## REPRESENTATIONS OF THE CATEGORY OF MODULES OVER POINTED HOPF ALGEBRAS OVER $S_3$ AND $S_4$

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## ABSTRACT

This is a joint work with Martín Mombelli. It has been submitted for publication. A preprint is available at arXiv:1006.1857v1[math.QA].

We shall denote by  $\Bbbk$  an algebraically closed field of characteristic zero.

Given a tensor category  $\mathcal{C}$ , an *exact module category* [EO] over  $\mathcal{C}$  is an Abelian category  $\mathcal{M}$  equipped with a biexact functor  $\otimes : \mathcal{C} \times \mathcal{M} \to \mathcal{M}$  subject to natural associativity and unity axioms, such that for any projective object  $P \in \mathcal{C}$  and any  $M \in \mathcal{M}$  the object  $P \otimes M$  is again projective.

We will recall the basic results on module categories over finite-dimensional Hopf algebras [AM]. We recall the main result of [M] that gives an isomorphism between Loewy-graded comodule algebras and a semidirect product of a twisted group algebra and an homogeneous coideal subalgebra inside the Nichols algebra. We will also recall the classification of finite-dimensional Hopf algebras with coradical  $\&S_3$  or  $\&S_4$  from [AHS], [GG], respectively.

Using these results, we will show that if n = 3, 4, 5 and H is a finite dimensional pointed Hopf algebra over  $S_n$ , then for every exact indecomposable module category  $\mathcal{M}$  over  $\operatorname{Rep}(\operatorname{gr} H)$  there exists

- a subgroup  $F < \mathbb{S}_n$  and a 2-cocycle  $\psi \in Z^2(F, \mathbb{k}^{\times})$ ,
- a subset  $Y \subseteq X$  invariant under the action of F,
- a family of scalars  $\{\xi_C\}$  compatible with  $(F, \psi, Y)$ ,

such that  $\mathcal{M} \simeq_{\mathcal{A}(Y,F,\psi,\xi)}\mathcal{M}$ , where  $\mathcal{A}(Y,F,\psi,\xi)$  is a left gr *H*-comodule algebra constructed from data  $(Y,F,\psi,\xi)$ . We also show a classification of connected homogeneous left coideal subalgebras  $\mathcal{A}(Y,F,\psi,\xi)$  of gr *H* and together with a presentation by generators and relations.

Finally we prove that if H is a finite-dimensional Hopf algebra with coradical  $\Bbbk S_3$  or  $\Bbbk S_4$  then H and gr H are cocycle deformations of each other, a result analogous to a theorem of Masuoka [Ma] for abelian groups. This implies that there is a bijective correspondence between module categories over  $\operatorname{Rep}(H)$  and  $\operatorname{Rep}(\operatorname{gr} H)$ .

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The work was partially supported by CONICET, FONCyT-ANPCyT, Secyt (UNC), Mincyt (Córdoba).