The CSL model as a mechanism for the quantum-to-classical transition of the primordial perturbations

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Successes of the Inflationary Paradigm

- **Inflation**: A phase of accelerated expansion in the early Universe ($\ddot{a} > 0$) induced by a type of matter satisfying $\rho + 3P < 0$. It is usually modeled by a scalar field $\phi$ called the **inflaton**.

- Solves the problems of the Big Bang model (Horizon, Flatness, etc.)

- The main success... **Quantum Fluctuations of the inflaton** results in theoretical predictions of the **spectrum of primordial inhomogeneities**. [Mukhanov, Chibisov 1982]

- Observational Data in agreement with such predictions
The Data

- The temperature anisotropies of the CMB photons emitted by the Last Scattering Surface are related to the **curvature perturbation** $\Psi$ (i.e., the **scalar metric perturbation**, in the appropriate gauge) by

$$\frac{\delta T}{T_0}(\theta, \varphi) \simeq \frac{1}{3} \Psi(\eta_D, \mathbf{x}_D)$$

(1)

- Moreover, one can use the spherical harmonic functions and write

$$\frac{\delta T}{T_0}(\theta, \varphi) = \sum_{m,l} a_{lm} Y_{lm}(\theta, \varphi).$$

Thus,

$$a_{lm} = \frac{1}{3} \int d\Omega \Psi(\eta_D, \mathbf{x}_D) Y_{lm}^*(\theta, \varphi)$$

(2)

This is the quantity that is measured.

- The inflationary paradigm is supposed to provide a physical mechanism to relate $\Psi$ with the quantum fluctuations of the matter fields in the early Universe.
Theoretical Predictions

- The QFT of inflation can be described by the quantization of the Mukhanov-Sasaki variable

\[ v \equiv a\delta \phi + \Psi \phi_0 a^2 / a' \]

where \( \phi(\mathbf{x}, \eta) = \phi_0(\eta) + \delta \phi(\mathbf{x}, \eta) \); \( a(\eta) \) is the scale factor and \( \eta \) is the conformal time.

- Quantization of \( v \) implies quantization of \( \delta \phi \) and \( \Psi \), i.e. \( \hat{\delta \phi} \) and \( \hat{\Psi} \).

- One proceeds to calculate the 2-point correlation function in the vacuum state \( |0\rangle \):

\[ \langle 0 | \hat{\Psi}(\eta, \mathbf{x}) \hat{\Psi}(\eta, \mathbf{y}) |0\rangle \]

(3)

- Once \( \langle 0 | \hat{\Psi}(\eta, \mathbf{x}) \hat{\Psi}(\eta, \mathbf{y}) |0\rangle \) is known, one proceeds to make the identification

\[ \langle 0 | \hat{\Psi}(\eta, \mathbf{x}) \hat{\Psi}(\eta, \mathbf{y}) |0\rangle = \Psi(\eta, \mathbf{x}) \Psi(\eta, \mathbf{y}) \]

(4)

- For slow roll inflation one defines the slow-roll parameters

\[ \epsilon \equiv 1 - \mathcal{H}' / \mathcal{H}^2 \simeq M_P^2 / 2 (\partial \phi V/V)^2 ; \quad \delta \equiv M_P^2 (\partial^2 \phi V/V) \].

The theoretical prediction is the (scalar) power spectrum

\[ P_{\Psi}(k) = \frac{1}{2\pi^2} |\Psi_k|^2 k^3 \propto \left( \frac{H^2}{M_P^2 \epsilon} \right) \left|_{k=aH} \right. ; \quad n_s - 1 \equiv \frac{d \ln P(k)}{d \ln k} = -6\epsilon + 2\delta \]

(5)
We rewrite the expression for the coefficients $a_{lm}$ in terms of the Fourier modes.

$$a_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3k}{(2\pi)^3} j_l(kR_D) Y^*_{lm}(\hat{k}) \Delta(k) \Psi_k,$$

with $j_l(kR_D)$ the spherical Bessel function of order $l$ and $R_D$ the comoving radius of the last scattering surface. Additionally, $\Delta(k)$ the transfer functions.

The classical scalar metric perturbation $\Psi_k$ corresponds to the primordial curvature perturbation.

How does the transition $\hat{\Psi}_k \rightarrow \Psi_k$ takes place?
The Standard Answer

- Evidently if one uses $\langle 0 | \hat{\Psi}_k | 0 \rangle = \Psi_k = 0$, then $a_{lm} = 0$ for all $l, m$. The standard argument is that it is incorrect to state that $a_{lm} = 0$; the correct statement is $\overline{a_{lm}} = 0$, i.e. the average over an “ensemble of universes.” Then, what is the prediction for $a_{lm}$ within our universe?

- **Official Answer:** The classic curvature perturbation $\Psi_k$ that is being used to calculate $a_{lm}$ is related with the quantum fluctuations of the vacuum, this is

$$\Psi_k = \sqrt{\langle 0 | \hat{\Psi}_k^2 | 0 \rangle} e^{i\alpha_k}$$

(7)

thus, the amplitude of the classical quantity $\Psi_k$ is equal to the quantum uncertainty $\sqrt{\langle 0 | \hat{\Psi}_k^2 | 0 \rangle}$ and $\alpha_k$ corresponds to a random phase.

- **Justification 1:** Because it works or *Shut up and calculate!*

- **Justification 2:** Decoherence, evolution of the vacuum state into a squeezed state, etc.
The fundamental problem

- **Remarkable**: The universe was originally described by a space-time which is homogeneous and isotropic, and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of $e^{-80}$), but the universe ended up with inhomogeneities that fit the experimental data.

- **Question** How do we end up in a situation which is not symmetric (the symmetry being the homogeneity and isotropy) given that there is nothing in the dynamics that breaks such symmetries?

- In other words...

**How did it happen?**

\[
|\text{symmetric}\rangle \xrightarrow{\text{Schröd. Eq.}} |\text{non-symmetric}\rangle \quad (\star)
\]
It is clear that the question (★) is closely related to the quantum measurement problem, which in the cosmological context, appears in an exacerbated manner.

We prefer the *Shut up and let me think* alternative lifestyle.

Often these issues are resolved as “just philosophy,” with no impact whatsoever on the theoretical predictions. We will see that such preconception is mistaken.
The inflationary paradigm

The quantum measurement problem and inflation

The CSL inflationary model

Comparing to the observational data

The transition and the collapse proposal

Our approach on attempting to answer the question (★) is described as:

- By invoking the collapse of the wave function, one could break the symmetry of the original state.

\[ |\text{symmetric}\rangle \longrightarrow \text{Collapse} \longrightarrow |\text{non-symmetric}\rangle \quad (8) \]

- According to standard QM the collapse of the wave function is the result of a **measurement**. But it is not clear how to apply this postulate to the cosmological setting?

**Phenomenological Model**: Some element intrinsic to the system induced the collapse of the wave function.

**The Continuous Spontaneous Localization (CSL) model** [Pearle,Ghirardi 1989] was proposed to provide a solution to the measurement problem. In particular, the “observers” do not play a role at all.

- Our goal is to apply the CSL model to the inflationary universe and obtain testable predictions.
The idea is to modify the Schrödinger equation in such a way that includes the reduction of the wave function. In particular:

- For **microscopic systems**, the wave function must behave like Schrödinger's wave function: it must **diffuse, superimpose, interfere, ...**
- For **macroscopic objects**, the wave function must be always well localized in space and behave like a point moving according to Newton's laws.
- In a measurement situation, one recovers **quantum probabilities** and **postulate of w.p. reduction**
Dynamics of the CSL model

The CSL model proposes the modification

\[ d|\psi\rangle = \left\{ \left[ -i\hat{H} - \frac{\lambda}{2} \left( \hat{A} - \langle \hat{A} \rangle \right)^2 \right] dt + \sqrt{\lambda} \left( \hat{A} - \langle \hat{A} \rangle \right) dW_t \right\} |\psi\rangle \quad (9) \]

- The stochastic behavior is codified in the noise \( W(t) \), formally known as a Wiener process, i.e. a process that satisfy
  \( \mathbb{E}(dW_t) = 0 \),
  \( \mathbb{E}(dW_t dW_{t'}) = \delta(t - t')dt^2 \), where \( \mathbb{E} \)
  is an ensemble average over possible realizations of \( W_t \).
- The operator \( \hat{A} \) is known as the collapse operator. The wave function evolves (stochastically) to one of the eigen-states of \( \hat{A} \).
- The parameter \( \lambda \) measures the “strength” of the collapse;
  \( \lambda \equiv (m/m_0)\lambda_{CSL} \)
- Lower Bound: \( \lambda_{CSL} = 10^{-16}\text{s}^{-1} \).
  Upper Bound: \( 10^{12}\lambda_{CSL} \)

\( \lambda_0 \text{ too large:} \) violation of known experimental data

\( \lambda_0 \text{ too small:} \) macro-objects are not localized

\[ \lambda_0 \text{ OK!} \]
Relating the classical quantities with the quantum objects

- Let us recall the expression for \( a_{lm} \),

\[
a_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3 k}{(2\pi)^3} j_l(kR_D) Y_{l m}^*(\hat{k}) \Delta(k) \Psi_k,
\]

- The CSL model evolves the vacuum state \( |0\rangle \) to a final state \( |\Theta\rangle \) which does not poses the same symmetries as the vacuum state.

- How to relate \( \Psi_k \) with \( \hat{\Psi}_k \)?
Relating the classical quantities with the quantum objects

- **First option:** Use semi-classical gravity \( G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle \).
- The geometry is always classical, only the matter fields are quantized.
- Einstein semi-classical equations lead to:

\[
\Psi_k(\eta) = \frac{H}{k^2 M_P} \sqrt{\frac{2}{\epsilon}} \langle \Theta(\eta) | \hat{\pi}_k | \Theta(\eta) \rangle.
\]  

[11]

[Cañate, Pearle & Sudarsky 2013], where \( \hat{\pi}_k \) is the canonical conjugated momentum to \( \hat{\phi}_k \).

- Under some circumstances the model is consistent with observational data.

- On the other hand, it has been shown that in the semi-classical gravity framework, the tensor-to-scalar ratio is [GL, L. Kraiselburd & S. Landau, 2015]

\[
r < 10^{-9} \epsilon^2
\]
Relating the classical quantities with the quantum objects

- **Second option** (our approach) [GL, G. Bengochea 2015]
- \( \Psi_k = \langle \hat{\Psi}_k \rangle \)
- At the beginning of inflation, the quantum state of inflaton is the vacuum \( |0\rangle \). At the end of inflation the state of each mode of the inflaton is \( |\Theta\rangle \). The CSL model drives the evolution \( |0\rangle \rightarrow |\Theta\rangle \). Thus,

\[
\Psi_k(\tau) = \langle 0 | \hat{\Psi}_k | 0 \rangle = 0 \quad (12)
\]

\[
\Psi_k(\eta) = \langle \Theta(\eta) | \hat{\Psi}_k | \Theta(\eta) \rangle \quad (13)
\]
CSL inflationary model

**Step I:**

- Write the Hamiltonian for the Mukhanov-Sasaki variable

\[
H_{k}^{R,I} = p_{k}^{R,I} p_{k}^{*R,I} + \frac{z'}{z} \left( v_{k}^{R,I} p_{k}^{*R,I} + v_{k}^{*R,I} p_{k}^{R,I} \right) + k^2 v_{k}^{R,I} v_{k}^{*R,I}, \tag{14}
\]

The indexes \( R, I \) denote the real and imaginary parts of the field and its momentum.

- The canonical conjugated momentum to \( v_{k} \) is

\[
p_{k} = v_{k}' - \left( \frac{z'}{z} \right) v_{k}, \tag{15}
\]

with \( z \equiv a \phi' / \mathcal{H} \) and \( \mathcal{H} \equiv a' / a \)

- Promote to quantum operators \( \hat{v}_{k} \) and \( \hat{p}_{k} \) with commutator

\[
[\hat{v}_{k}^{R,I}, \hat{p}_{k'}^{R,I}] = i \delta(k - k').
\]
CSL inflationary model

**Step II:**

- Write the wave functional (it is convenient to work in the momentum representation) \( \Phi[p(\eta, x)] = \Pi_k \Phi_k[p^R_k, p^I_k] = \Pi_k \Phi^R_k(p^R_k, \eta) \Phi^I_k(p^I_k, \eta) \).

- The wave functional associated to each model of the real and imaginary part

\[
\Phi^{R,I}(\eta, p_k^{R,I}) = \exp[-A_k(\eta)(p_k^{R,I})^2 + B_k(\eta)p_k^{R,I} + C_k(\eta)] \tag{16}
\]

evolves according to the CSL-Schrödinger equation, the initial conditions are \( A_k(\tau) = 1/2k, B_k(\tau) = C_k(\tau) = 0 \) corresponding to the Bunch-Davies vacuum and \( \tau \) is the conformal time at the beginning of inflation.
CSL Inflationary model

**Step III:** Choose the collapse operator.
- Einstein perturbed equations $\delta G_{ab} = 8\pi G \delta T_{ab}$ lead to

$$\Psi_k = \langle \hat{\Psi}_k \rangle = \sqrt{\frac{\epsilon}{2 k^2 M_P}} \langle \hat{p}_k \rangle = \sqrt{\frac{\epsilon}{2 k^2 M_P}} \left[ \langle \hat{p}_k^R \rangle + i \langle \hat{p}_k^I \rangle \right], \quad (17)$$

- The collapse operator is $\hat{p}_k$. 
CSL Inflationary model

Step IV:

- Solve the CSL-Schrödinger equation

\[
|\Phi_{R,I}^{k,R,I}(\eta)\rangle = \hat{T} \exp\left\{-\int_{\tau}^{\eta} d\eta'\left[i\hat{H}_{R,I}^{k,R,I} + \frac{1}{4\lambda_k}(W(\eta') - 2\lambda_k\hat{p}_{R,I}^{k,R,I})^2\right]\right\}|\Phi_{R,I}^{k,R,I}(\tau)\rangle
\]

(18)

- We can now compute

\[
\Psi_k = \langle \hat{\Psi}_k \rangle = \sqrt{\frac{\epsilon}{2k^2M_P}} \langle \hat{p}_k \rangle = \sqrt{\frac{\epsilon}{2k^2M_P}} [\langle \hat{p}_R^k \rangle + i\langle \hat{p}_I^k \rangle],
\]

(19)

- Additionally, one has

\[
[\Delta^2 \hat{\Psi}_{R,I}^k(\eta)] \rightarrow 0 \text{ and also } \langle \hat{\Psi}_{R,I}^k \rangle \neq 0
\]

(20)

as \(a(\eta)\) is increasing.
Step V:

- Obtain the theoretical predictions, in particular, we can calculate the scalar power spectrum defined as:

\[ \Psi_k \Psi_{k'}^* \equiv \frac{2\pi^2}{k^3} P_s(k) \delta(k - k') \]  

(21)

- Once the power spectrum is known, the model yields a prediction for the observational quantity \( C_l = (2l + 1)^{-1} \sum_m |a_{lm}|^2 \).

\[ C_l = 4\pi \int_0^{\infty} \frac{dk}{k} j_l^2(k R_D) \Delta^2(k) P_s(k) \]  

(22)
Theoretical Predictions

- Our approach predicts

\[ \mathcal{P}_s(k) = A_s k^{n_s - 1} F(\lambda_k) \]  

where

\[ A_s = \frac{H^2}{M_P^2 \epsilon}, \quad n_s - 1 = -6 \epsilon + 2 \delta, \]

- If \( \lambda_k = 0 \) then \( F(0) = 0 \), i.e. the space-time is perfectly homogeneous and isotropic \( \Psi_k = \langle 0 | \hat{\Psi}_k | 0 \rangle = 0 \).
- The function \( F(\lambda_k) \) can be made nearly scale invariant if

\[ \lambda_k = \frac{1}{k |\tau|} \equiv \frac{\lambda_0}{k} \]

- The conformal time at the beginning of inflation \( \tau \) depends on the e-foldings of inflation \( N \equiv \ln(a_f/a_i) \) and the energy scale associated to inflation \( V^{1/4} \).
The Power Spectrum

- The standard prediction for the PS is $P_s(k) = A_s k^{n_s - 1}$, with $n_s = 0.96$
- Our prediction is $P_s(k) = A_s k^{n_s - 1} F(\lambda_k)$, with $\lambda_k = \lambda_0/k$. The value of $\lambda_0$ depends on $N$ and $V^{1/4}$.
- We assume $E_{GUT} = 10^{16}$ GeV and variate $N$
- Plot $P_s(k)$ vs. $k$
Plot of the angular power spectrum (TT) $l(l + 1)C_l$ vs. $l$
Tensor modes

- The prediction for the tensor modes are

\[ P_t(k) = A_t k^{n_t} F(\lambda_k) \]  \hspace{1cm} (26)

with

\[ A_t = \frac{H^2}{M_P^2}, \quad n_t = -2\epsilon \]  \hspace{1cm} (27)

- The tensor-to-scalar ratio \( r \)

\[ r \equiv \frac{P_t}{P_s} = 16\epsilon \]  \hspace{1cm} (28)
Conclusions

- When applying the CSL model to inflation, we obtain a modification for the power spectrum codified in the function $F(\lambda_k)$.

- If $\lambda_k = \lambda_0 / k$ is possible to match the standard prediction under some circumstances. In particular, $\lambda_0$ depends on $N$ and $V^{1/4}$.

- On the other hand, it is possible to explore small differences between our prediction and the standard one by considering $\lambda_k = [\lambda_0 / k]^\alpha$ (might behave as a running of the spectral index).

- On the other hand, the CSL model is not relativistic; therefore, there is no way to justify, from first principles, the choice of the collapse operator and the recipe $\lambda_k = \lambda_0 / k$.

- Introducing a collapse mechanism violates the energy conservation, which might induces divergences in the energy-momentum tensor (future work).

- In our opinion, the CSL model can be applied, at least in a Phenomenological way, to the inflationary universe. That is, in a way that is consistent with the observational data. Nevertheless, there is future work to be done...
Gracias! - Obrigado!