TECHNIQUES FOR CLASSIFYING HOPF ALGEBRAS AND APPLICATIONS TO DIMENSION p^3

MARGARET BEATTIE AND GASTÓN ANDRÉS GARCÍA

ABSTRACT. The classification of all Hopf algebras of a given finite dimension over an algebraically closed field of characteristic 0 is a difficult problem. If the dimension is a prime, then the Hopf algebra is a group algebra. If the dimension is the square of a prime then the Hopf algebra is a group algebra or a Taft Hopf algebra. The classification is also complete for dimension 2p or $2p^2$, p a prime. Partial results for some other cases are available. For example, for dimension p^3 the classification of the semisimple Hopf algebras was done by Masuoka, and the pointed Hopf algebras were classified by Andruskiewitsch and Schneider, Caenepeel and Dăscălescu, and Ștefan and van Oystaeyen independently. Many classification results for the nonsemisimple, nonpointed, non-copointed case have been proved by the second author but the classification in general for dimension p^3 is still incomplete, up to now even for dimension 27.

In this paper we outline some results and techniques which have been useful in approaching this problem and add a few new ones. We give some further results on Hopf algebras of dimension p^3 and finish the classification for dimension 27.

Department of Mathematics and Computer Science Mount Allison University Sackville, NB E4L 1E6 Canada

FACULTAD DE MATEMÁTICA, ASTRONOMÍA Y FÍSICA &
FACULTAD DE CIENCIAS EXACTAS, FÍSICAS Y NATURALES,
UNIVERSIDAD NACIONAL DE CÓRDOBA. CIEM - CONICET.
MEDINA ALLENDE S/N
(5000) CIUDAD UNIVERSITARIA, CÓRDOBA, ARGENTINA
E-mail address: mbeattie@mta.ca
E-mail address: ggarcia@famaf.unc.edu.ar

Date: August 30, 2011.

²⁰¹⁰ Mathematics Subject Classification. 16T05.

M. Beattie's research was partially supported by an NSERC Discovery Grant. G. A. García's research was partially supported by ANPCyT-Foncyt, CONICET, Ministerio de Ciencia y Tecnología (Córdoba) and Secyt (UNC).