REPRESENTATION EQUIVALENCE AND P-SPECTRUM OF CONSTANT CURVATURE SPACE FORMS

E. A. LAURET, R. J. MIATELLO AND J. P. ROSSETTI

Abstract. We study the p-spectrum of a locally symmetric space of constant curvature $\Gamma\backslash X$, in connection with the right regular representation of the full isometry group G of X on $L^2(\Gamma\backslash G)_{\tau_p}$, where τ_p is the complexified p-exterior representation of $\mathrm{O}(n)$ on $\bigwedge^p(\mathbb{R}^n)_{\mathbb{C}}$. We give an expression of the multiplicity $d_\lambda(p,\Gamma)$ of the eigenvalues of the p-Hodge-Laplace operator in terms of multiplicities $n_\Gamma(\pi)$ of specific irreducible unitary representations of G.

As a consequence, we extend results of Pesce for the spectrum on functions to the p-spectrum of the Hodge-Laplace operator on p-forms of $\Gamma\backslash X$, and we compare p-isospectrality with τ_p-equivalence for $0 \leq p \leq n$. For spherical space forms, we show that τ-isospectrality implies τ-equivalence for a class of τ's that includes the case $\tau = \tau_p$. Furthermore we prove that $p-1$ and $p+1$-isospectral implies p-isospectral.

For nonpositive curvature space forms, we give examples showing that p-isospectrality is far from implying τ_p-equivalence, but a variant of Pesce's result remains true. Namely, for each fixed p, q-isospectrality for every $0 \leq q \leq p$ implies τ_q-equivalence for every $0 \leq q \leq p$. As a byproduct of the methods we obtain several results relating p-isospectrality with τ_p-equivalence.