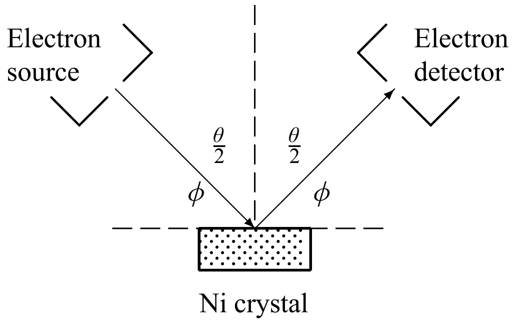

Figure 1.6 Davisson–Germer experiment: electrons strike the crystal's surface at an angle ϕ; the detector, symmetrically located from the electron source, measures the number of electrons scattered at an angle θ, where θ is the angle between the incident and scattered electron beams.

and a maximum at $\theta = 50^\circ$; that is, the bulk of the electrons scatter only in well-specified directions. They showed that the pattern persisted even when the intensity of the beam was so low that the incident electrons were sent one at a time. This can only result from a constructive interference of the scattered electrons. So, instead of the diffuse distribution pattern that results from material particles, the reflected electrons formed diffraction patterns that were identical with Bragg's X-ray diffraction by a *grating*. In fact, the intensity maximum of the scattered electrons in the Davisson–Germer experiment corresponds to the first maximum ($n = 1$) of the Bragg formula,

$$n\lambda = 2d \sin\phi, \tag{1.46}$$

where d is the spacing between the Bragg planes, ϕ is the angle between the incident ray and the crystal's reflecting planes, θ is the angle between the incident and scattered beams (d is given in terms of the separation D between successive atomic layers in the crystal by $d = D\sin\theta$). (debe corresponder a otro dibujo)

For an Ni crystal, we have $d = 0.091$ nm, since $D = 0.215$ nm. Since only one maximum is seen at $\theta = 50^\circ$ for a mono-energetic beam of electrons of kinetic energy 54 eV, and since $2\phi + \theta = \pi$ and hence $\sin\phi = \cos(\theta/2)$ (Figure 1.6), we can obtain from (1.46) the wavelength associated with the scattered electrons:

$$\lambda = \frac{2d}{n}\sin\phi = \frac{2d}{n}\cos\frac{1}{2}\theta = \frac{2 \times 0.091\ \text{nm}}{1}\cos 25^\circ = 0.165\ \text{nm}. \tag{1.47}$$

Now, let us look for the numerical value of λ that results from de Broglie's relation. Since the kinetic energy of the electrons is $K = 54$ eV, and since the momentum is $p = \sqrt{2m_eK}$ with $m_ec^2 = 0.511$ MeV (the rest mass energy of the electron) and $\hbar c \simeq 197.33$ eV nm, we can show that the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{2\pi\hbar c}{\sqrt{2m_ec^2K}} = 0.167\ \text{nm}, \tag{1.48}$$

which is in excellent agreement with the experimental value (1.47).

We have seen that the scattered electrons in the Davisson–Germer experiment produced interference fringes that were identical to those of Bragg's X-ray diffraction. Since the Bragg formula provided an accurate prediction of the electrons' interference fringes, the motion of an electron of momentum $\vec{p}$ must be described by means of a plane wave

$$\psi(\vec{r}, t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)} = Ae^{i(\vec{p}\cdot\vec{r}-Et)/\hbar}, \tag{1.49}$$