of the corresponding operator $\hat{A}$:

$$|\psi\rangle = \sum_n |\psi_n\rangle\langle\psi_n|\psi\rangle = \sum_n a_n |\psi_n\rangle. \tag{3.30}$$

According to Postulate 4, the act of measuring A changes the state of the system from $|\psi\rangle$ to one of the eigenstates $|\psi_n\rangle$ of the operator $\hat{A}$, and the result obtained is the eigenvalue a_n. The only *exception* to this rule is when the *system is already in one of the eigenstates of the observable being measured.* For instance, if the system is in the eigenstate $|\psi_n\rangle$, a measurement of the observable A yields with certainty (i.e., with probability = 1) the value a_n without changing the state $|\psi_n\rangle$.

Before a measurement, we do not know in advance with certainty in which eigenstate, among the various states $|\psi_n\rangle$, a system will be after the measurement; only a probabilistic outcome is possible. Postulate 4 states that the probability of finding the system in one particular nondegenerate eigenstate $|\psi_n\rangle$ is given by

$$P_n = \frac{|\langle\psi_n|\psi\rangle|^2}{\langle\psi|\psi\rangle}. \tag{3.31}$$

Note that the wave function does not predict the results of individual measurements; it instead determines the probability distribution, $P = |\psi|^2$, over measurements on many identical systems in the same state.

Finally, we may state that quantum mechanics is the mechanics applicable to objects for which measurements necessarily interfere with the state of the system. Quantum mechanically, we cannot ignore the effects of the measuring equipment on the system, for they are important. In general, certain measurements cannot be performed without major disturbances to other properties of the quantum system. In conclusion, *it is the effects of the interference by the equipment on the system which is the essence of quantum mechanics*.

3.5.2 Expectation Values

The expectation value $\langle\hat{A}\rangle$ of $\hat{A}$ with respect to a state $|\psi\rangle$ is defined by

$$\langle\hat{A}\rangle = \frac{\langle\psi|\hat{A}|\psi\rangle}{\langle\psi|\psi\rangle}. \tag{3.32}$$

For instance, the energy of a system is given by the expectation value of the Hamiltonian: $E = \langle\hat{H}\rangle = \langle\psi|\hat{H}|\psi\rangle/\langle\psi|\psi\rangle$.

In essence, the expectation value $\langle\hat{A}\rangle$ represents the average result of measuring $\hat{A}$ on the state $|\psi\rangle$. To see this, using the complete set of eigenvectors $|\psi_n\rangle$ of $\hat{A}$ as a basis (i.e., $\hat{A}$ is diagonal in ψ_n), we can rewrite $\langle\hat{A}\rangle$ as follows:

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$$\langle\hat{A}\rangle = \frac{1}{\langle\psi|\psi\rangle}\sum_{nm}\langle\psi|\psi_m\rangle\langle\psi_m|\hat{A}|\psi_n\rangle\langle\psi_n|\psi\rangle = \sum_n a_n \frac{|\langle\psi_n|\psi\rangle|^2}{\langle\psi|\psi\rangle}, \tag{3.33}$$

where we have used $\langle\psi_m|\hat{A}|\psi_n\rangle = a_n\delta_{nm}$. Since the quantity $|\langle\psi_n|\psi\rangle|^2/\langle\psi|\psi\rangle$ gives the probability P_n of finding the value a_n after measuring the observable A, we can indeed interpret $\langle\hat{A}\rangle$ as an *average* of a series of measurements of A:

$$\boxed{\langle\hat{A}\rangle = \sum_n a_n \frac{|\langle\psi_n|\psi\rangle|^2}{\langle\psi|\psi\rangle} = \sum_n a_n P_n.} \tag{3.34}$$