Atomic and nuclear physics
X-ray physics
Detection of x-rays

Determining the ion dose rate of the x-ray tube with molybdenum anode

Objects of the experiment
- To introduce and to explain the terms ion dose (exposure dose) and ion dose rate for quantifying the action of x-rays.
- To determine the ion dose rate in an air-filled plate capacitor by measuring the ionization current.

Principles
Dosimetry is the quantitative measurement of the effects that x-rays cause when passing through matter and which can be used to detect x-rays. Dosimetry is thus important for medical and technical applications as well as in radiation protection. It does not represent a measurement of the actual intensity of the x-radiation. Such a determination would require e.g. calorimetric measurements in which the entire x-radiation is absorbed and converted into heat. However, with suitable calibration, measurements of dose and time can be used as a measure of the radiation intensity.

Dose and dose rate:
In terms of radiation, the dose can be defined on the basis of both the ionizing action and the energy absorption of the x-rays when they pass through matter. The first case is the measure of the ion dose (also called the exposure dose) and the second the absorbed dose.

The ion dose (exposure dose)
\[ J = \frac{dQ}{dm} \]  

is the quotient of the charge \( dQ \) generated in air by charge carriers of one sign due to irradiation, and the mass \( dm \) of the irradiated volume element. Its derived SI unit is the coulomb per kilogram (C kg\(^{-1}\)): 1 C kg\(^{-1}\) = 1 As kg\(^{-1}\)

The absorbed dose
\[ K = \frac{dW}{dm} \]  

is the quotient of the energy \( dW \) absorbed by the irradiated material and the mass \( dm \) of the irradiated volume element. Its derived SI unit is the gray (Gy): 1 Gy = 1 J kg\(^{-1}\)

Fig. 1 Diagram of beam path in plate capacitor, for calculating the irradiated volume \( V \)
The effective intensity of the x-rays is defined as the quotient of dose and time. The ion dose rate is defined as
\[ j = \frac{dI}{dt} \tag{III} \]
measured in A kg\(^{-1}\), and the absorbed dose rate is defined as
\[ k = \frac{dK}{dt} \tag{IV} \]
measured in Gy s\(^{-1}\) = W kg\(^{-1}\).

### Determining the ion dose rate:

The ion dose rate can be measured in an air-filled plate capacitor by measuring the saturation value of the ionization current \( I_C \) (see experiment P6.3.1.3). This is determined as
\[ I_C = \frac{dQ}{dt} \tag{V} \]
using (I) and (II), this gives us:
\[ j = \frac{dI_C}{dm} \tag{V} \]
As x-rays diverge as they propagate and are attenuated in air, the ion dose rate \( j \) is a location-dependent quantity, and would require a great deal of effort to measure. It is easier to measure the mean ion dose rate
\[ j > = \frac{I_C}{m^2} \tag{VI} \]
for which we need to determine the total ionization current \( I_C \) and the mass
\[ m = \rho \cdot V \tag{VII} \]
of the total irradiated volume \( V \).

The density \( \rho \) of air is calculated as
\[ \rho = \rho_0 \cdot \frac{T_0}{T} \cdot \frac{p}{p_0} \tag{VIII} \]
with \( \rho_0 = 1.293 \text{ kg m}^{-3}, T_0 = 273 \text{ K} \) and \( p_0 = 1013 \text{ hPa} \) from the temperature \( T \) and barometric pressure \( p \) in the experiment chamber. The volume \( V \) can be calculated with the aid of Fig. 1.

### Calculating the irradiated volume \( V \):
In Fig. 1, the focal spot of the x-ray tube is presumed to closely approximate a point. The rectangular diaphragm in front of the plate capacitor shapes the radiation cone of the x-ray tube into a beam which penetrates the volume \( V \) of air to be calculated.

The distance between the focal spot and the rectangular diaphragm is \( s_0 = 15.5 \text{ cm} \). The dimensions of the diaphragm are \( a_0 = 4.5 \text{ cm} \) and \( b_0 = 0.6 \text{ cm} \). The x-rays propagate in a straight line, and thus illuminate at any given distance \( s \) from the focal spot a rectangle behind the diaphragm with the dimensions
\[ a(s) = \frac{s}{s_0} \cdot a_0 \text{ and } b(s) = \frac{s}{s_0} \cdot b_0 \tag{IX} \]
The irradiated volume of air in the plate capacitor is thus equivalent to the integral
\[ V = \int_{s_1}^{s_2} a(s) \cdot b(s) \cdot ds \tag{X} \]
with the integral limits
\[ s_1 = s_0 + d \text{ and } s_2 = s_0 + d + D \tag{XI} \]
\( d = 2.5 \text{ cm}: \) distance from diaphragm to plate capacitor
\( D = 16.0 \text{ cm}: \) length of plate capacitor.

This gives us
\[ V = \frac{1}{3} \cdot a_0 \cdot b_0 \cdot \left( \frac{s_2^2 - s_1^2}{s_0^2} \right) \]
and thus
\[ V = a_0 \cdot b_0 \cdot D \cdot \left( \frac{s_2^2 + s_2s_1 + s_1^2}{s_0^2} \right) = 125 \text{ cm}^3 \tag{XII} \].

### Safety notes
The x-ray apparatus fulfills all regulations governing an x-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 80797 Rö).

The built-in protection and screening measures reduce the local dose rate outside of the x-ray apparatus to less than 1 μSv/h, a value which is on the order of magnitude of the natural background radiation.

- Before putting the x-ray apparatus into operation inspect it for damage and to make sure that the high voltage is shut off when the sliding doors are opened (see Instruction Sheet for x-ray apparatus).
- Keep the x-ray apparatus secure from access by unauthorized persons.
- Do not allow the anode of the x-ray tube Mo to overheat.
- When switching on the x-ray apparatus, check to make sure that the ventilator in the tube chamber is turning.
Setup

Set up the experiment as shown in Fig. 2. Fig. 3 shows the electrical connections of the plate capacitor and the electrometer amplifier for determining the ionization current.

Mechanical setup:
- If necessary, demount the collimator of the x-ray apparatus and remove any experiment equipment from the chamber of the x-ray apparatus.
- Connect the adapter cable BNC/4 mm to the bottom capacitor plate (BNC socket) with the BNC plug and connect the connecting lead to the top capacitor plate (safety socket) of the plate capacitor x-ray.
- Lift the plate capacitor into the experiment chamber of the x-ray apparatus and insert the mounting plugs in the mounting sockets. Check to make sure that the capacitor plates are aligned parallel to the base plate of the x-ray apparatus, and correct as necessary.
- Feed the two cables into the free channel until they reappear on the right side of the x-ray apparatus.

Electrical assembly:
- Connect the connecting lead to the positive pole of the 450 V DC power supply and connect the adapter cable BNC/4 mm to the electrometer amplifier fitted with the 1 GΩ resistor.
- Ground the electrometer amplifier to the negative terminal of the 450 V DC amplifier.
- Connect a voltmeter to measure the capacitor voltage $U_C$ and the output voltage of the electrometer amplifier $U_E$.
- Plug in the x-ray apparatus to the mains power and switch it on.

Carrying out the experiment

- Determine the temperature $\theta$ and the barometric pressure $p$ in the experiment chamber and use these to calculate the irradiated mass $m$ according to equations (VII) and (VIII).

a) Saturation ionization current $I_C$ as a function of the emission current $I$:
- Set the tube high voltage to $U = 35$ kV.
- Set the capacitor voltage $U_C \geq 140$ V, so that the saturation value of the ionization current $I_C$ is reached.

To record a measurement series, increase the emission current $I$ in steps from 0 mA to 1 mA and determine the ionization current $I_C$ for each step from the voltage $U_E$ at the output of the electrometer amplifier:

$$I_C = \frac{U_E}{1 \text{ G}\Omega}$$

- Write down your measuring results and the calculated mean ion dose rate.

b) Saturation ionization current $I_C$ as a function of the tube high voltage $U$:
- Set the emission current $I = 1.0$ mA.
- Set the capacitor voltage $U_C \geq 140$ V.

Increase the tube high voltage $U$ in steps from 5 kV to 35 kV and determine the corresponding ionization current $I_C$.

- Write down your measuring results and the calculated mean ion dose rate.

Measuring example and evaluation

$T = 303$ K and $p = 1017$ hPa:

From equation (VIII) we can calculate $p = 1.17 \text{ kg m}^{-3}$ and from (VII) and (XII) $m = 0.147 \times 10^{-3}$ kg

Thus, in accordance with (VI) the mean ion dose rate is

$$\frac{<j>}{\mu A \cdot \text{kg}^{-1}} = \frac{I_C}{nA} \frac{1}{0.147 \text{ kg}}$$
a) Measurements as a function of the emission current $I$:

Tab. 1: Saturation value of the ionization current $I_C$ and the mean ion dose rate $<j>$ as a function of the emission current $I$ of the x-ray tube, tube high voltage $U = 35$ kV.

<table>
<thead>
<tr>
<th>$I$ (mA)</th>
<th>$I_C$ (nA)</th>
<th>$&lt;j&gt;$ ($\mu$A $\cdot$ kg$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>0.1</td>
<td>0.48</td>
<td>3.27</td>
</tr>
<tr>
<td>0.2</td>
<td>0.92</td>
<td>6.27</td>
</tr>
<tr>
<td>0.3</td>
<td>1.30</td>
<td>8.86</td>
</tr>
<tr>
<td>0.4</td>
<td>1.72</td>
<td>11.7</td>
</tr>
<tr>
<td>0.5</td>
<td>2.10</td>
<td>14.3</td>
</tr>
<tr>
<td>0.6</td>
<td>2.45</td>
<td>16.7</td>
</tr>
<tr>
<td>0.7</td>
<td>2.80</td>
<td>19.1</td>
</tr>
<tr>
<td>0.8</td>
<td>3.20</td>
<td>21.8</td>
</tr>
<tr>
<td>0.9</td>
<td>3.55</td>
<td>24.2</td>
</tr>
<tr>
<td>1.0</td>
<td>3.90</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Tab. 2: Saturation value of ionization current $I_C$ and mean ion dose rate $<j>$ as a function of the tube high voltage $U$, emission current $I = 1.0$ mA.

<table>
<thead>
<tr>
<th>$U$ (kV)</th>
<th>$I_C$ (nA)</th>
<th>$&lt;j&gt;$ ($\mu$A $\cdot$ kg$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>7.5</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>10.0</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>12.5</td>
<td>0.03</td>
<td>0.20</td>
</tr>
<tr>
<td>15.0</td>
<td>0.10</td>
<td>0.68</td>
</tr>
<tr>
<td>17.5</td>
<td>0.26</td>
<td>1.77</td>
</tr>
<tr>
<td>20.0</td>
<td>0.49</td>
<td>3.34</td>
</tr>
<tr>
<td>22.5</td>
<td>0.81</td>
<td>5.52</td>
</tr>
<tr>
<td>25.0</td>
<td>1.25</td>
<td>8.52</td>
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<td>27.5</td>
<td>1.82</td>
<td>12.4</td>
</tr>
<tr>
<td>30.0</td>
<td>2.40</td>
<td>16.4</td>
</tr>
<tr>
<td>32.5</td>
<td>3.10</td>
<td>21.1</td>
</tr>
<tr>
<td>35.0</td>
<td>3.90</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Fig. 4  Mean ion dose rate $<j>$ as a function of the emission current $I$ of the x-ray tube, $U = 35$ kV

Fig. 5  Mean ion dose rate $<j>$ as a function of the tube high voltage $U$, $I = 1.0$ mA

Results

At the maximum operating parameters of the x-ray tube ($U = 35$ kV, $I = 1.0$ mA), the mean ion dose rate in the plate capacitor is

$<j> = 26.6 \mu$A kg$^{-1}$.

Additional information

In addition to the ion dose and the absorbed dose, the dose equivalent $D$ is a further important quantity. It is measured in sieverts (Sv): $1$ Sv $= 1$ J kg$^{-1}$.

A radiation with a specific dose equivalent has the same biological effect on tissues as hard x-rays generated with a voltage of $200$ kV and having the corresponding absorbed dose. In x-ray and $\gamma$ radiation, the absorbed dose $K$ and the dose equivalent $D$ are identical when $K$ is measured in Gy and $D$ in Sv. The conversion factor for the ion dose $J$ is: $1$ Sv $\equiv 0.0308$ As kg$^{-1}$.

The mean ion dose rate of $26.6$ A kg$^{-1}$ in the plate capacitor thus corresponds to a dose equivalent rate of $864$ $\mu$Sv s$^{-1}$ resp. $3.11$ Sv h$^{-1}$.

For comparison: in the measurements for type approval of the x-ray apparatus, a dose equivalent rate of over $10$ Sv h$^{-1}$ was measured in the radiation cone of the x-ray tube.