FaMAF
Facultad de Matemática, Astronomía y Física
Universidad Nacional de Córdoba
GTMC
Grupo de Teoría de la Materia Condensada
CONICET

Long-term ordering kinetics of the two-dimensional q-state Potts model

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PRE **76** 031108 (2007)

ÉCOLE DE PHYSIQUE
LES HOUCHES

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Ferromagnetic q states Potts model

$$H = -J \sum_{nn} \delta(s_i, s_j)$$

$$s_i = 1, \ldots, q$$

$$\delta(s, s') \begin{cases} 1 & \text{if } s = s' \\ 0 & \text{otherwise} \end{cases}$$

$$k_B = J = 1$$

$$J > 0$$

2D →

- 2nd order phase transition for q = 2,3,4
- 1st order phase transition for q ≥ 5

$$\frac{k_B T_c}{J} = [ln(1 + \sqrt{q})]^{-1}$$

T. Kihara, Y. Midzuno, T. Shizume, *J. Phys. Soc. Japan*. **9**, 681 (1954).

Ordering kinetics: Single spin-flip dynamics

Relaxation after a quench from $T = \infty$ to $T < T_t$ for $q > 4$

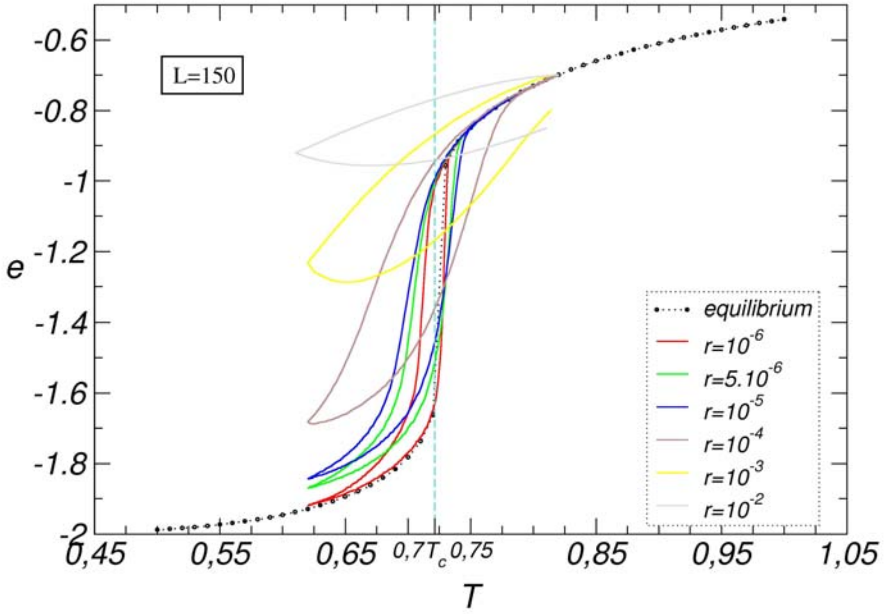
Monte Carlo simulations on a squares lattice with $N = L \times L$ sites (periodic boundary conditions) for $q = 9$:

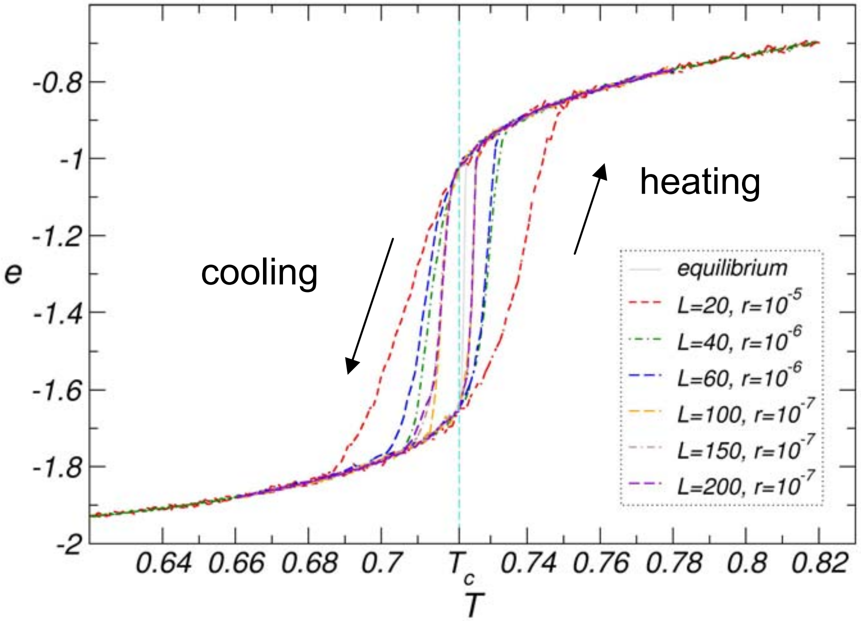
- Metropolis algorithm.
- Continuous time MC algorithm (n-fold way algorithm).

M.K. Bortz, J. Lebowitz, *J. Comp. Phys* **17**, 10 (1975).

Cooling-heating cicles at constant rate *r*

T(t) = T(0) ± r t

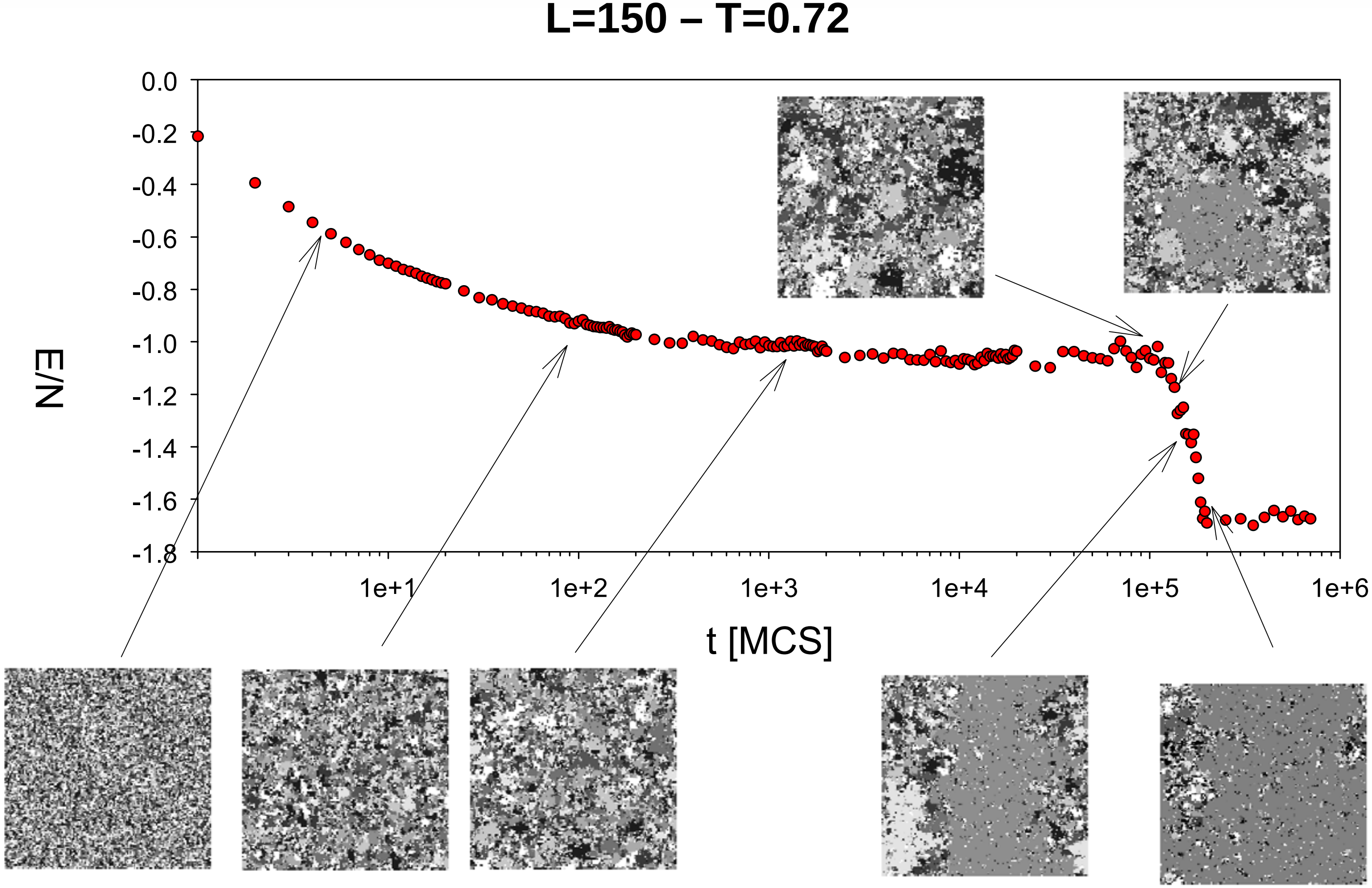




Spinodal temperature $T_{sp} \approx 0.715$

$T_c(q=9) = 1/\ln(4) \approx 0.72135$

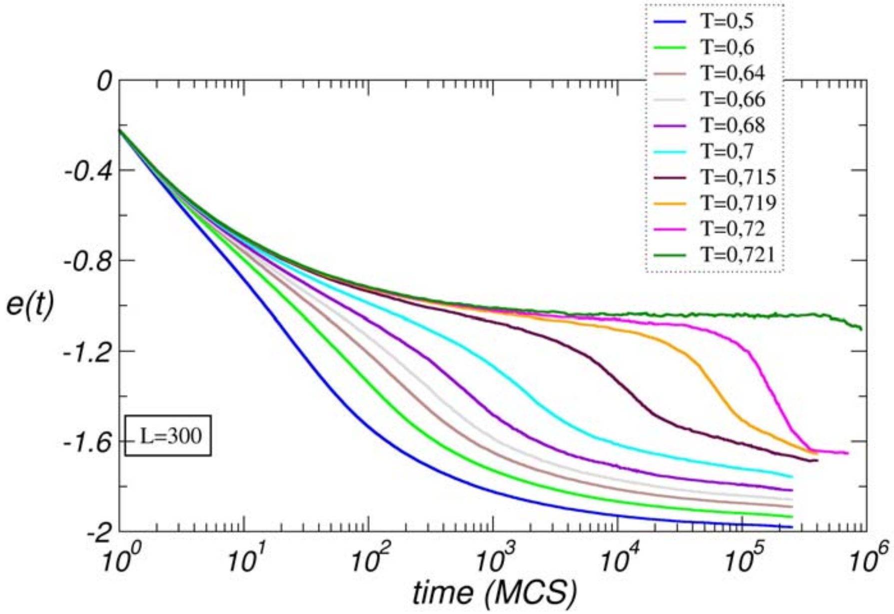
Relaxation at $T_{sp} < T < T_c$



$T_c \approx 0.72135$

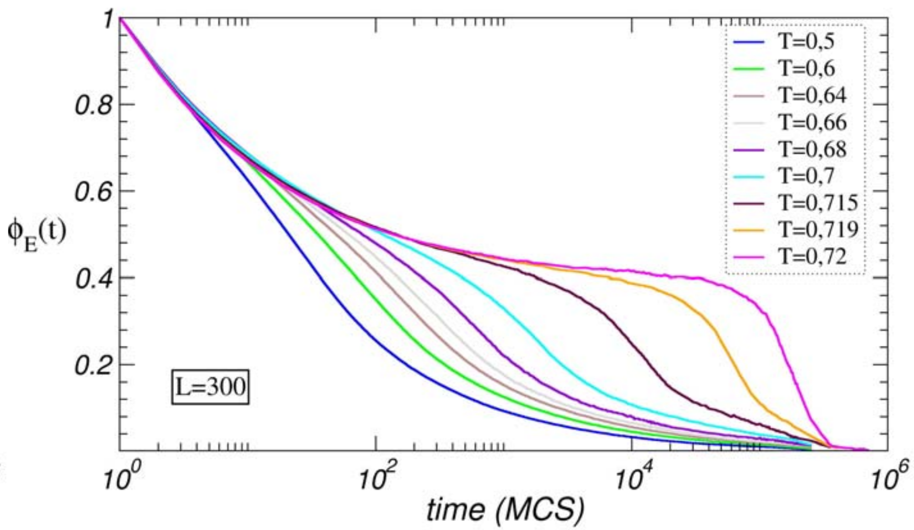
Average energy per spin :

$$e(t) \equiv \langle H(t) \rangle / N$$



Relaxation function :

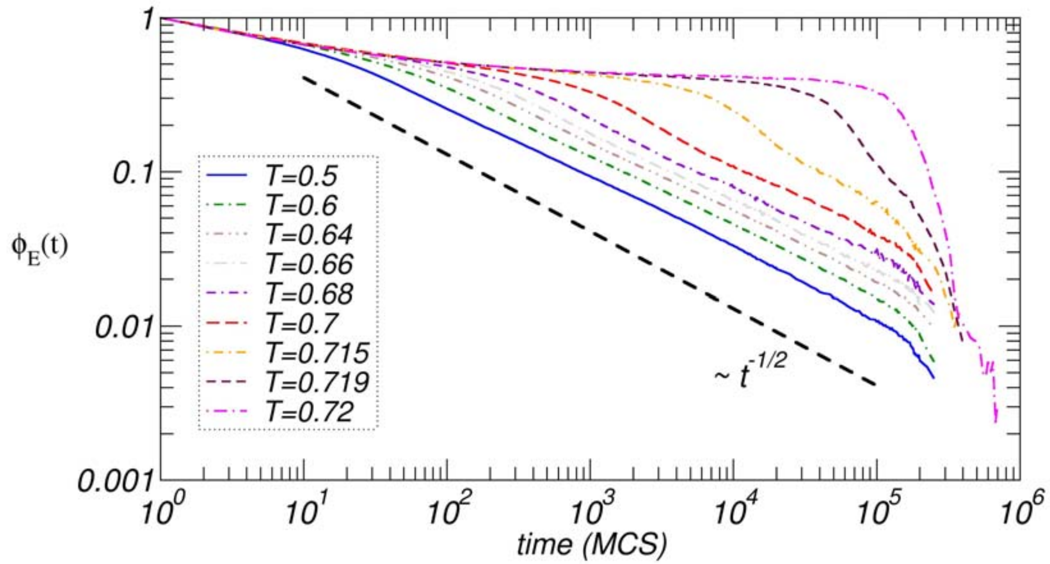
$$\phi_E(t) \equiv \frac{e(t) - e(\infty)}{e(0) - e(\infty)}$$

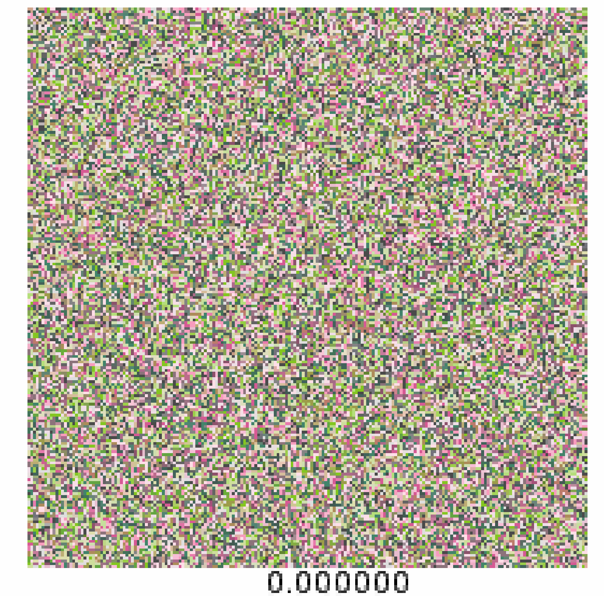


$T_{sp} \approx 0.715$ $T_c \approx 0.72135$

Relaxation at $0 << T < T_{sp}$

L=300





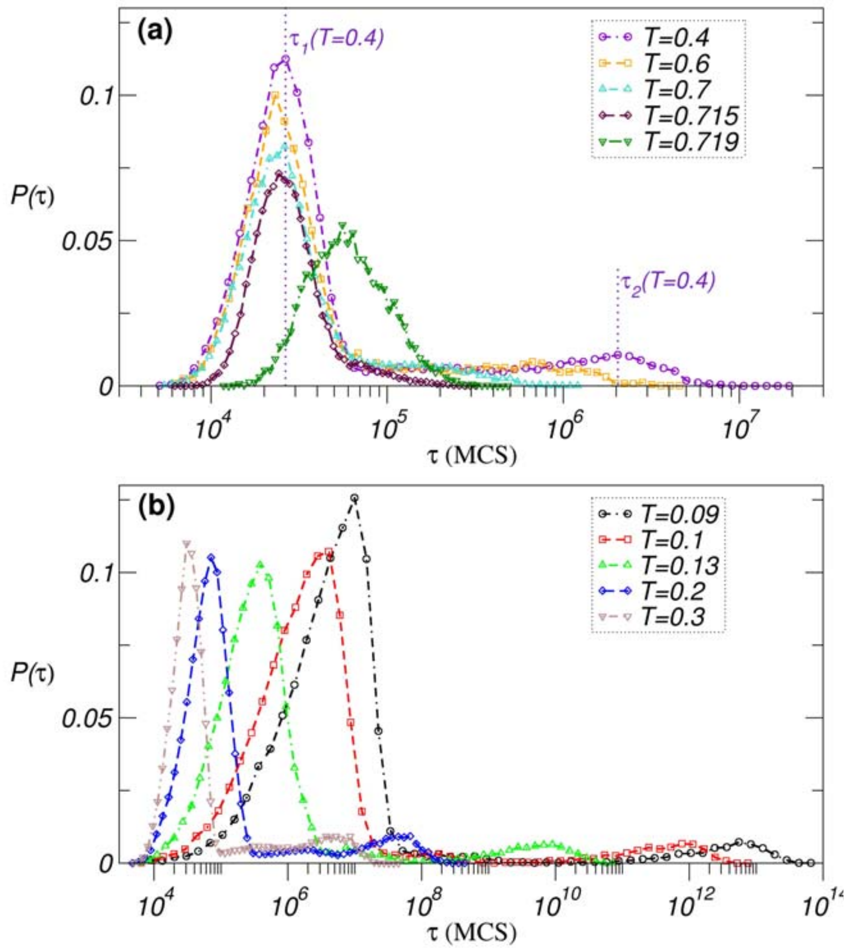
L=200, T=0.2

Normal Coarsening : Average linear domain size: $l(t) \sim t^{1/2}$

$$\phi(t) \sim 1/l(t) \longrightarrow \phi(t) \sim t^{-1/2}$$

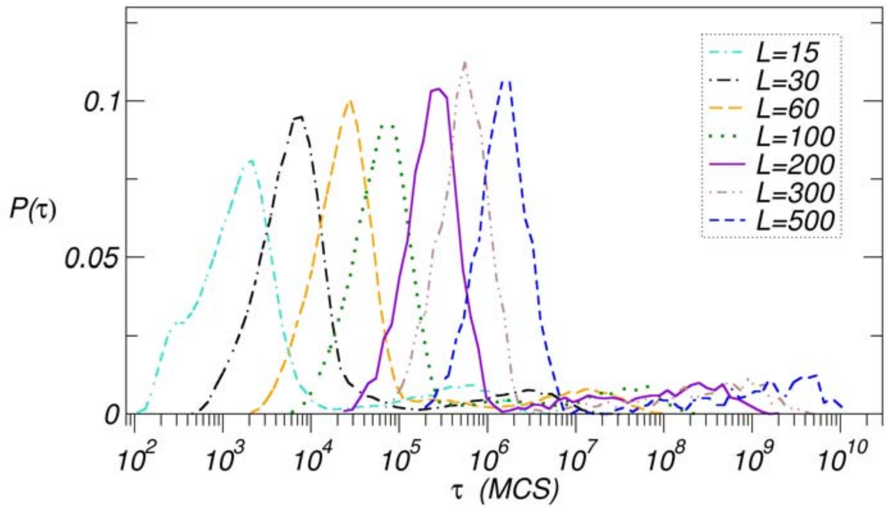
Equilibration time distribution

L = 100

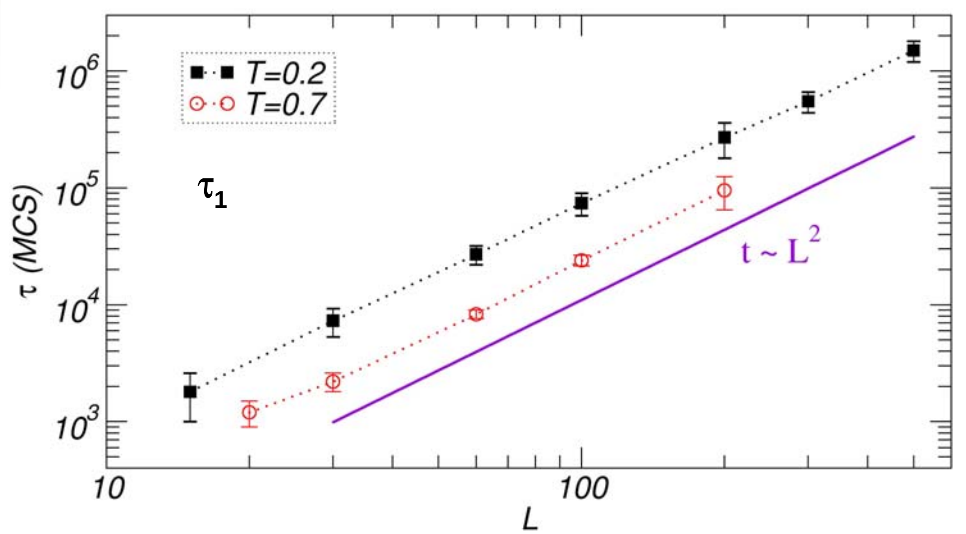


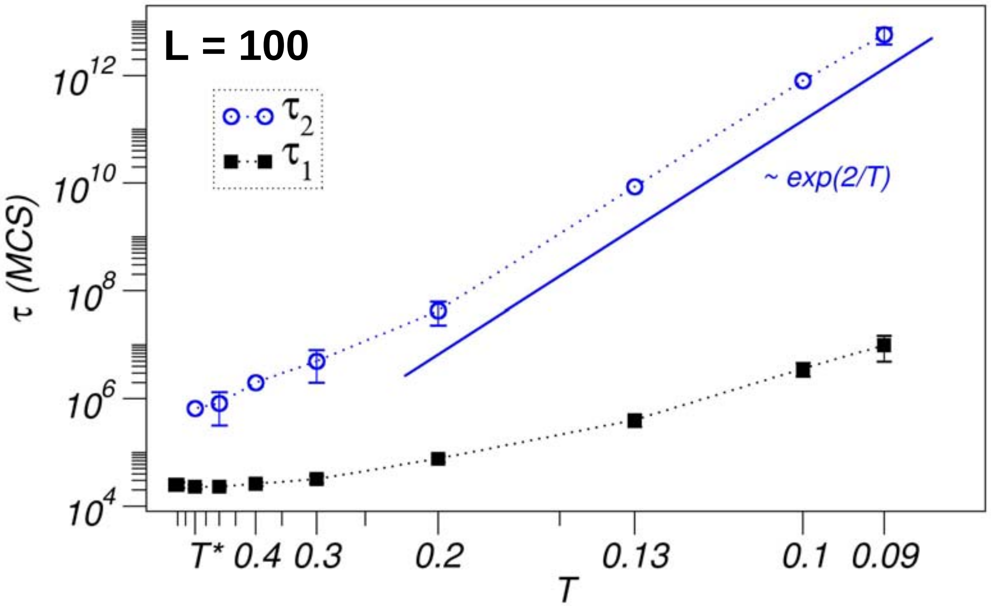
Equilibration time τ:

$$e(\tau) - e(\infty) < \sigma(L, T)$$



Multiple-peak structure for T < T* ≈ 0.6





Coarsening:

$l(t) \sim t^{1/2}$: average domain size

$$\delta e(\tau) \ll 1 \Rightarrow L \sim l(\tau) \sim \tau^{1/2}$$

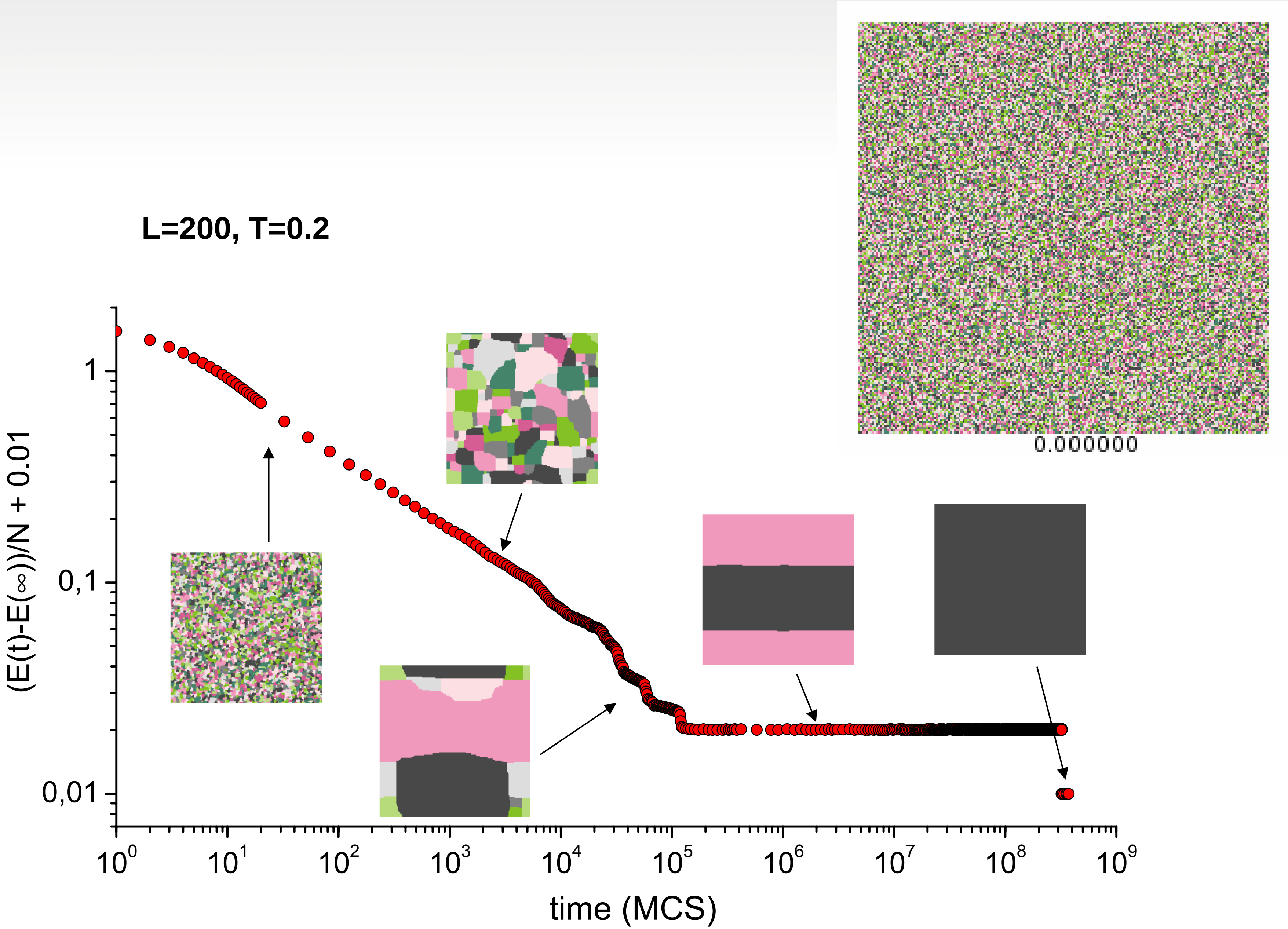
↓

$\tau \sim L^2$

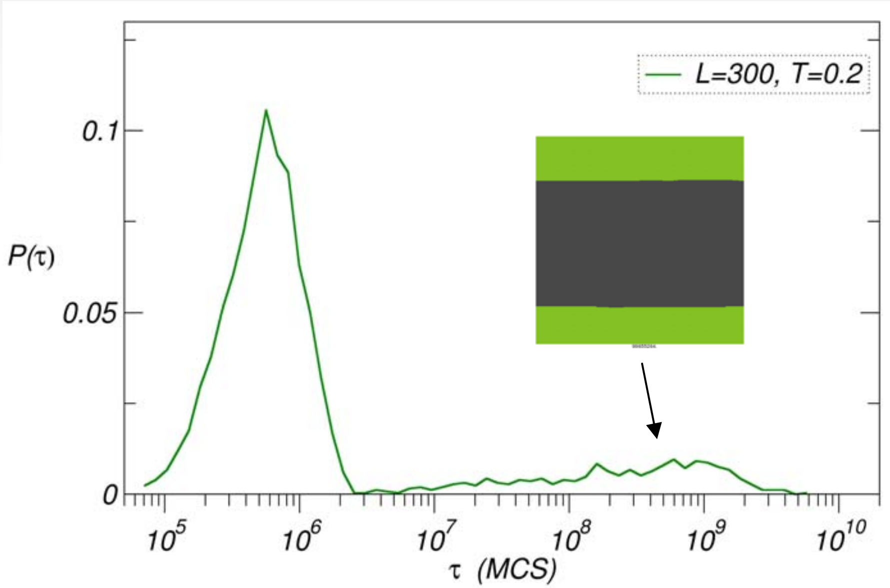
Activation:

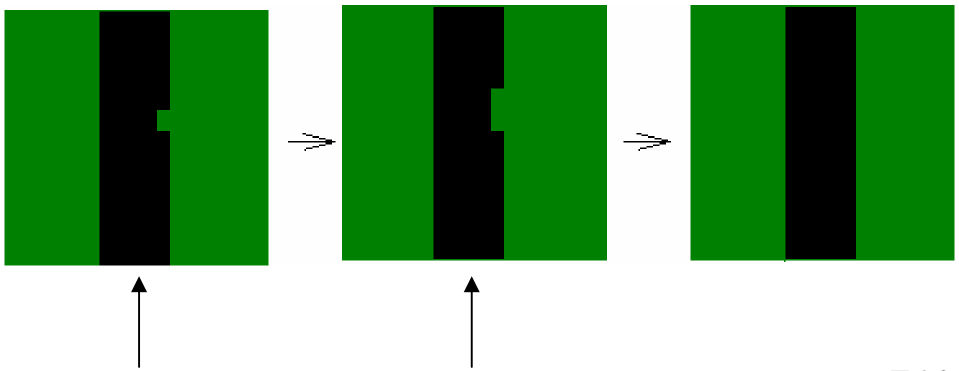
↓

τ_2 → Stripe states



L=200, T=0.2
0.000000
(E(t)-E(∞))/N + 0.01
1
0,1
0,01
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
time (MCS)





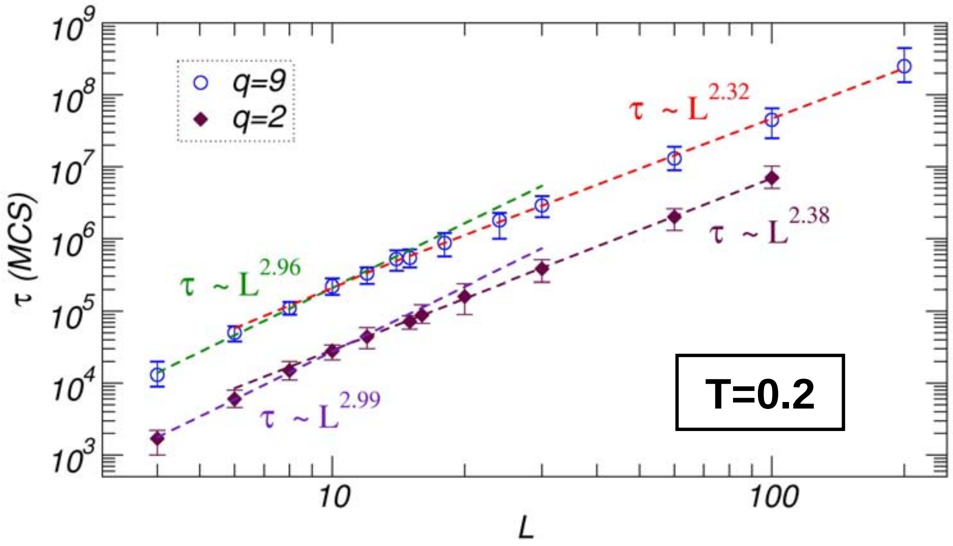
energy cost = 2

energy cost = 0

$$\tau \approx L^{\omega} e^{2/T} \ , \ \omega = 3$$

q = 2, T = 0: V. Spirin, P.L. Krapivsky & S. Redner, “Freezing in Ising ferromagnets”, PRE 65, 016119 (2001)

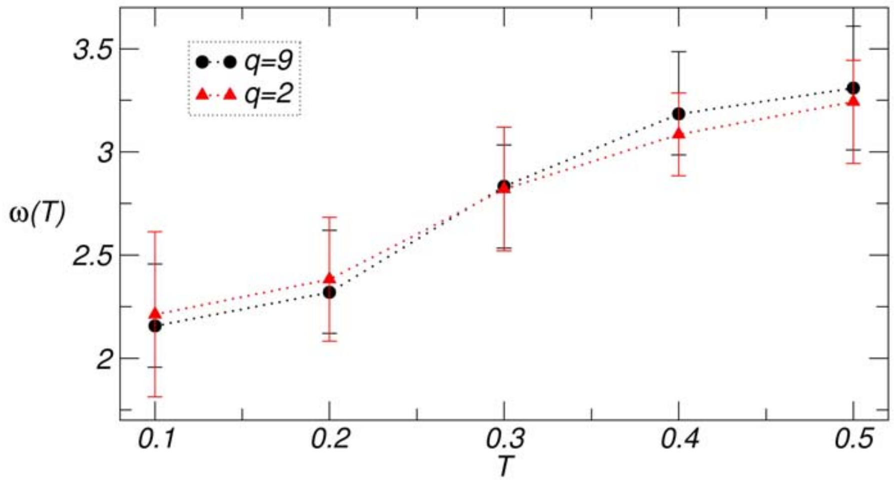
A. Lipowski “Anomalous phase-ordering kinetics in the Ising model”, Physica A **268**, 6 (1999)



V. Spirin et al. → $\omega = 3$

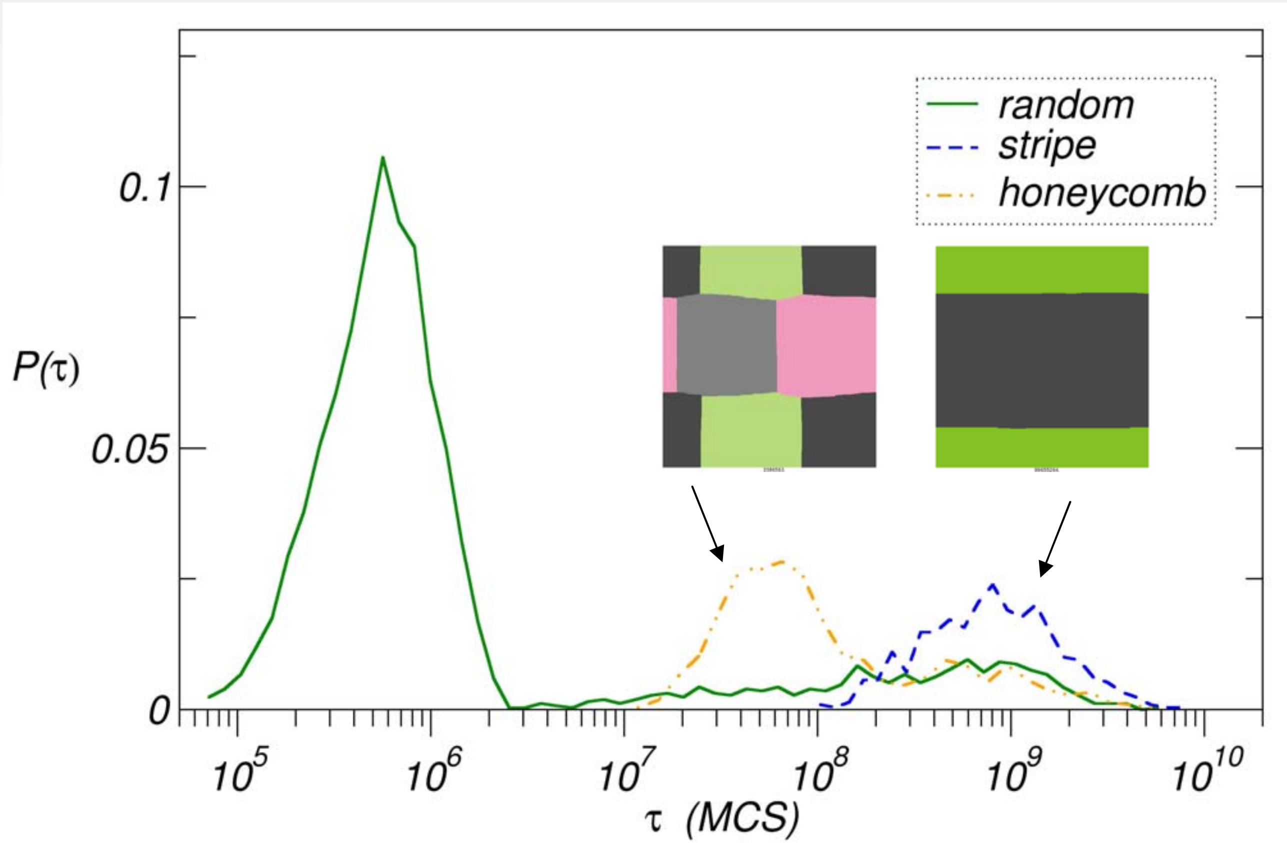
valid for small L and low T

For greater sizes: more than one dent is created.



For high temperatures: elastic lines:

$\omega = \omega(T)$



Lifshitz states: q ≥ d + 1

I.M. Lifshitz, Soviet Physics JETP **15**, 939 (1962)

SOVIET PHYSICS JETP VOLUME 15, NUMBER 5 NOVEMBER, 1962

KINETICS OF ORDERING DURING SECOND-ORDER PHASE TRANSITIONS

I. M. LIFSHITZ

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor December 22, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 42, 1354-1359 (May, 1962)

The kinetics of ordering during second-order phase transitions is investigated. It is found that reorganization of the lattice does not occur via uniform relaxation or by a nucleation mechanism, but as a result of a peculiar process of formation of web-like ordered regions and their subsequent swelling. The intermediate stages of this process are investigated and its speed is determined as a function of time and of the characteristic parameters of the problem.

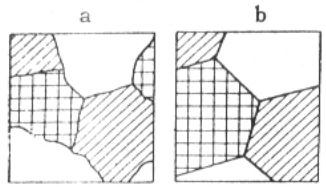
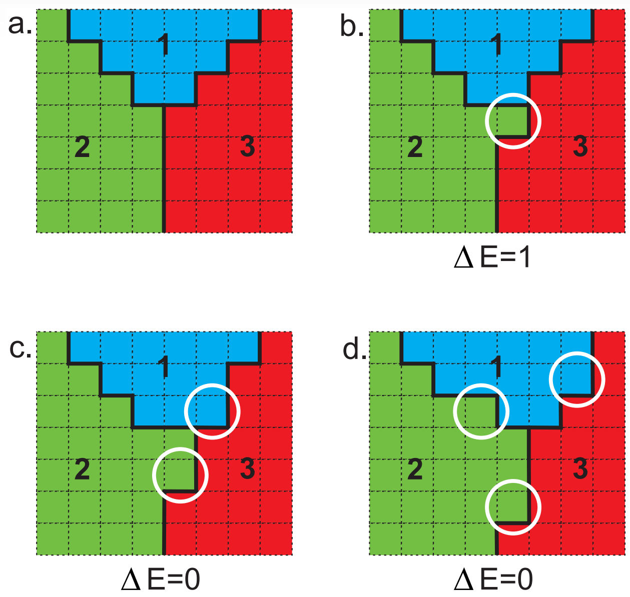
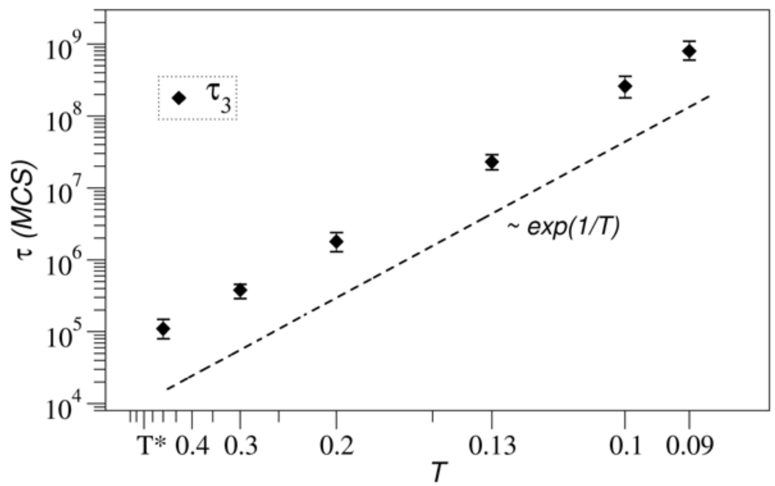


FIG. 3



$$\tau_3 \propto L^{3.06} e^{1/T}$$



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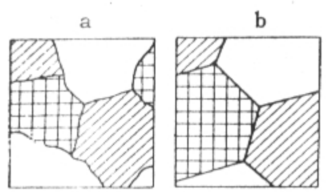
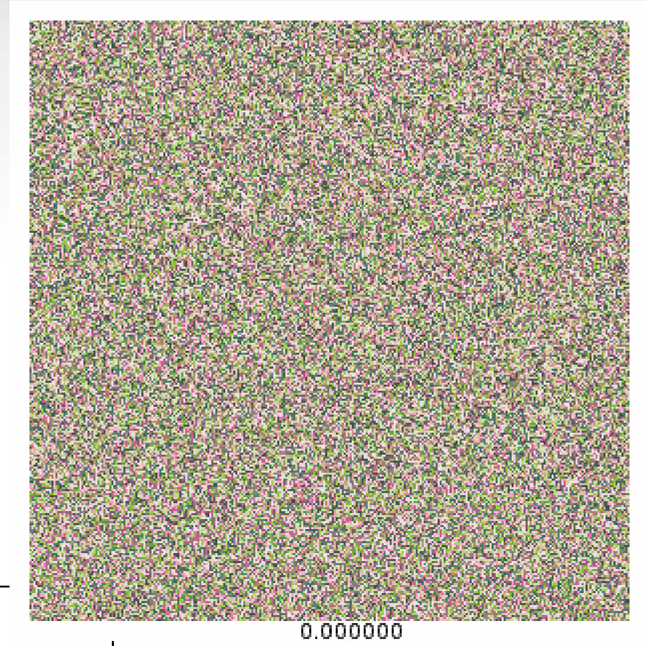
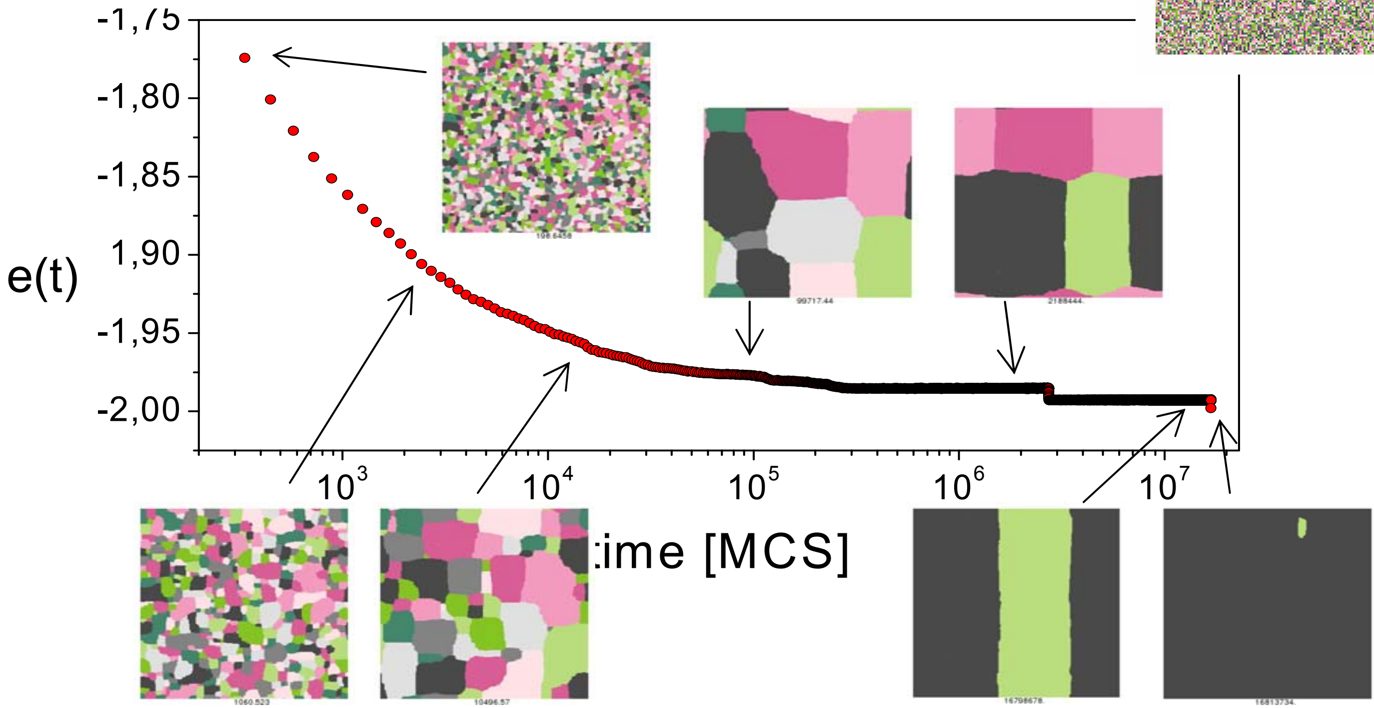


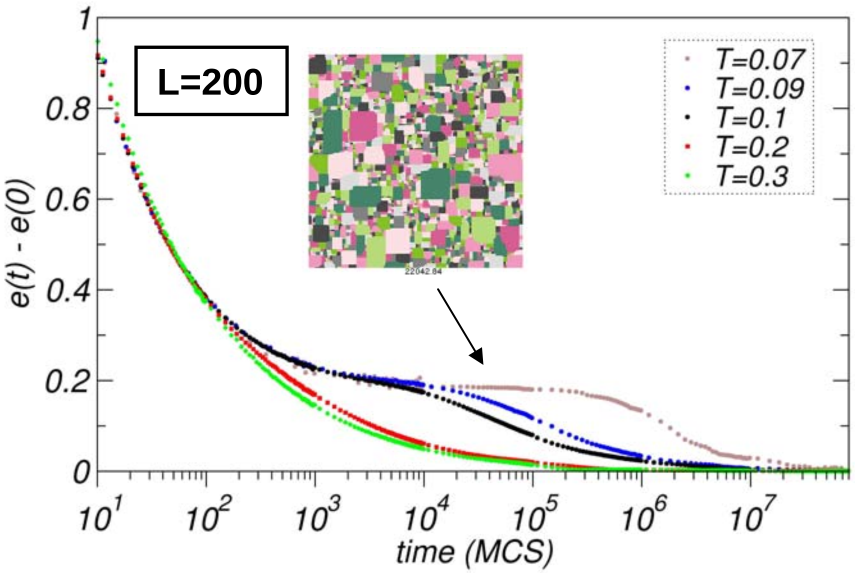
FIG. 3

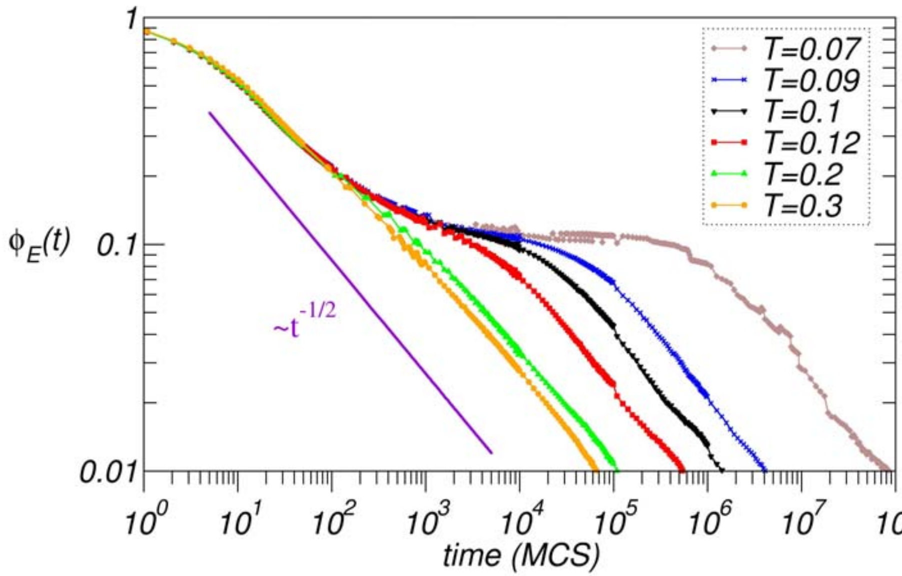
L = 300, T = 0.3





Relaxation at T << T_c





disordered state with finite life time, $\tau \sim exp(A/T)$, for $T < T_g$; $0.1 < T_g < 0.2$ (q = 9)

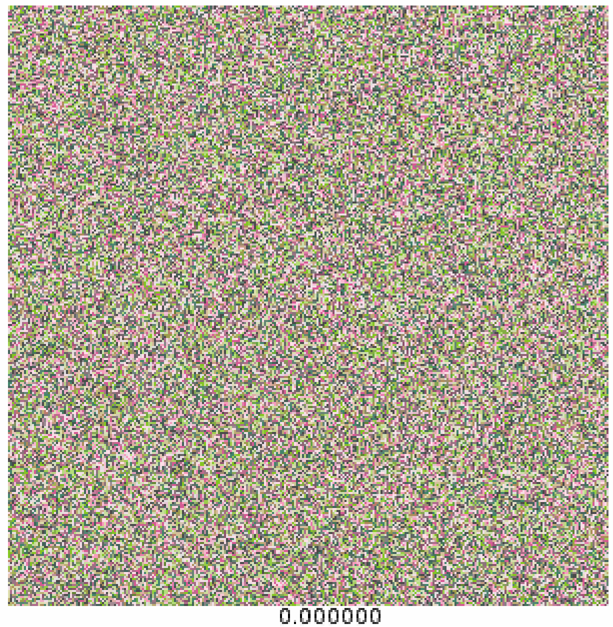
↓

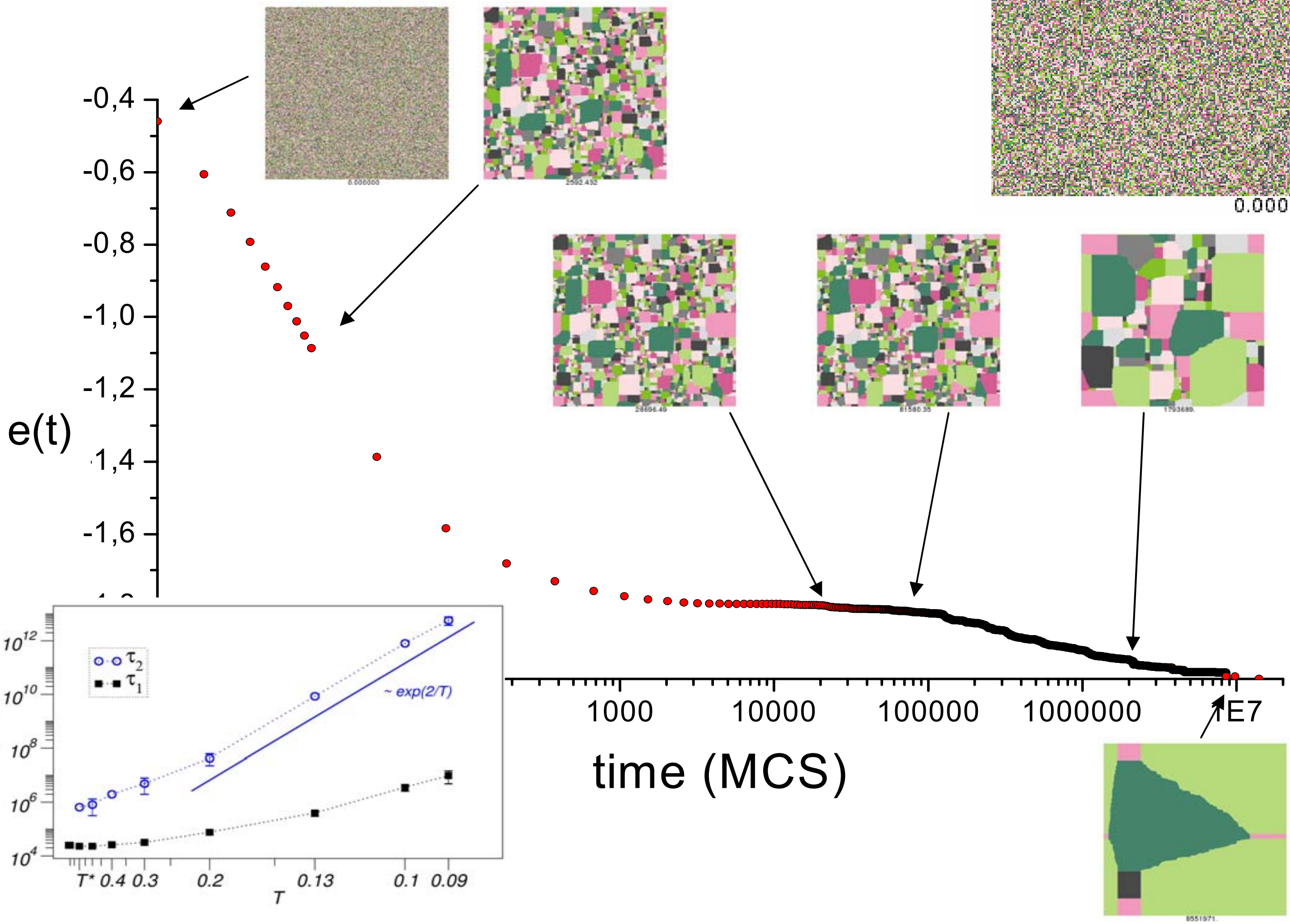
glassy state

q = 7 : M.J. de Oliveira, A. Petri & T. Tomé, *Europhys. Lett.* **65**, 20 (2004)

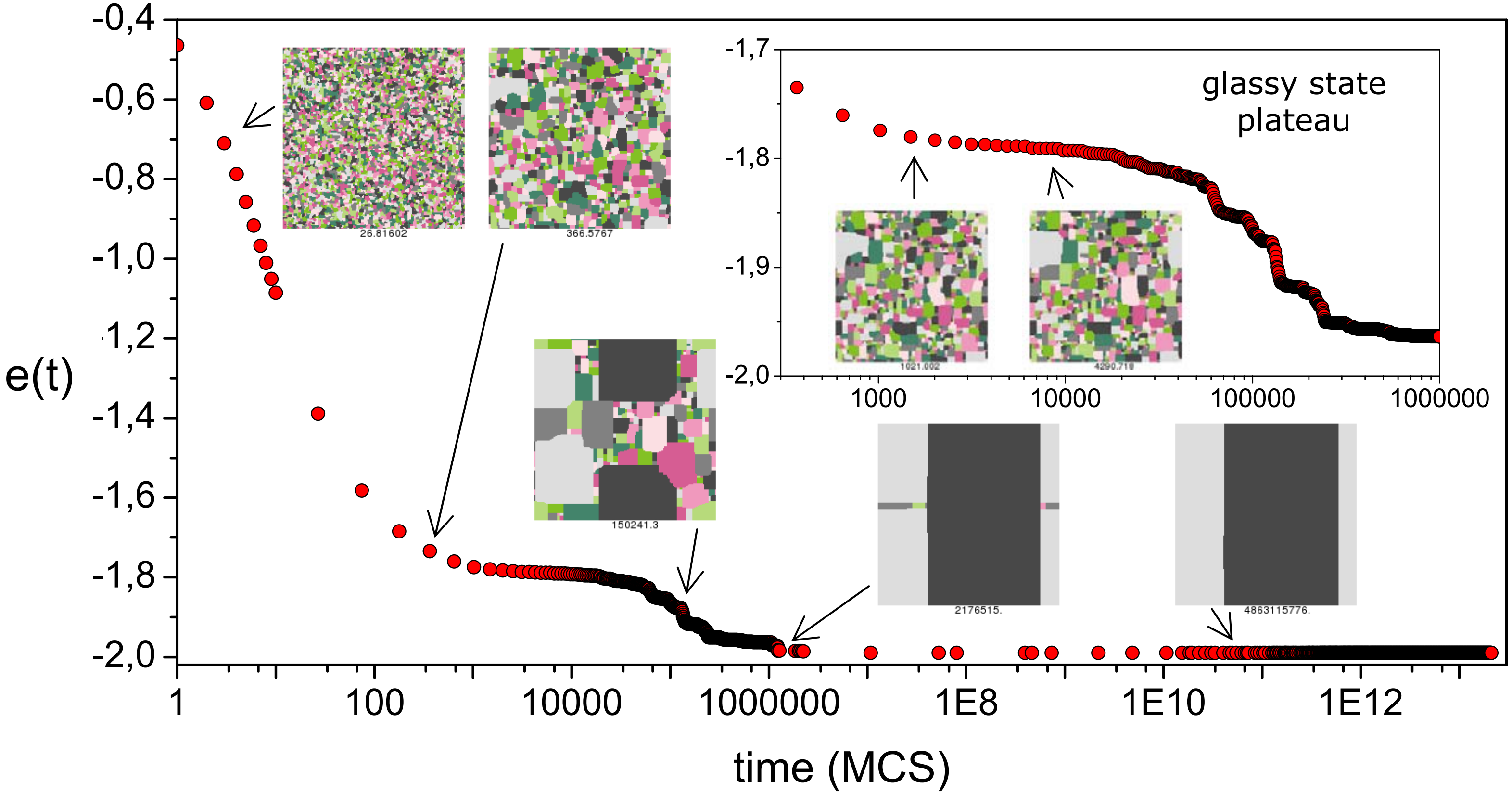
M. Ibañez de Berganza, V. Loreto & A. Petri, *Philos. Mag.* **87**, 779 (2007)

L=300, T=0.08


0.000000


-0,4
-0,6
-0,8
-1,0
-1,2
e(t)
-1,4
-1,6
1000
10000
100000
1000000
1E7
time (MCS)
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
τ (MCS)
τ_2
τ_1
~ exp(2/T)
T* 0.4 0.3
0.2
0.13
0.1 0.09
T

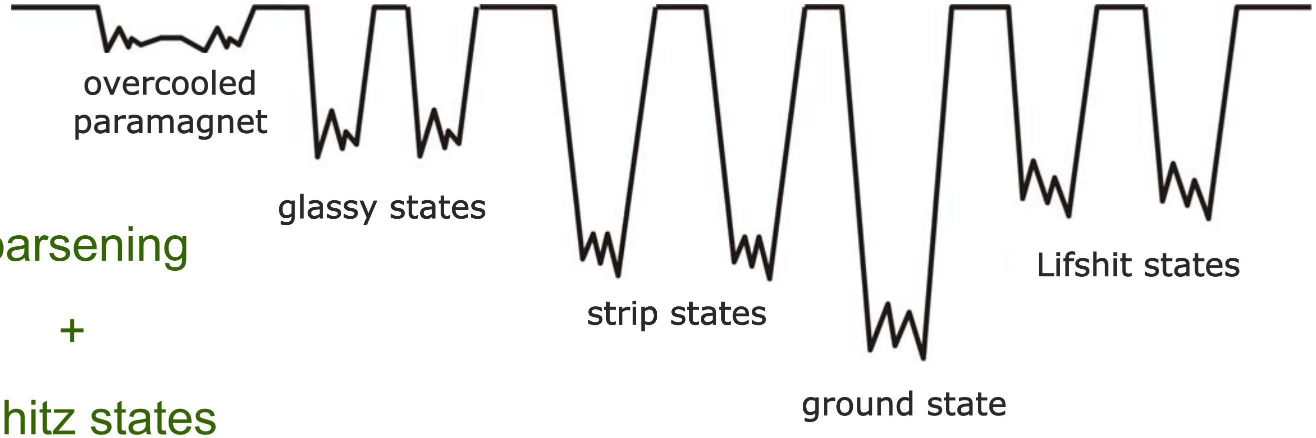
L=200, T=0.09



e(t)
time (MCS)
-0,4
-0,6
-0,8
-1,0
-1,2
-1,4
-1,6
-1,8
-2,0
1
100
10000
1000000
1E8
1E10
1E12
26.81602
366.5767
150241.3
2176515.
4863115776.
glassy state plateau
-1,7
-1,8
-1,9
-2,0
1000
10000
100000
1000000
1021.002
4290.718

Conclusions

$$H = -J \sum_{nn} \delta(s_i, s_j)$$

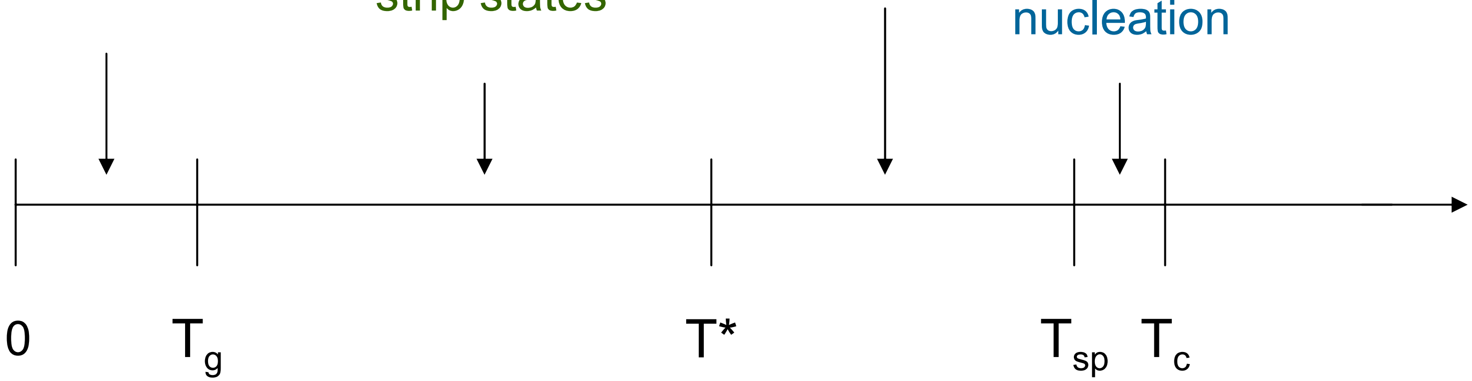


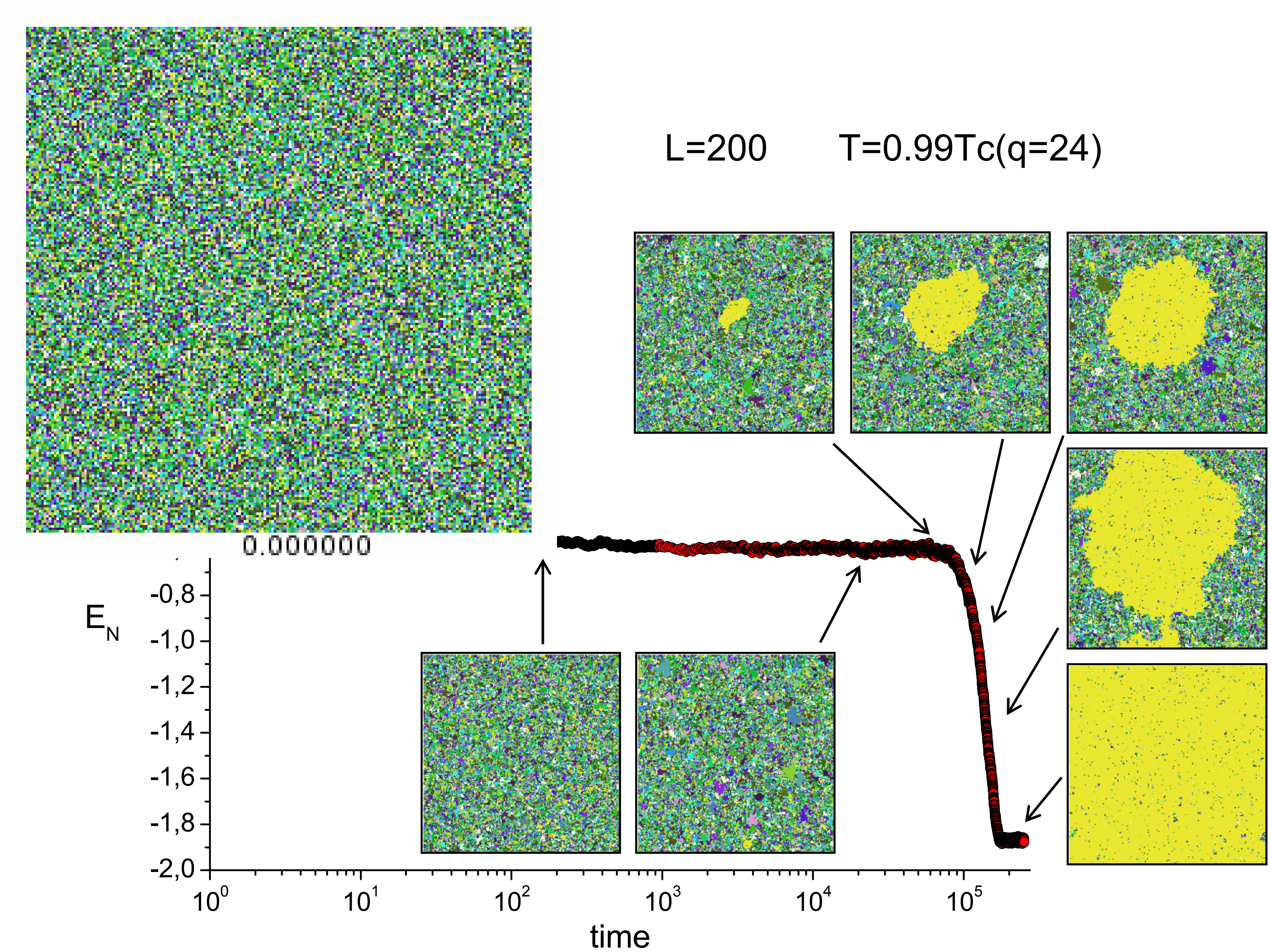
coarsening
+
glassy state
+
strip states

coarsening
+
Lifshitz states
+
strip states

coarsening

nucleation





0.000000
L=200
T=0.99Tc(q=24)
E_N
-0,8
-1,0
-1,2
-1,4
-1,6
-1,8
-2,0
10^0
10^1
10^2
10^3
10^4
10^5
time