

Robustness of dynamical decoupling sequences

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Active protection of quantum states is an essential prerequisite for the implementation of quantum computing. Dynamical decoupling (DD) is a promising approach that applies sequences of control pulses to the system in order to reduce the adverse effect of system-environment interactions. Since every hardware device has finite precision, the errors of the DD control pulses can themselves destroy the stored information rather than protect it. We experimentally compare the performance of different DD sequences in the presence of an environment that was chosen such that all relevant DD sequences can equally suppress its effect on the system. Under these conditions, the remaining decay of the qubits under DD allows us to compare very precisely the robustness of the different DD sequences with respect to imperfections of the control pulses.

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I. INTRODUCTION

Quantum computers can execute certain tasks more efficiently than classical computers by processing information according to the laws of quantum mechanics. In analogy to a classical bit, which can assume the values 0 or 1, quantum mechanical two-level systems like a spin-1/2 can be used as quantum bits by identifying their eigenstates with these values, e.g., $|0\rangle$ for spin up and $|1\rangle$ for spin down. In quantum information processing and quantum memory applications, it is very important to keep the information isolated from the environment: uncontrolled interactions with the environment tend to degrade the quantum information. This environment-induced loss of quantum information is called “decoherence” [1–3].

If one is able to control the system in such a way to reduce the detrimental effect of the system-environment (SE) interaction, one can preserve the quantum state for a longer time. This way of fighting decoherence by applying fast and strong pulses has been termed dynamical decoupling (DD) [4–9]. The main attraction of DD is that it requires few additional resources. In contrast to quantum error correction [10–12], e.g., it does not require additional qubits. DD can be traced back to Hahn’s “spin echo” experiment, where a refocusing pulse induces a time reversal of the SE interaction of nuclear spins [13]. This increases the decay time or decoherence time of the stored information in the qubit. The technique has evolved significantly since then, and its efficiency was studied and demonstrated in many different systems [14–30].

It has been shown that the type as well as the spectral density of the SE interaction play a significant role for finding the optimal DD sequences [7,25,31–38]. Moreover, unavoidable errors in the control pulses are also an important source of decoherence [18,24,39–49]. Thus optimal DD sequences must be able to reduce the effective SE interaction while compen-

sating the effects of nonideal control fields [22,24,43,46–51]. Reference [48] is a recent review of this subject.

In this article we compare the performance of different DD sequences in a system where pulse errors are the dominant source of decoherence; pulse errors become significant when most of the effects of the SE interaction have been eliminated. To this end, we prepare a system where the spectral density of the SE coupling has two main contributions. One source of noise is almost static and can therefore be refocused by all tested DD sequences. The other contribution is a rapidly fluctuating noise, whose correlation time is much shorter than the time required for an inversion of the spins. This type of noise cannot be refocused by any DD sequence. Therefore the main difference between the performance of the DD sequences is their susceptibility to pulse errors.

In a previous work [18,24], we compared the performance of different DD sequences, and in particular we found that the “Carr-Purcell-Meiboom-Gill” (CPMG) sequence [52,53] performed particularly well for specific initial states. In this case the decoherence time was one order of magnitude longer than for robust sequences that reduced decoherence symmetrically with respect to arbitrary initial states. We found that this difference arose because the system qubit, a ^{13}C spin, interacted with neighboring ^{13}C spins. While the CPMG sequence was able to reduce the effect of ^{13}C - ^{13}C couplings, the robust sequences were not [36]. Since the tested DD sequences were not designed for eliminating the effect of homonuclear couplings, the longer decoherence times for CPMG applied to certain initial conditions cannot be taken as a measure of its performance. In this work we therefore use a system that does not exhibit homonuclear interactions. This allows us to show that the robust sequences can also achieve the optimal decoherence time observed under CPMG, and this performance is independent of the initial condition.

The article is organized as follows. In Sec. II we define the system, in Sec. III we introduce the DD sequences to be compared, and in Sec. IV we compare their robustness against pulse errors effects. In Sec. V we give a qualitative theoretical analysis based on average Hamiltonian theory to explain the experimental results. In the last section we draw some conclusions.

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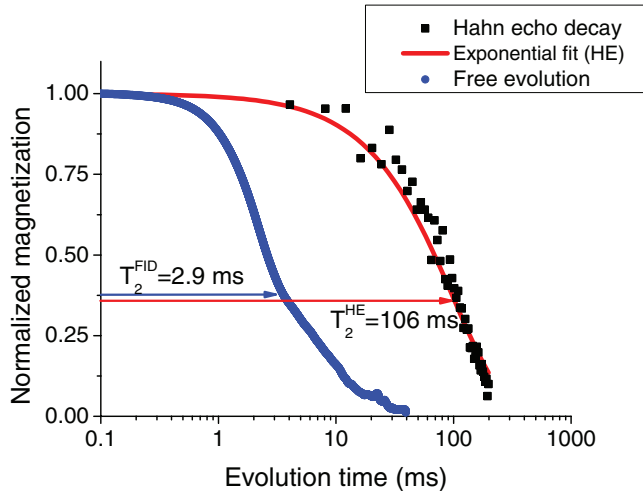


FIG. 1. (Color online) Decay of the magnetization of the ^1H spin system under free evolution and in a Hahn spin-echo (HE) sequence.

II. THE SYSTEM

The experimental system is an ensemble of noninteracting spins $1/2$. They consist of the protons of a water sample to which we added 5 mg/100 ml CuSO_4 to reduce the T_1 relaxation time to 287 ms. This results in faster repetition times and shorter overall duration of the experiments. The sample was placed in a static magnetic field along the z direction and its Hamiltonian is

$$\mathcal{H}_s = \omega_s S_z, \quad (1)$$

where ω_s is the Zeeman frequency and \hat{S}_z is the system spin operator along the z axis. The inhomogeneities of the static field correspond to a static perturbation, and molecular motion makes this perturbation time-dependent on a time scale that is slow compared with the delays between the DD pulses used in our experiments. This makes it possible to refocus this perturbation very effectively.

The second major source of noise is the fluctuating dipole-dipole interaction, whose correlation time is the molecular reorientation time (≈ 35 fs), much faster than any conceivable control fields for nuclear spins and therefore not amenable to DD. On the other hand, these fluctuations are so fast that their average effect on the system is relatively small [54].

Experiments were performed on a home-built NMR spectrometer with a ^1H resonance frequency of 360 MHz. The radio frequency field strength was $2\pi \times 13.3$ kHz, which corresponds to a π pulse duration of $37.5 \mu\text{s}$. An initial state $\propto I_x$ or I_y was prepared by rotating the I_z equilibrium state with a resonant $\pi/2$ pulse. The free evolution decay of the transversal magnetization of our system has a decay time of 2.9 ms (free induction decay). A simple Hahn-echo sequence [13] increases this time to 106 ms, as shown in Fig. 1.

III. DYNAMICAL DECOUPLING SEQUENCES

DD sequences consist of repetitive trains of π pulses. The delays between the pulses and their phases are important parameters for improving the performance of the DD sequences [4–6,50,52,53,55,56]. In particular the relative phases, which

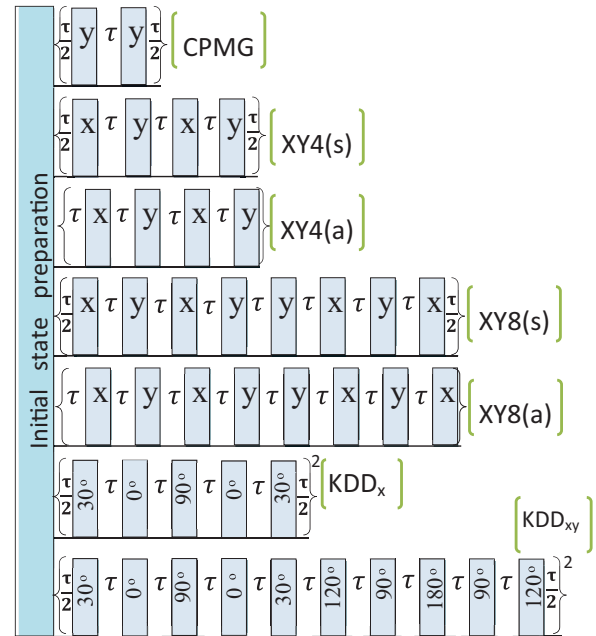


FIG. 2. (Color online) Dynamical decoupling pulse sequences tested in this work.

correspond to the directions of the rotation axes, are important for making the sequences robust against pulse imperfections and unwanted environmental interactions [18,22,24].

Figure 2 gives an overview of the sequences that we examined for this work. It shows a single cycle for each sequence, which is repeated as often as required. τ is the delay between the pulses. The Carr-Purcell (CP) sequence [52] and the version by Meiboom and Gill [53], known as CPMG, use the same sequence of refocusing pulses; they differ only with respect to the state to which they are applied. In the case of the CP sequence, the initial state is perpendicular to the rotation axis of the inversion pulses; in the CPMG version, it is parallel. Errors in the flip angles destroy the perpendicular component, but they leave the longitudinal component unscathed [18,53]. The sequence XY4 was introduced by Maudsley [55], and it reduces the effect of pulse imperfections for arbitrary initial states [18,55,56]. It consists of four pulses with phases x - y - x - y (Fig. 2). An asymmetric version of the XY4 sequence was introduced by Viola *et al.* [4], which we designate XY4(a). The XY8-sequences are symmetrized versions of the XY4 sequences [46,56]. Two DD sequences that are particularly robust against flip-angle and resonance offset errors are the KDD_x and KDD_{xy} sequences [24,47,48]. They were designed by combining the rotation pattern of the XY4 sequence with that of a robust composite pulse [57].

In earlier works using these sequences, the conditions were chosen such that the dominant perturbation was the environmental noise [18,24,44,46,47]. In this work, we focus on a system that allow us to make a comparison between these sequences in a regime where all sequences perform equally well at eliminating the environmental noise, and any differences in their performance can be attributed directly to their robustness, i.e., to their efficiency in suppressing the effect of pulse imperfections.

IV. ROBUSTNESS COMPARISON

To compare the sensitivity of the sequences to pulse imperfections, we prepared two orthogonal initial states I_x and I_y and then measured their decay as a function of time under the application of the different DD sequences described in the previous section. Figure 3 shows the echo train of a CPMG sequence. From these data, we extracted the signal (I_x magnetization in this example) at the end of each DD cycle (marked by blue squares in the figure). The decay of the echoes was mostly exponential, with some exceptions discussed below.

The extracted echoes (blue squares in Fig. 3) were fitted with an exponential function to obtain the decay time of the magnetization. Experiments were repeated and the decay times plotted as a function of the delay between pulses.

Figure 4 compares the decay times of different DD sequences as a function of the delay between pulses. For the CPMG sequence, we present the decay of the I_x and I_y magnetization separately, marked as CP and CPMG, respectively. For the other sequences, whose performance is quite symmetric with respect to the initial condition, we present the decay times averaged over the two initial conditions. For long delays between the pulses, the observed decay times reach a limiting value of ≈ 276 ms, irrespective of the sequence and the initial condition, and very close to the measured value of T_1 . This is a verification of the assumption that all sequences can effectively decouple the slowly fluctuating environment.

For shorter pulse delays (i.e., more pulses in a given time interval), the signal decays more rapidly. This is most prominent for the CP sequence. In this situation, pulse imperfections add coherently and generate a rapid loss of magnetization [18].

As the pulse delays become shorter than 0.5 ms, which corresponds to 864 pulses during the 0.5 s measurement time, the other sequences also start to generate shorter decay times,

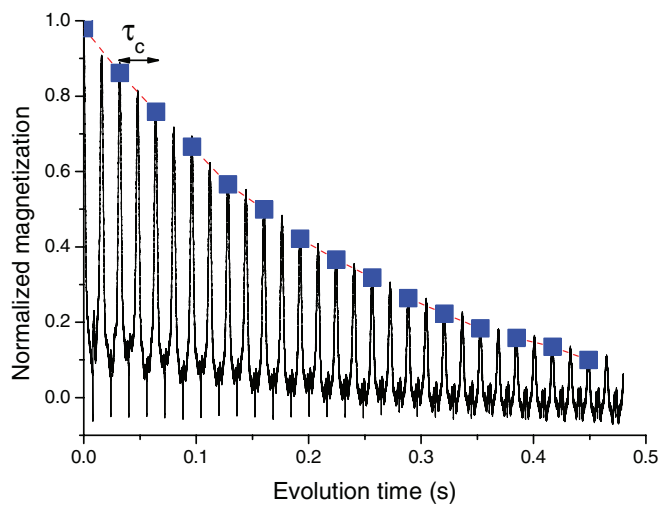


FIG. 3. (Color online) Time evolution of the spin-system magnetization under the application of a CPMG sequence. The black solid line shows the evolution of the magnetization, and the blue squares mark the echo amplitude at the end of a CPMG cycle. We use the echo maxima for measuring the CPMG decay time. The cycle time was $\tau_c = 32$ ms.

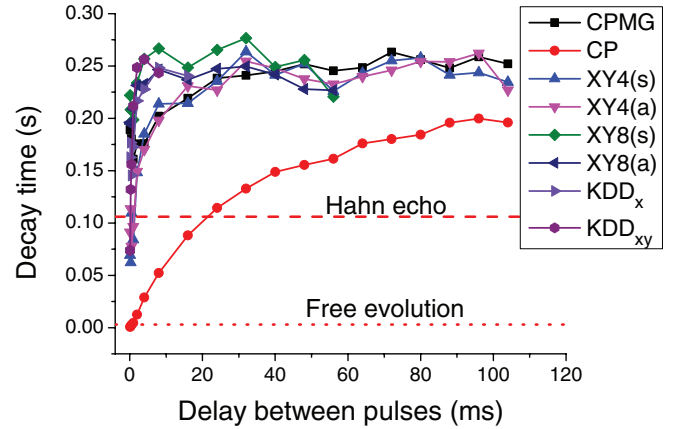


FIG. 4. (Color online) Average decay times as a function of the delay τ between pulses for different DD sequences.

and their decays become nonexponential. Figure 5 shows a representative example of such a signal. It can be fitted with a double exponential

$$s(t) = a e^{-t/T_2^f} + b e^{-t/T_2^s} \quad (2)$$

with two decay times T_2^f and T_2^s .

Figure 6 shows the decay times fitted with Eq. (2) for different average delays between pulses. For the fast decay times (T_2^f), which are represented by empty symbols in Fig. 6, the performance of all DD sequences is quite similar. For the slow component (T_2^s , represented by filled symbols), XY8, CPMG, and KDD_x perform better than XY4.

As we stated before, the decay time is dominated by the effect of pulse errors for short delays. The resulting average Hamiltonian projects the magnetization onto its eigenbase; this results in the fast decay component. After this projection, the remaining magnetization, which is not significantly affected by the pulse imperfections, decays on a slower time scale, which is dominated by the environment.

For a quantitative comparison of the different pulse sequences, we calculate the average magnetization decay

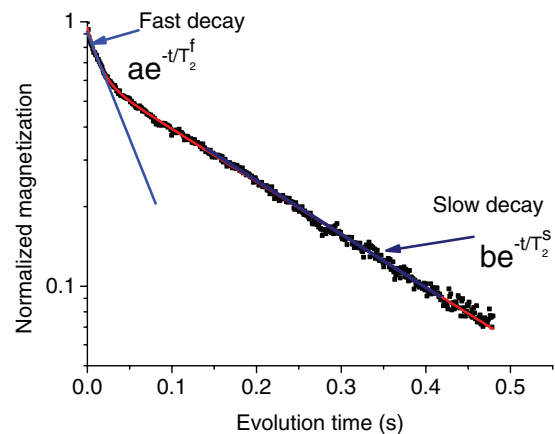


FIG. 5. (Color online) Normalized spin magnetization as a function of the evolution time for short delays ($\tau = 100 \mu\text{s}$) between the pulses for an XY8(s) sequence. Pulse errors dominate here, inducing a multiexponential decay. The red solid line is a fit to Eq. (2).

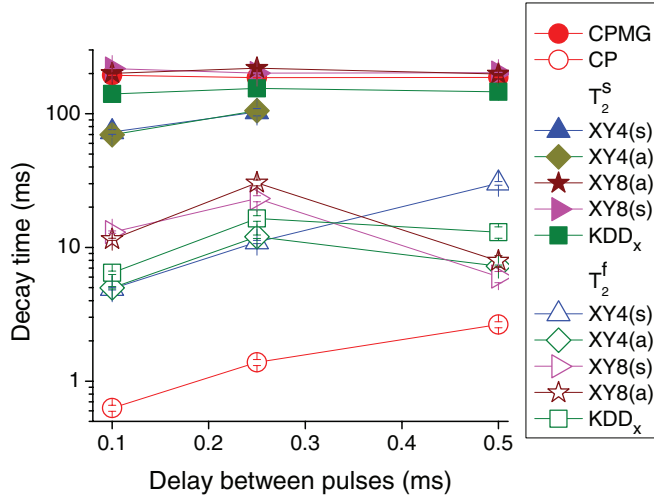


FIG. 6. (Color online) Fitted decay times for the double-exponential decay.

resulting per pulse of the sequence. For this evaluation, we consider only the short-time component described by T_2^f .

Since the pulse error is the dominant source of decay, we quantify its effect by measuring the fractional decay of the magnetization per pulse. The pulses are the same for all the DD sequences, but their effect, averaged over full cycles, shows how well the sequence is able to cancel the imperfections of the individual pulses.

Figure 7 shows the average decay per pulse for the different sequences, plotted against the number of pulses. For these data, the interpulse delay was $\tau = 100 \mu\text{s}$. The most conspicuous feature is that CP performs very badly and CPMG very well. The compensated sequences lie between these two extremes, and we find that the higher order sequences (XY8, KDD) perform better than the lower order sequences (XY4). For unknown initial conditions, KDD shows the best performance. Under the present conditions, sequences that differ only with respect to time reversal symmetry perform quite similarly, in contrast to other cases discussed earlier [46].

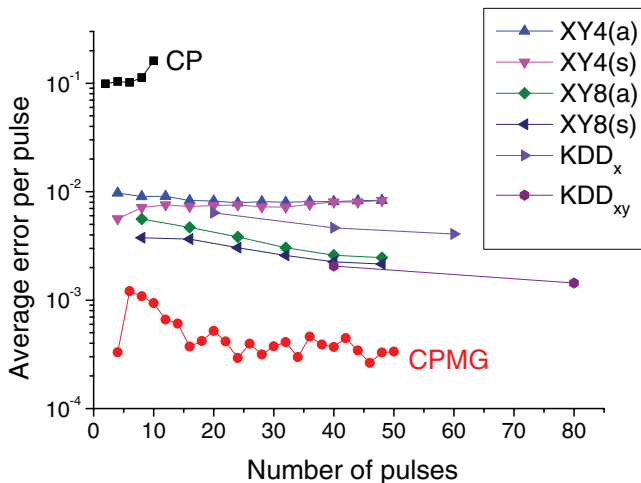


FIG. 7. (Color online) Average error per pulse for different DD sequences with delay $\tau = 100 \mu\text{s}$.

V. THEORETICAL ANALYSIS BY AVERAGE HAMILTONIAN THEORY

Average Hamiltonian theory (AHT) can be used to describe the effect of DD sequences applied to the system during an interval of time $t_1 < t < t_2$. If the evolution of the system is governed by a time-dependent Hamiltonian $\mathcal{H}(t)$, the effective evolution can be described by an average Hamiltonian $\tilde{\mathcal{H}}(t_1, t_2)$. If the Hamiltonian $\mathcal{H}(t)$ is periodic with a cycle time τ_c , i.e. $\mathcal{H}(t) = \mathcal{H}(t + \tau_c)$ and the observation is stroboscopic and synchronized with the period τ_c , the evolution operator for each cycle is $\exp\{-i\tilde{\mathcal{H}}(0, \tau_c)\tau_c\}$ [58,59].

As we discussed above, the remaining environmental noise fluctuates so rapidly that its effects cannot be reduced by DD. It is a good approximation to describe it as a classical field affecting the precession frequency of the spins [60]. Therefore Eq. (1) can be written as

$$\mathcal{H}_{SE} = \Delta\omega_z(t)S_z, \quad (3)$$

where the average of the random precession frequency is $\langle \Delta\omega_z(t) \rangle = 0$ for every t . It causes an exponential attenuation e^{-t/T_2} independent of the delay between the pulses.

We can write the pulse propagator as a composition of the product of the ideal pulse propagator $R_\phi = e^{-i\pi S_\phi}$ and two additional evolutions for flip angle errors as

$$R_\phi = e^{-i(1+\epsilon)\pi S_\phi} = e^{-iH_\phi t_p/2} e^{-i\pi S_\phi} e^{-iH_\phi t_p/2}, \quad (4)$$

where $H_\phi = \frac{\epsilon\pi}{t_p} S_\phi$ and t_p is the pulse length. For the sequences XY4(s) and XY4(a) the effect of the flip angle error vanishes in the zero-order average Hamiltonian, while the first-order term for both sequences is [46]

$$\tilde{\mathcal{H}}_1^{XY4(s)} = \tilde{\mathcal{H}}_1^{XY4(a)} = \frac{5\epsilon^2\pi^2}{16\tau} S_z. \quad (5)$$

This shows that there is no difference between symmetric and asymmetric sequences of XY4 up to first order of the average Hamiltonian, which is in good agreement with the experimental results of Fig. 7.

We consider now the XY8(s) and XY8(a) sequences. The zero-order and the first-order average Hamiltonian terms in Ref. [46] vanish if we consider only the flip angle error effects. The first nonzero term of the average Hamiltonian is then the second-order term, which is again equal for both versions of the sequence:

$$\tilde{\mathcal{H}}_2^{XY8(s)} = \tilde{\mathcal{H}}_2^{XY8(a)} = \frac{13\epsilon^3\pi^3}{1536\tau} (S_x + S_y). \quad (6)$$

This is also in excellent agreement with Fig. 7 where the symmetric and asymmetric version behave similarly, but they are more robust than the XY4 sequences.

For CPMG and CP, the zeroth-order and first-order average Hamiltonians are

$$\tilde{\mathcal{H}}_0^{\text{CPMG}} = \frac{\epsilon\pi S_y}{\tau}, \quad (7)$$

and

$$\tilde{\mathcal{H}}_1^{\text{CPMG}} = 0. \quad (8)$$

In the CPMG experiment, the initial condition is $\propto S_y$, which commutes with the average Hamiltonian and is therefore

not affected by pulse errors. In the CP experiment, the initial condition is $\propto S_x$, which is dephased by the pulse errors, in agreement with the data in Fig. 7 [18].

In the case of the KDD_x pulse sequence, the zero- and first-order average Hamiltonians vanish. The higher order terms were kept small by the design of the sequence [57,61]. As shown in Ref. [62], this makes it robust against several systematic errors because it is a geometric quantum gate. In Ref. [24], we showed by numerical simulation and experimental data that the KDD_{xy} sequence is more robust against flip angle errors than the other DD sequences tested. Overall, the experimental comparison between the different sequences is in good agreement with the numerical simulations and analytical results based on average Hamiltonian theory.

VI. CONCLUSION

We have tested the robustness of different DD sequences by comparing them in an environment that interacts with the spins in such a way that the decoherence time under the application

of DD sequences with ideal pulses is independent of the delay between the pulses. This allowed us to study the robustness of the different DD sequences by isolating the effects of the pulse errors. We found that the decoherence time of the most robust sequences, the KDD family, is the longest for arbitrary initial states. This is consistent with the measured error per pulse averaged over many cycles of a DD sequence, where the KDD sequences have the lowest effective error. In the regime studied, the time symmetrization on the cycles does not play a significant role for reducing decoherence, since only the phases of the pulses are important for reducing the effect of pulse errors. Our experimental results for pulse errors are in good agreement with previous numerical simulations and predictions of average Hamiltonian theory.

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