fitting ones.

**INTRODUCTION**

When a collimated electron beam enters a specimen, the intensity of characteristic radiation generated for line $j$ of element $i$, may be written as

$$I_j^i = n_j^i \int_{0}^{\infty} \phi_j^i(\rho z) d(\rho z)$$

(1)

where $\phi_j^i(\rho z)$ is the depth distribution of $j$ characteristic production for element $i$ ($\rho z$ is the mass depth), normalized with respect to the number $n_j^i$ of ionizations of the same type generated in an isolated layer of identical composition (Fig. 1).

If $\psi$ is the take-off angle, the emerging intensity results:

$$I_j^i = \frac{\Delta \Omega}{4\pi} n_j^i \int_{0}^{\infty} \phi_j^i(\rho z) e^{-\chi \rho z} d(\rho z)$$

(2)

where $\chi = \mu \cot \psi$, $\mu$ is the mass absorption coefficient of the specimen to the observed radiation and $\Delta \Omega$ is the solid angle subtended by the detector. In order to avoid factors depending on the particular detector, geometry and atomic properties, this intensity is compared with that of a standard of known composition:

$$K = \frac{I_j}{I_i}$$

(henceforth the indices $i$ and $j$ are omitted in order to simplify the notation, and an index 0 will always refer to the standard) As a rough approximation, we could say that this ratio corresponds to the value of the concentration $C_i$ of the observed element in the specimen with respect to its concentration $C_0$ in the standard:

$$K = \frac{C_i}{C_0}$$

This approximation requires several corrections. Firstly, it should be noted that ionizations produced by incident electrons are not identically distributed in the sample and the standard; this effect, called the 'atomic number effect,' is corrected by a factor $\mathcal{Z}$. Second, radiation from the sample suffers an attenuation different to that from the standard, and an absorption correction factor $\mathcal{A}$ must be applied. Finally, there may exist an element excited by the incident electrons that emits photons with sufficient energy to ionize in turn the element of interest, thus giving an enhancement of primary radiation, also possible with photons from the continuum. The difference in this fluorescence enhancement between the specimen and standard is corrected by a factor $\mathcal{F}$. Then,

$$K = \frac{C_i}{C_0} \mathcal{Z} \mathcal{A} \mathcal{F}$$

$\mathcal{A}$ may take values very distant from unity, so that it is the most important factor; $\mathcal{Z}$ varies between 0.8 and 1.2 and $\mathcal{F}$ usually takes values very close to unity.

**ABSORPTION CORRECTION**

The behaviour of the absorption correction will be evaluated making use of the function

$$f(\chi) = \frac{\int_{0}^{\infty} \phi_j^i(\rho z) e^{-\chi \rho z} d(\rho z)}{\int_{0}^{\infty} \phi_j^i(\rho z) d(\rho z)}$$

(3)

![Figure 1. Characteristic x-ray production at depth \( \rho z \) from a layer \( d(\rho z) \).](image-url)
so that \( d \) is the ratio of \( f(\chi) \) evaluated for the specimen and for the standard. For this purpose several expressions for \( \phi(pz) \) are taken into account:

(a) Philibert\(^1\) obtained \( f(\chi) \) assuming an exponential attenuation of electrons [with the additional simplification \( \phi(0) = 0 \)]:

\[
\phi(pz) \propto e^{-\eta p z / h}
\]

where \( \sigma \) is the Lenard coefficient,\(^2\) which is essentially dependent on the energy of the incident electrons, and \( h \) is related to the mean ionization depth. This distribution produces

\[
f(\chi) = \frac{1}{\left(1 + \frac{\chi}{\sigma}ight)} \left(1 + \frac{h}{1 + h} \cdot \frac{\chi}{\sigma}\right)^{-1}
\]

This expression is one of the most commonly used in the computation programs for microanalysis, generally employing the parameters \( \sigma \) and \( h \) proposed by Heinrich:\(^3\)

\[
\sigma = \frac{4.5 \times 10^5}{E_o^{1.65} - E_j^{1.65}}
\]

\[
h = 1.2 \frac{A}{Z^2}
\]

where \( E_o \) is the incident electron energy, \( E_j \) the excitation energy for the shell of interest and \( A \) and \( Z \) are the atomic weight and number, respectively.

(b) Bishop\(^4\) suggested a uniform distribution function centred in the mean depth \( \bar{z} \) of x-ray generation (square model); thus, the following expression is obtained:

\[
f(\chi) = \frac{1 - \exp \left(2\chi \rho \bar{z} \right)}{2\chi \rho \bar{z}}
\]

The value of \( \bar{z} \) is assessed for the Philibert distribution:

\[
\rho \bar{z} = \frac{2h + 1}{h + 1} \frac{1}{\sigma}
\]

and with an expression obtained by Love and Scott\(^5\) based on Monte Carlo simulations:

\[
\rho \bar{z} = \frac{A}{Z} \frac{G(\eta) \ln U_o}{H(\eta) + \ln U_o}
\]

\[
\times (0.787 \times 10^{-5} J^{0.5} E_o^{1.5} + 0.735 \times 10^{-6} E_o^2)
\]

where \( G \) and \( H \) are polynomials, \( \eta \) is the electron backscattering coefficient, \( U_o \) is the overvoltage \( E_0 / E_j \), and \( J \) is the mean excitation energy.

(c) Packwood and Brown\(^6\) proposed the following expression for the distribution of characteristic x-ray production:

\[
\phi(pz) = \gamma_0 \left[1 - \frac{\gamma_0 - \phi_0}{\gamma_0} e^{-\phi_0 pz} \right] e^{-\alpha(pz) \bar{z}}
\]

in which

\[
\alpha = 2.14 \times 10^7 \frac{Z^{1.16}}{AE_j^{0.25}} \left[ \frac{\ln(1.166 E_o / J)}{E_o - E_j} \right]^{0.5}
\]

where \( J \) is the mean excitation energy.

Tirira \textit{et al.}\(^7\) improved the parameter \( \beta \) and obtained better results than those given in previous publications\(^6,8\) with the following expression:

\[
\beta = 109,000 \frac{Z^{1.5}}{(E_o - E_j)^4 A}
\]

and the original expressions for \( \gamma_0 \) and \( \phi_0 \) were improved by theoretical calculations by Tirira and Riveros,\(^8\) who arrived at

\[
\gamma_0 = \frac{U_o \ln U_o}{U_o - 1} (1 + \eta)
\]

\[
\phi_0 = 1 + \eta \frac{U_o \ln U_o}{U_o - 1}
\]

Then, substituting in Eqn (3), we have

\[
f(\chi) = \frac{\gamma_0 - \phi_0}{\gamma_0} R \left( \frac{\chi + \beta}{2\alpha} \right)
\]

\[
1 - \gamma_0 - \phi_0 R \left( \frac{\beta}{2\alpha} \right)
\]

where \( R(\chi) \) is as specified in the Appendix.

**ATOMIC NUMBER CORRECTION**

This factor may be expressed in terms of the ionization cross-section \( Q_j \) of the j shell of element i:

\[
\mathcal{Z} = \frac{R_j \int_{E_j}^{E_o} Q_j / S \, dE}{R_j \int_{E_j}^{E_o} Q_j / S \, dE}
\]

where \( S \) is the stopping power and \( R \) takes into account the loss of ionization caused by electron backscattering. Since \( Q_j / S \) varies smoothly with \( E \), it is usual to replace the integrals by the integrands evaluated for a certain energy \( EM \) between \( E_j \) and \( E_o \). Thus we obtain

\[
\mathcal{Z} = \frac{R_j \frac{S_j}{S}}{R_j}
\]

(4)

Different expressions exist for \( EM \) that may be chosen for the calculation of \( S_j / S \) and \( S \).

It is also possible to obtain an expression for \( \mathcal{Z} \) making use of the distribution function \( \phi(pz) \). According to Eqn (1),

\[
\mathcal{Z} = \int_0^\infty \phi(pz) d(pz)
\]

(5)

This expression will only be evaluated for the Gaussian distribution suggested by Packwood and Brown\(^6\) with the improved coefficients mentioned above, yielding a 'Gaussian atomic number correction,' \( \mathcal{Z}_G \).

However, in Eqn (4) several parameters will be employed instead for the evaluation of the 'conventional' model \( \mathcal{Z}_C \), taking into account various expressions\(^10,11\) for the backscattering factor \( R \) with values for the electron backscattering coefficient \( \eta \) published by Bishop,\(^12\) and for the stopping power \( S \),\(^11,13,14\) in which the models of Wilson\(^15\) and Bloch\(^16\) are used for the mean excitation energy. The corresponding expressions are given in the Appendix.

The parameters \( R \) and \( S \) for the specimen are obtained by computing a mean value from those corresponding to each element, weighted with the respective concentrations.
**EVALUATION OF MODELS IN QUANTITATIVE MICROANALYSIS**

**FLUORESCENCE CORRECTION**

Fluorescence enhancement is taken into account by a factor \( 1 + I_2/I_1 \), where \( I_2 \) is the secondary radiation emerging from the specimen and \( I_1 \) the primary radiation, given by Eqn (2). In order to find \( I_2 \) it is assumed that in a layer \( d(pz) \) at depth \( pz \), the amount of radiation of element \( s \) capable of exciting the \( j \) shell of element \( i \) generated isotropically in all directions is \( n_i \phi_i(pz) d(pz) \). The portion of these photons absorbed by the element \( i \) in the layer \( d(py) \) (see Fig. 2) between the directions \( \theta \) and \( \theta + d\theta \) is given by

\[
\frac{1}{2} \sin \theta d\theta \left[ 1 + \frac{\mu_y}{\mu_i} \sec \theta \right] C_i \mu_i^{\frac{1}{2}} \frac{d(py)}{\cos \theta}
\]

where \( \mu_y \) and \( \mu_i \) are the mass absorption coefficients of element \( i \) and the specimen to \( s \) radiation, respectively. Of these ionizations, only a fraction \( (r_i - 1)/r_i \) occurs in the \( j \) shell, and of all these, a fraction \( \omega_i \) is converted into characteristic radiation. The portion of these photons that leaves the specimen towards the detector is \( \exp \left[ -\chi_s(pyz) \right] \), so the following equation is obtained:

\[
\mathcal{F} = \frac{1}{2} C \omega_i \frac{r_i - 1}{r_i} \frac{A_i}{A_s} \frac{\mu_i^{\frac{1}{2}}}{\chi_i} \frac{E_j^{\frac{1}{2}}}{E_k^{\frac{1}{2}}} \frac{U_0^{\frac{1}{2}}}{U_0^{\frac{1}{2}}} \left( \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \right) \frac{\ln(1 + b_n \mu_i^{\frac{1}{2}} + \mu_z \ln(1 + \chi_i))}{\ln(1 + \chi_i)}
\]

\[+ \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \]

\[+ \gamma \left[ 1 - \exp \left( -\chi_s(pyz) \right) \right]
\]

where \( \mathcal{F} \) is the fluorescence yield of element \( s \) of the \( k \) shell in question, \( A_i \) and \( A_s \) are the respective atomic weights and \( f(x) \) is a function related to radiation absorption, here evaluated with the Gaussian distribution (see Appendix). The subscript \( l \) represents the element to which \( \phi(pz) \) corresponds. This expression for \( \mathcal{F} \) is almost independent of \( b_n \) and \( d_n \), values for these coefficients are given in the Appendix.

(b) Reed gave an expression in which an exponential attenuation for electrons \( \phi(pz) \approx e^{-\alpha s} \) is assumed, obtaining

\[
\mathcal{F} = 1 + C \phi(pz) \left( \frac{U_0^{\frac{1}{2}} - 1}{U_0^{\frac{1}{2}} - 1} \right)^{1.67} \mu_i^{\frac{1}{2}} \frac{\ln(1 + x) + \ln(1 + y)}{x}
\]

where \( x = \chi_i/\mu_s \), \( y = \sigma/\mu_s \) and \( J(i) \) is a factor taking into account the fluorescence yields and the shells involved in the enhancement; this factor is tabulated in Reed’s publications.

(c) If the Gaussian distribution for \( \phi(pz) \) is used, the factor \( \mathcal{F} \) becomes

\[
\mathcal{F} = 1 + C \omega_i \frac{r_i - 1}{r_i} \frac{A_i}{A_s} \frac{\mu_i^{\frac{1}{2}}}{\chi_i} \frac{E_j^{\frac{1}{2}}}{E_k^{\frac{1}{2}}} \frac{U_0^{\frac{1}{2}}}{U_0^{\frac{1}{2}}} \left( \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \right) \frac{\ln(1 + b_n \mu_i^{\frac{1}{2}} + \mu_z \ln(1 + \chi_i))}{\ln(1 + \chi_i)}
\]

\[+ \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \]

\[+ \gamma \left[ 1 - \exp \left( -\chi_s(pyz) \right) \right]
\]

where \( S(a, x) \) and \( p \) are given in the Appendix.

(d) A simplified expression for \( \phi(pz) \) may be used; for example, taking \( \phi(pz) \approx \delta(x - \bar{z}) \), where \( \delta \) is the Dirac delta function, i.e. assuming that all production of \( s \) radiation occurs at one single depth \( \bar{z} \). In this way, the following expression for \( \mathcal{F} \) is obtained:

\[
\mathcal{F} = 1 + C \omega_i \frac{r_i - 1}{r_i} \frac{A_i}{A_s} \frac{\mu_i^{\frac{1}{2}}}{\chi_i} \frac{E_j^{\frac{1}{2}}}{E_k^{\frac{1}{2}}} \frac{U_0^{\frac{1}{2}}}{U_0^{\frac{1}{2}}} \left( \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \right) \frac{\ln(1 + b_n \mu_i^{\frac{1}{2}} + \mu_z \ln(1 + \chi_i))}{\ln(1 + \chi_i)}
\]

\[+ \sum_{n=1}^{3} \frac{d_n}{b_n + \chi_i} \]

\[+ \gamma \left[ 1 - \exp \left( -\chi_s(pyz) \right) \right]
\]

where \( \gamma = 0.577216 \) is Euler’s constant. The value of \( \bar{z} \) used here corresponds to the expression

\[
\bar{z} = \frac{\int_0^\infty \phi(pz) d(pz)}{\int_0^\infty \phi(pz) d(pz)}
\]

in which \( \phi(pz) \) is the Gaussian distribution.

**PROCEDURE FOR EVALUATION OF THE DIFFERENT CORRECTION MODELS**

The procedure usually followed in order to evaluate the accuracy of the correction models consists in studying...
in preceding works only those data measured with a Jeol JXA-50A microprobe (ψ = 35°) were taken into account, since the corresponding values for the same set of specimens obtained with a Cambridge Microscan 1 (ψ = 20°) were considered to be wrong.

For the oxygen lines, the mass absorption coefficients proposed by Ruste and Gantois were used, except for Pb, where their value differs considerably from the experimental values, and was therefore replaced with one suggested by Robinson.

In the evaluation of Eqn (4) for the conventional Z factor, EM was chosen as the arithmetic mean value of $E_0$ and $E_f$ for its simplicity and accuracy, i.e. $EM = (E_0 + E_f)/2$.

**RESULTS AND DISCUSSION**

As considered above, the $\alpha$ factor is the most important correction, so it has to be evaluated first. For this purpose, some models for $Z$ were utilized ($Z_3$ and some conventional ones) and also the fluorescence correction factor suggested by Reed.

When evaluating $Z_3$, a certain dependence on the stopping power of both the sample and the standard appeared in the parameters $\alpha$ and $\alpha_0$; in the case of the specimen this parameter was averaged using mass concentrations and also atomic fractions, and noticeable differences were found in the results.

The parameters $R$ and $S$ were also tested for atomic fractions, but the best results were always obtained when mass concentrations were used.

Table 1 shows that every model worsens on passing from heavy to light elements. In this range, conventional models of $\alpha$ do not behave well, since they show a marked tendency to overestimate $K$ (generally symmetries greater than 3 and mean values greater than 1), except for model 10; this trend is opposite to that observed with heavy elements. On the other hand, $Z_3$ [absorption correction using the Gaussian expression for $\phi(\rho z)$] gives symmetries smaller than $ca$ 2.

For heavy element analysis, the best fitting models were the Gaussian and Philibert's simplified models. This was to be expected because, if absorption was moderate, the same values for $f(\chi)$ were assessed with the respective models of $\phi(\rho z)$ (see Fig. 4). For high absorption ($\chi > 1000 \text{ cm}^2 \text{ g}^{-1}$), the values for $f(\chi)$ were different, and the performances of the two models were different for light elements.

Bishop's square model, in spite of its roughness, showed good results, particularly model 10, whose symmetry for light elements was 1.69.

Next, the different atomic number correction models were evaluated, with Reed's factor $F$ and $Z_3$ (using Wilson's mean excitation energy), since the latter showed the best behaviour for both sets of specimens. Table 1 shows that for heavy elements, $Z_3$ with mass averaging was the best fitting model. The other models gave similar results among themselves, except those for the Love stopping power, with worse symmetry (models 16 and 17).

For oxide determinations, the histograms showed much greater asymmetry; the models utilizing Love's
Table 1. Evaluation of the different $Z$ and $A$ factors using Reed’s $F$

<table>
<thead>
<tr>
<th>Method</th>
<th>Model for $\phi(x)$</th>
<th>Parameters</th>
<th>Atomic number correction model</th>
<th>Mean excitation energy</th>
<th>Averaging</th>
<th>Rejected data</th>
<th>$K/K$</th>
<th>Symmetry</th>
<th>$\sigma_4$(%)</th>
<th>Rejected data</th>
<th>$K/K$</th>
<th>Symmetry</th>
<th>$\sigma_4$(%)</th>
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<td>Philibert simplified$^1$</td>
<td>$\sigma$ and $h$ by Heinrich$^3$</td>
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<td>Mass</td>
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<td>—</td>
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<td>$\sigma$ and $h$ by Love et al.$^{50}$</td>
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<td>6.39</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Gaussian</td>
<td>$\rho \bar{z}$ by Love et al.$^{56}$</td>
<td>S: Livingston$^{14}$, R: Springer</td>
<td>Mass</td>
<td>20</td>
<td>1.000</td>
<td>0.97</td>
<td>4.84</td>
<td>3</td>
<td>0.970</td>
<td>0.63</td>
<td>7.45</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Gaussian</td>
<td>$\rho \bar{z}$ by Love et al.$^{56}$</td>
<td>S: Livingston$^{14}$, R: Springer</td>
<td>Mass</td>
<td>20</td>
<td>1.000</td>
<td>1.00</td>
<td>4.96</td>
<td>2</td>
<td>0.978</td>
<td>0.70</td>
<td>6.88</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Comparison of the different models for the depth distribution of characteristic production with experimental data for copper (25 keV). —, Gaussian; ——-, Philibert’s simplified expressions$^1$; ---, Bishop.

Table 2. Fluorescence correction models with Gaussian $\mathcal{F}$ and $\mathcal{A}$ factors

<table>
<thead>
<tr>
<th>Model</th>
<th>$K/K$</th>
<th>Symmetry</th>
<th>$\sigma_4$(%)</th>
<th>Rejected data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(x-z)$</td>
<td>0.998</td>
<td>0.77</td>
<td>5.73</td>
<td>12</td>
</tr>
<tr>
<td>Gaussian</td>
<td>0.996</td>
<td>0.67</td>
<td>5.68</td>
<td>14</td>
</tr>
<tr>
<td>Criss and Birks$^{18}$</td>
<td>0.995</td>
<td>0.67</td>
<td>5.64</td>
<td>14</td>
</tr>
<tr>
<td>Reed$^{19}$</td>
<td>0.992</td>
<td>0.63</td>
<td>5.83</td>
<td>13</td>
</tr>
</tbody>
</table>

In order to evaluate the different $\mathcal{F}$ models, only 191 data were chosen from the list of 471, since in the remainder this correction was insignificant. $\mathcal{F}$ and $\mathcal{A}$ corrections were evaluated for the Gaussian expressions (using Wilson’s $J$ and mass averaging). The results for different $\mathcal{F}$ are shown in Table 2. As this is the smallest correction, it was expected that the different models would produce almost the same fitting; thus it was observed that a model as rough as $\delta(x-z)$ fitted data as accurately as Packwood and Brown’s distribution$^6$, which adjusted the experimental data for radiation stopping power and $\mathcal{F}_c$, with atomic averaging were the best fitting ones.

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distribution to a great extent. Every model improved the results corresponding to Reed’s model, the most frequently used one.

In conclusion, it may be said that it is convenient to choose the $\mathcal{F}$ expression which simplifies numerical calculations, since the differences are very small compared with the errors in $\mathcal{E}$ and $\mathcal{S}$ corrections.

**Acknowledgement**

The authors thank Lic. Horacio Brizuela for useful discussions of several problems; they are also indebted to the C.I.M. (Centro de Investigación de Materiales) for providing the access to its equipment. They acknowledge support from the CONICET (Consejo Nacional de Investigaciones Científicas y Técnicas) and the CONICOR (Consejo de Investigaciones Científicas y Técnicas de la Provincia de Córdoba).

**REFERENCES**


**APPENDIX**

When dealing with Gaussian $\phi(\rho z)$, $R(x)$ appearing in $f(x)$ is related to the complementary error function:

$$ R(x) = \sum_{i=0}^{\infty} \frac{a_i}{1 + px} $$

and the constants have the following values: $a_1 = 0.254820952$, $a_2 = 0.284496736$, $a_3 = 1.421414374$, $a_4 = -1.453152027$, $a_5 = 1.061405429$ and $p = 0.3275911$

The expressions for $R$ involved in the atomic number correction are as follows:

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$$ R = \sum_{i=0}^{\infty} c_i(E_i/E_0)^i $$

where the coefficients $c_i$ are tabulated in Ref. 10;

**Love et al.**:

$$ R = 1 - \eta [I(U_0) + \eta G(U_0) + \rho z]^{1/67} $$

where $I(U_0)$ and $U_0 G(U_0)$ are polynomials in $\ln U_0$.

The corresponding expressions for $S$ are as follows: Love et al.:

$$ S = \frac{Z}{A \left[ 1 + 16.05 \left( \frac{J}{F_i} \right)^{0.5} \left( \frac{U_0^{0.5} - 1}{U_0 - 1} \right)^{0.07} \right]} $$

Bethe:

$$ S = 7.85 \times 10^3 \frac{Z \ln \left( \frac{1.166 EM/\rho}{EM A} \right)}{A} $$

Livingston: the same expression as Bethe’s, but taking into account shell effects.

Fluorescence correction due to Criss and Birks involves the following function:

$$ f_i(x) = \frac{\gamma_0(0) - \phi_0}{2 \alpha x} \left[ \frac{2 \alpha x}{2 \alpha} \right] $$

in which $R(x)$ has just been specified. The corresponding coefficients for $\phi(\rho z)$ were fitted by the authors: $d_1 = -0.4780091$, $d_2 = 14.857378$, $d_3 = -13.08157$, $b_1 = 2301.5987$ cm$^{-2}$ g$^{-1}$, $b_2 = 3951.1108$ cm$^{-2}$ g$^{-1}$ and $b_3 = 5389.2626$ cm$^{-2}$ g$^{-1}$.

$S(a, x)$ involved in the Gaussian fluorescence correction model is as follows:

$$ S(a, x) = \sum_{n=0}^{\infty} \frac{a_n}{n!} \left[ \ln \left( \frac{1 + x}{x} \right) - \sum_{j=1}^{n} \frac{1}{j!} \frac{1}{j!} \frac{1}{(1 + x)^j} \right] $$

$$ + \sum_{j=1}^{n} \frac{1}{j!} \frac{1}{(1 + x)^j} \frac{1}{\mu_j} \left( \frac{1 + x}{x} \right) $$

The coefficients $p$ and $a_n$ appeared above in $R(x)$.

The following expression is obtained for the mean depth when dealing with the Gaussian $\phi(\rho z)$:

$$ \rho z = \frac{1}{\alpha \sqrt{\pi}} \left[ \frac{2 \alpha x}{2 \alpha} \sum_{n=1}^{\infty} \frac{a_n}{(1 + p/2 \cdot \beta/a)^n} \right] $$

$$ - \frac{1}{\alpha \sqrt{\pi}} \left[ \frac{2 \alpha x}{2 \alpha} \sum_{n=1}^{\infty} \frac{a_n}{(1 + p/2 \cdot \beta/a)^n} \right] $$

$$ + \frac{1}{\alpha \sqrt{\pi}} \left[ \gamma_0(0) - \phi_0 \left( \sum_{n=1}^{\infty} \frac{a_n}{(1 + p/2 \cdot \beta/a)^n} \right) \right] $$