IRREDUCIBLE REPRESENTATIONS OF LIFTINGS OF QUANTUM PLANES

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In this note, the irreducible representations of a lifting of a quantum plane are determined.

1. Introduction

In [5, 1], the structure of the coradicals of the duals of liftings of some quantum linear spaces was studied, and several examples were explicitly constructed. In this note, we describe the irreducible representations for any lifting of a quantum plane. If A is the lifting of a quantum linear plane, then the approach in [5, 1] was via the coradical of A^*. The coalgebra $A^* = C$ was written as a direct sum of certain sub-coalgebras $C(\eta)$ with $\eta \in \hat{\Gamma}$ and the coradicals of the $C(\eta)$ were determined. Here we write the lifting A as a product of algebras $A(\xi)$, $\xi \in \hat{\Gamma}$, and determine irreducible representations of each of six possible types of algebra $A(\xi)$ that can arise.

Notation

Let k be an algebraically closed field of characteristic 0. If G is a finite group then ${}^G_G\mathcal{Y}D = {}^{k[G]}_{k[G]}\mathcal{Y}D$ will denote the category of Yetter-Drinfel'd modules over the group algebra $k[G]$.

Throughout, we let Γ be a finite abelian group and let $\hat{\Gamma}$ denote the group of characters of Γ. For $V \in_{\Gamma}^{\Gamma} \mathcal{YD}$, $g \in \Gamma, \chi \in \hat{\Gamma}$, we write V_g^{χ} for the set of $v \in V$ with the action of Γ on v given by $h \rightharpoonup v = \chi(h)v$ and the coaction by $\delta(v) = g \otimes v$. Since Γ is an abelian group, $V = \oplus_{g\in\Gamma,\chi\in\widehat{\Gamma}} V_g^{\chi}$ [2, Section 2].

The coradical of a coalgebra C is denoted C_0.

If v is a complex number and a is a non-negative integer, then we set as usual

$$(0)_v = 0, \qquad (a)_v = 1 + v + \dots v^{a-1} = 1 + v(a-1)_v = (a-1)_v + v^{a-1}.$$

2. Preliminaries

Let A be a finite-dimensional pointed Hopf algebra; let V be the infinitesimal braiding of A [2]. Recall that $V \in_{G(A)}^{G(A)} \mathcal{YD}$ is the Yetter-Drinfeld module of right coinvariant elements in the $kG(A)$-Hopf module A_1/A_0.

Let Λ be a subgroup of $G(A)$ central in A. For $\xi \in \widehat{\Lambda}$, let

$$e_{\xi} = |\Lambda|^{-1} \sum_{g\in\Lambda} \xi^{-1}(g) g$$

denote the minimal idempotent corresponding to ξ. If also $\eta \in \widehat{\Lambda}$ then

$$e_{\eta} \leftharpoonup \xi = \langle \xi, e_{\eta,(1)} \rangle e_{\eta,(2)} = e_{\xi^{-1}\eta}. \tag{1}$$

Clearly, Ae_{ξ} is a two-sided ideal of A and $A = \oplus_{\xi\in\widehat{\Lambda}} Ae_{\xi}$. Furthermore, let $J_{\xi} = \oplus_{\xi\neq\zeta\in\widehat{\Lambda}} Ae_{\zeta}$ and let $A(\xi) := A/J_{\xi}$; we have an isomorphism of algebras $A \simeq \prod_{\xi\in\widehat{\Lambda}} A(\xi)$. Hence, we have an isomorphism of coalgebras $C := A^* \simeq \oplus_{\xi\in\widehat{\Lambda}} C(\xi)$ where $C(\xi) := A(\xi)^* = J_{\xi}^{\perp}$. Clearly, $C(\epsilon)$ is a Hopf subalgebra of C; it is dual to the quotient Hopf algebra $A/J_{\epsilon} = A/Ak[\Lambda]^+$. It turns out that a useful approach to the description of the coalgebra structure of C is via the coalgebras $C(\xi)$ for a well-chosen subgroup Λ. Indeed, $C_0 := \oplus_{\xi\in\widehat{\Lambda}} C(\xi)_0$ by [9, 9.0.1].

Assume that $\xi \in \widehat{\Lambda}$ is the restriction of $\widetilde{\xi} \in G(A)$. Then $\widetilde{\xi} C(\zeta) = C(\xi\zeta)$ for any $\zeta \in \widehat{\Lambda}$. Indeed, $\widetilde{\xi} C(\zeta) \subset (Ae_{\eta})^{\perp}$ for any $\eta \in \widehat{\Lambda}$, $\eta \neq \xi\zeta$, by (1); thus $\widetilde{\xi} C(\zeta) = J_{\xi\zeta}^{\perp}$.

Let us consider the following hypotheses.

(a) The group $\Gamma := G(A)$ is abelian. Then there exists a basis $v_1, \dots v_{\theta}$ of V with $v_i \in V_{g_i}^{\chi_i}$ for all i. Let $r_i > 1$ be the order of $\chi_i(g_i)$.

(b) V is a quantum linear space; that is, $\chi_i(g_j)\chi_j(g_i) = 1$ for $i \neq j$.

Furthermore, assuming (a) and (b), we choose the subgroup Λ of Γ as follows:

(c) $\Lambda = \{g \in \Gamma : \chi_i(g) = 1 \text{ for } 1 \leq i \leq \theta\}$.

Note that by the definition in (c), Λ is central in A, and if $\chi_j^{r_j} = \epsilon$ or if $\chi_i\chi_j = \epsilon$ for all $i \neq j$, then

$$g_j^{r_j} \in \Lambda \text{ and } \chi_j \in \widehat{\Gamma/\Lambda}. \tag{2}$$

Also since the sequence $1 \to \widehat{\Gamma/\Lambda} \to \widehat{\Gamma} \to \widehat{\Lambda} \to 1$ is exact, we can always choose a preimage $\widetilde{\xi}$ in $\widehat{\Gamma}$ of $\xi \in \widehat{\Lambda}$; then $C(\xi) = \widetilde{\xi}C(\epsilon)$.

Proposition 2.1. *Let A be a finite-dimensional pointed Hopf algebra; let V be the infinitesimal braiding of A. Assume (a) and (b) above, i.e. A is a lifting of a quantum linear space with abelian group of grouplikes. Then A is generated by the grouplike elements $h \in \Gamma$ and by $(1, g_i)$-primitives $x_i, 1 \leq i \leq \theta$ with defining relations*

$$\begin{aligned} hx_i &= \chi_i(h)x_ih; \\ x_i^{r_i} &= \alpha_{ii}(g_i^{r_i} - 1); \\ x_ix_j &= \chi_j(g_i)x_jx_i + \alpha_{ij}(g_ig_j - 1), \end{aligned}$$

for all $h \in \Gamma$, $1 \leq i \leq \theta$. We have dim $A = |\Gamma|r_1 \ldots r_\theta$.

We may assume $\alpha_{ii} \in \{0, 1\}$ and then we must have that

$$\begin{aligned} \alpha_{ii} &= 0 \text{ if } g_i^{r_i} = 1 \text{ or } \chi_i^{r_i} \neq \epsilon; \\ \alpha_{ij} &= 0 \text{ if } g_ig_j = 1 \text{ or } \chi_i\chi_j \neq \epsilon. \end{aligned}$$

Note that $\alpha_{ji} = -\chi_j(g_i)^{-1}\alpha_{ij} = -\chi_i(g_j)\alpha_{ij}$. Thus the lifting A is described by the *lifting matrix* $\mathcal{A} = (\alpha_{ij})$ with 0's or 1's on the diagonal and with $\alpha_{ji} = -\chi_i(g_j)\alpha_{ij}$ for $i \neq j$.

Proof. By the same argument as in [4], A is generated by group-like and skew-primitive elements. See [3] for the rest of the proof. These liftings were independently constructed in [6] by iterated Ore extensions. □

In the rest of the paper we assume that A is a lifting of a quantum linear space with notation as in Proposition 2.1.

Since Γ is finite abelian, there exist elements $h_1, \dots, h_t$ in Γ, and non-negative integers $a_1, \dots, a_t$, $b_1, \dots, b_t$ with $a_u = \operatorname{ord} h_u$, $\Gamma = \langle h_1 \rangle \oplus \cdots \oplus \langle h_t \rangle$ and $\Lambda = \langle h_1^{b_1} \rangle \oplus \cdots \oplus \langle h_t^{b_t} \rangle$.

Let $\xi \in \widehat{\Lambda}$; the algebra $A(\xi)$ is then generated by x_i, $1 \le i \le \theta$, h_u, $1 \le u \le t$, and relations

$$\begin{gathered} h_u h_\ell = h_\ell h_u, \quad h_u^{b_u} = \xi(h_u^{b_u}), \quad h_u x_i = \chi_i(h_u) x_i h_u, \\ x_i^{r_i} = \alpha_{ii}(g_i^{r_i} - 1), \quad x_i x_j = \chi_j(g_i) x_j x_i + \alpha_{ij}(g_i g_j - 1), \end{gathered}$$

$1 \le u, \ell \le t$, $1 \le i \le \theta$. Let V be any finite-dimensional $A(\xi)$-module; we denote the action of the elements x_i, h_u on V by the same letters. Thus V is a Γ-module, since $A(\xi)$ is a quotient of A; and

$$V = \oplus_{\eta \in F(\xi)} V^\eta$$

where

$$F(\xi) = \{\eta \in \widehat{\Gamma} : \eta|_\Lambda = \xi\} = \widetilde{\xi}\, \widehat{\Gamma/\Lambda}; \tag{3}$$

see above. We have $\dim A(\xi) = \dfrac{|\Gamma| r_1 \dots r_\theta}{|\Lambda|}$.

The case when the rank $\theta = 1$ is known [8, 5]. We shall investigate the case $\theta = 2$ in the next Section. Consider the condition

(d) The rank $\theta > 2$ and $\operatorname{ord} r_i > 2$, $1 \le i \le \theta$.

We say that $i, j \in I := \{1, \dots, \theta\}$ are *linked* [4] if $\alpha_{ij} \neq 0$. By (d), if i is linked to j and k then $j = k$ [4]. Thus I is a disjoint union of the set of vertices which are linked, which has even cardinal, and the rest. Roughly speaking, the representation theory of $A(\xi)$ looks like the "tensor product" of representation theories of similar algebras with rank $\theta = 1$ or 2.

3. The rank 2 case

In this section we assume that $\theta = 2$ and write $x = x_1$, $y = x_2$, $r = r_1$, $s = r_2$. The lifting matrix of A has the form $\mathcal{A} = \begin{bmatrix} \alpha_{11} & \nu \\ -\chi_2(g_1)\nu & \alpha_{22} \end{bmatrix}$.

Let $\xi \in \widehat{\Lambda}$; we shall determine the irreducible representations of the algebra $A(\xi)$ generated by x, y, h_u, $1 \le u \le t$, and relations

$$h_u h_\ell = h_\ell h_u, \quad h_u^{b_u} = \xi(h_u^{b_u}); \tag{4}$$

$$h_u x = \chi_1(h_u) x h_u; \tag{5}$$

$$h_u y = \chi_2(h_u) y h_u; \tag{6}$$

$$x^r = \alpha_{11}(g_1^r - 1); \text{ we denote } \alpha_{11}(\xi(g_1^r) - 1) = \alpha; \tag{7}$$

$$y^s = \alpha_{22}(g_2^s - 1); \text{ we denote } \alpha_{22}(\xi(g_2^s) - 1) = \beta; \tag{8}$$

$$xy = \chi_2(g_1) yx + \nu(g_1 g_2 - 1). \tag{9}$$

We have $\dim A(\xi) = \dfrac{|\Gamma| rs}{|\Lambda|}$.

Let $q = \chi_1(g_1)$. If $\chi_1\chi_2 = \epsilon$, then $q = \chi_2(g_1)^{-1} = \chi_1(g_2) = \chi_2(g_2)^{-1}$ (hence $r = s$), and relation (9) becomes $yx = q\,(xy - \nu(g_1 g_2 - 1))$.

We distinguish six cases; up to change of variables, these six cases cover all possibilities for $A(\xi)$.

(I) $\alpha = \beta = 0$, $\nu = 0$.

(II) $\alpha = 1$, $\beta = 0$, $\nu = 0$. Necessarily, $\chi_1^r = \epsilon$ and $g_1^r \neq 1$.

(III) $\alpha = \beta = 1$, $\nu = 0$. Necessarily, $\chi_1^r = \chi_2^s = \epsilon$, $g_1^r \neq 1$ and $g_2^s \neq 1$.

(IV) $\alpha = \beta = 0$, $\nu = 1$. Necessarily, $\chi_1\chi_2 = \epsilon$ and $g_1 g_2 \neq 1$.

(V) $\alpha = 1$, $\beta = 0$, $\nu = 1$. Necessarily, $\chi_1^r = \epsilon$ and $g_1^r \neq 1$; $\chi_1\chi_2 = \epsilon$ and $g_1 g_2 \neq 1$.

(VI) $\alpha = \beta = 1$, $\nu \neq 0$. Necessarily, $\chi_1^r = \chi_2^s = \epsilon$, $g_1^r \neq 1$, $g_2^s \neq 1$; $\chi_1\chi_2 = \epsilon$ and $g_1 g_2 \neq 1$.

Now we examine each of the 6 cases listed above.

Case (I) Here the lifting is trivial. In this case we have the following well-known theorem.

Theorem 3.1. *The irreducible representations of $A(\xi)$ have dimension one and are parametrized by $\widehat{\Gamma/\Lambda}$. Indeed, $A(\xi) \simeq \mathcal{B}(V)\#k[\Gamma]/(\mathcal{B}(V)\#k[\Gamma])e_\xi$.*

Case (II) Here $\alpha = 1$ and $\beta = \nu = 0$. Thus $\chi_1^r = \epsilon$ and since $q = \chi_1(g_1)$ is a primitive $r-$th root of unity, r is the order of χ_1.

It is now convenient to consider the algebra $B(\xi)$ presented by generators x, h_u, $1 \le u \le t$, and relations

$$\begin{aligned} h_u h_\ell &= h_\ell h_u, & h_u^{b_u} &= \xi(h_u^{b_u}), \\ h_u x &= \chi_1(h_u) x h_u, & x^r &= 1. \end{aligned}$$

The representation theory of $B(\xi)$ is well-known; we include it for completeness. (For example, see [10], [8].)

Lemma 3.2. *Let $\eta \in F(\xi)$. Let $W(\eta)$ be a vector space with a basis f_i, $0 \le i \le r-1$, with subscripts $i \in \mathbb{Z}/r$. There is a representation of $B(\xi)$ on $W(\eta)$ defined by the following rules:*

$$x.f_i = f_{i+1}, \quad h_u.f_i = (\chi_1^i \eta)(h_u) f_i, \quad 1 \le u \le t$$

where $i \in \mathbb{Z}/r$ means that $x.f_{r-1} = f_0$. Furthermore, $W(\eta)$ is irreducible; all irreducibles are of this kind; $W(\eta) \simeq W(\eta')$ if and only if $\eta = \eta' \chi_1^m$ for some m; and $B(\xi)$ is semisimple.

Proof. It is straightforward to verify that $W(\eta)$ is a representation of $B(\xi)$. We see that $W(\eta)$ is irreducible because the f_i's belong to different isotypical components for Γ.

Let V be an irreducible representation of $A(\xi)$. Since $V = \oplus_{\eta \in F(\xi)} V^\eta$, we can choose $v \in V^\eta - 0$ for some $\eta \in \widehat{\Gamma}$. Let t be the order of x in V; clearly $x^i.v \in V^{\chi_1^i \eta} - 0$, $0 \le i \le t$; and $x^t.v = v$. Then $r|t|r$, thus $t = r$ and $V \simeq W(\eta)$. If $\phi : W(\eta) \to W(\eta')$ is an isomorphism of $B(\xi)$-modules, then $\phi(f_0) \in k f'_m$; thus $\eta = \eta' \chi_1^m$. Conversely, assume that $\eta = \eta' \chi_1^m$ and define $\phi : W(\eta) \to W(\eta')$ by $\phi(f_i) = f'_{m+i}$; ϕ is an isomorphism of $B(\xi)$-modules. Finally, $\dim B(\xi) \le |\Gamma/\Lambda| r$ by the defining relations, but the dimension of the quotient of $B(\xi)$ by its Jacobson radical is $\ge |\Gamma/\Lambda| r$ by what we have already proved; thus $B(\xi)$ is semisimple. □

Theorem 3.3. *Let $A(\xi)$ be such that $\alpha = 1$, $\beta = \nu = 0$. Let $\pi : A(\xi) \to B(\xi)$ be the algebra map sending y to 0. Then any irreducible representation of $A(\xi)$ factorizes through π. In particular, all irreducible representations of $A(\xi)$ are described by Lemma 3.2.*

Proof. Let V be an irreducible representation of $A(\xi)$. Since $0 \ne \ker y$ is stable under the action of Γ and x, we see that y acts as 0 on V. □

Case (III) Here $\alpha = \beta = 1$ and $\nu = 0$. Let $w := \chi_2(g_1)$. Since $\alpha = \beta = 1$, as noted in previous cases, we have that χ_1 has order r and χ_2 has order s. Thus $w^r = w^s = 1$.

Theorem 3.4. *$A(\xi)$ is semisimple.*

Proof. There exists a unique algebra automorphism Y of $B(\xi)$ such that $Y(h_u) = \chi_2(h_u)^{-1}h_u$, $1 \le u \le t$, and $Y(x) = w^{-1}x$; clearly Y^s =id. It is well-known that the smash product $B(\xi)\#k\,Y$ is semisimple, see [7]. But $B(\xi)\#k\,Y$ is isomorphic as an algebra to $A(\xi)$, say by dimension counting. □

The explicit description of all simple $A(\xi)$-modules can be obtained by means of Clifford theory, see [7].

Case (IV) In the next three cases, we have $\nu \neq 0$ so that $\chi_1\chi_2 = \epsilon$ and so $r = s$. For all of the remaining cases, we will want to define a set of scalars c_i by the following recursive definition.

Assume that $\chi_1\chi_2 = \epsilon$. Fix $c = c_0$ and for $\eta \in F(\xi)$ and $i > 0$, define

$$c_i = q\left(c_{i-1} - \nu\eta\chi_1^{i-1}(g_1g_2 - 1)\right) = q\left(c_{i-1} + \nu - \nu q^{2(i-1)}\eta(g_1g_2)\right). \quad (10)$$

The second equality follows from the fact that $q = \chi_1(g_1) = \chi_1(g_2)$. A simple induction shows that for $i > 0$, we have

$$c_i = q^i c + q(i)_q\nu\left(1 - q^{i-1}\eta(g_1g_2)\right). \quad (11)$$

Thus if q is a primitive r-th root of unity, if $i \equiv k \mod r$, then $c_i = c_k$.

In this case, $\alpha = \beta = 0$ and $\nu = 1$. Here, the representation theory is similar to that of a Frobenius-Lusztig kernel of type $\mathfrak{sl}(2)$.

Theorem 3.5. *Let $\eta \in F(\xi)$ as defined in (3). Let $(c_i)_{i\ge 0}$ be scalars defined recursively by (10) with $c_0 = 0$.*

Let N be the least positive integer such that $c_N = 0$. Note that since $(r)_q = 0$, then $N \le r$. Let $L(\eta)$ be a vector space with a basis $(v_i)_{0\le i\le N-1}$; set $v_{-1} = v_N = 0$ in $L(\eta)$. Then there exists a representation of $A(\xi)$ on $L(\eta)$ given by

$$h_u.v_i = \eta\chi_1^i(h_u)v_i, \quad 1 \le u \le t, \qquad y.v_i = c_iv_{i-1}, \qquad x.v_i = v_{i+1}. \quad (12)$$

Furthermore, $L(\eta)$ is irreducible. Also any irreducible $A(\xi)$-module is isomorphic to $L(\eta)$ for some η; and $L(\eta)$ is isomorphic to $L(\eta')$ only when $\eta = \eta'$.

Proof. The verification that (12) defines a representation of $A(\xi)$ is straightforward. The fact that $y.v_0 = 0 = x.v_{N-1}$ ensures that relations (7) and (8) hold while (10) guarantees that relation (9) is respected. We leave the reader to check the details and to check that $A(\xi)$ is irreducible.

Let $\rho : A(\xi) \to \operatorname{End} V$ be an irreducible representation. Since $\ker y \neq 0$ and is Γ-stable, there exists $v \in \ker y - 0$, $v \in V^{\eta}$ for some $\eta \in F(\xi)$. Set $v_0 = v$, $v_i = x^i.v$, $i > 0$; $v_i \in V^{\eta\chi_1^i}$ for all i. It follows from the fact that relation (9) must be respected and from a simple induction that for $i > 0$, we have $y.v_i = d_i v_{i-1}$, where the d_i's satisfy the recursive relation (10).

Now, $v_0, v_1 \ldots v_{m-1}$ generate a submodule of V where $v_m = x^m.v_0 = 0$ and $m \leq r$. If $m > N$, then $v_{m-1}, \ldots, v_N$ is a submodule of V. Thus $m = N$ and since $d_i = c_i$, $i \geq 0$, then V coincides with the submodule generated by the v_i's, $i \geq 0$ which is isomorphic to $L(\eta)$.

Finally, $L(\eta)$ is presented as $A(\xi)$-module by generator v_0 with relations $h_u(v_0) = \eta(h_u)v_0$, $1 \leq u \leq t$, $y.v_0 = 0$, $x^N.v_0 = 0$. Thus, $L(\eta) \simeq L(\eta')$ implies $\eta = \eta'$. $\square$

Case (V) In this case, we have $\alpha = \nu = 1$ and $\beta = 0$.

Theorem 3.6. *Let $\eta \in F(\xi)$. Let $W(\eta)$ be the $B(\xi)$-module defined in Lemma 3.2 with basis $f_0, \ldots, f_{r-1}$ with subscripts taken modulo r. Set $c_i = 0$ for $i = 0$ and define scalars $(c_i)_{0<i\leq r}$ recursively by (10). Define an operator y on $W(\eta)$ by $y.f_i = c_i f_{i-1}$. Then this defines a representation of $A(\xi)$ and we denote this $A(\xi)$-module by $L(\eta)$. Then $L(\eta)$ is irreducible; all irreducibles are of this kind; If $L(\eta) \simeq L(\eta')$ then $\eta = \eta'\chi_1^m$ for some m.*

Proof. As usual, the verification that $L(\eta)$ is an irreducible representation is straightforward. Recall from (11) that it makes sense to compute subscripts modulo r.

Next, let V be an irreducible $A(\xi)$-module. Then there exists $\eta \in F(\xi)$ and $v \neq 0$ such that $v \in \ker y \cap V^{\eta}$. Set $f_0 = v$, $f_i = x^i v$. Arguing as in Lemma 3.2 we see that the f_i's span a $B(\xi)$-submodule U isomorphic to $W(\xi)$. Relation (9) implies the description of the action of y on U by $y.f_i = \alpha_i f_{i-1}$, where the α_i's are defined by (10). Hence $V = U \simeq L(\xi)$. $\square$

Case (VI) In this case, $\alpha = \beta = 1$ and $\nu \neq 0$.

Theorem 3.7. *Let $\eta \in F(\xi)$. Let $W(\eta)$ be the $B(\xi)$-module defined in Lemma 3.2. Set $c_0 = c \neq 0$ and define a family of scalars $(c_i)_{0\leq i\leq r-1}$ inductively by (10). Define an operator y on $W(\eta)$ by $y.f_i = c_i f_{i-1}$.*

(i). This defines a representation of $A(\xi)$ if and only if c is a solution of the equation

$$c_0 c_1 \dots c_{r-1} = 1. \tag{13}$$

If this is the case, we denote this $A(\xi)$-module by $L(\eta, c)$. Furthermore, $L(\eta, c)$ is irreducible.

(ii). If $L(\eta, c) \simeq L(\eta', c')$ if and only if $\eta = \eta' \chi_1^m$ for some m, and $c = c'_m$.

(iii). Assume that the equation (13), a polynomial in c, has simple roots. Then all irreducibles are of this kind and $A(\xi)$ is semisimple.

Proof. (i). We have to check the relations (6), (8) and (9). Here (6) is clear and (8) is equivalent to (13). We evaluate both sides of (9) on f_i; if $0 \leq i < r-1$ the equality follows from the defining condition (10); otherwise it follows from (11). The irreducibility is clear.

(ii). Left to the reader.

(iii). In general, the dimension of the semisimple quotient of $A(\xi)$ corresponding to all the representations of the type $L(\eta, c)$ is $\frac{|\Gamma| \#\{\text{solutions of (13)}\} r^2}{|\Lambda| r}$, and this equals $\dim A(\xi)$ if and only if the equation (13), a polynomial in c, has simple roots. □

4. CONCLUSIONS

Let A be a lifting of a quantum plane. Then $A \simeq \prod_{\xi \in \widehat{\Lambda}} A(\xi)$; hence the irreducible representations of A are the union of the irreducible representations of $A(\xi)$, $\xi \in \widehat{\Lambda}$. We have determined the last ones in Section 3, up to a finite number of exceptions in Case (VI). As a consequence, we can also determine the coradical of the dual Hopf algebra $C = A^*$.

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