Gas de Bose-Einstein (Reichl-Huang)

$$Z_\mu^{(\mathrm{BE})}(T,V) = \sum_{n_o=0}^{\infty} \cdots \sum_{n_\infty=0}^{\infty} e^{-\beta \sum_{j=0}^{\infty} n_j(\epsilon_j-\mu)} = \prod_{j=0}^{\infty}\left[\sum_{n_j=0}^{\infty} e^{-\beta(\epsilon_j-\mu)n_j}\right]$$

$\mu < \epsilon_j \ \ \forall j \quad \left(t^n = e^{-\beta(\epsilon_j-\mu)\,n} < 1\right)$

$$\hookrightarrow \quad Z_\mu^{(\mathrm{BE})}(T,V) = \prod_{j=0}^{\infty}\left[\frac{1}{1-e^{-\beta(\epsilon_j-\mu)}}\right]$$

ϵ_j depende solo de $\boldsymbol{p}_\ell \quad \rightarrow \quad j=(\ell, m_s) \qquad m_s = -s, -s+1, \ldots, s$

proyección del espín s

$Z_\mu^{(\mathrm{BE})}$: $g_s = 2s+1$ factores idénticos ...

$$Z_\mu^{(\mathrm{BE})}(T,V) = \prod_{\ell=0}^{\infty}\left[\frac{1}{1-e^{-\beta(\epsilon_\ell-\mu)}}\right]^{g_s}$$

$$(s=0 \quad \leftrightarrow \quad g_s = 1) \qquad \hookrightarrow \quad \Omega_{\mathrm{BE}}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1-e^{-\beta(\epsilon_\ell-\mu)}\right]$$

Gas de Bose-Einstein

$$\Omega_{\rm BE}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell - \mu)}\right]$$

$$\langle N \rangle = -\left(\frac{\partial \Omega_{\rm BE}}{\partial \mu}\right)_{T,V} = \sum_{\ell=0}^{\infty} \frac{1\; g_s}{e^{\beta(\epsilon_\ell-\mu)} - 1} = \sum_{\ell=0}^{\infty} \langle n_\ell \rangle$$

$\uparrow$

número medio de partículas en $\boldsymbol{p}_\ell$

fugacidad $\quad z \equiv e^{\beta\mu} \quad \rightarrow \quad \langle n_\ell \rangle = \dfrac{z\; g_s}{e^{\beta\epsilon_\ell} - z} \geq 0 \quad \Rightarrow \quad e^{\beta\epsilon_\ell} > z \;\; \forall\, \ell$

$\hookrightarrow$ mínima ϵ_ℓ : $0 \quad \rightarrow \quad \boldsymbol{0 < z < 1} \quad \Rightarrow \quad \boldsymbol{\mu < 0}$

estado fundamental : $\quad \langle n_o \rangle = \dfrac{z\; g_s}{1 - z}$

¿ $z \rightarrow 1$?

Gas de Bose-Einstein

$$\Omega_{\rm BE}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell-\mu)}\right] \qquad \langle N\rangle = \sum_{\ell=0}^{\infty} \frac{1\; g_s}{e^{\beta(\epsilon_\ell-\mu)} - 1}$$

partículas libres : $\epsilon = \dfrac{\boldsymbol{p}^2}{2m} = \dfrac{\hbar^2\boldsymbol{k}^2}{2m}$ condiciones periódicas : $k_{x,y,z}L = 2\pi\,\ell_{x,y,z}$

$$L\,\Delta k_{x,y,z} = 2\pi\,\Delta\ell_{x,y,z} \quad\Rightarrow\quad \Delta k_{x,y,z} = \frac{2\pi}{L}\Delta\ell_{x,y,z}$$

límite termodinámico : $\langle N\rangle$ muy grande $\leftrightarrow$ V muy grande $\Rightarrow$ $\boldsymbol{p} = \hbar\boldsymbol{k}$ continuas

$$\sum_{\ell} f(\boldsymbol{\ell})\,\Delta\ell_x\,\Delta\ell_y\,\Delta\ell_z \;\to\; \frac{V}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{k}\; f(\boldsymbol{k}) = \frac{V}{h^3}\int \mathrm{d}^3\boldsymbol{p}\; f(\boldsymbol{p})$$

dependencia a través de $\epsilon = p^2/(2m)$:

$$\sum_{\ell} f(\boldsymbol{\ell})\,\Delta\ell_x\,\Delta\ell_y\,\Delta\ell_z \;\to\; \frac{4\pi V}{h^3}\int \mathrm{d}p\; p^2\; f(p) = \frac{4\sqrt{2}\pi m^{3/2}V}{h^3}\int \mathrm{d}\epsilon\;\sqrt{\epsilon}\; f(\epsilon)$$

Gas de Bose-Einstein

$$\Omega_{\mathrm{BE}}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell - \mu)}\right] \qquad \langle N\rangle = \sum_{\ell=0}^{\infty} \frac{1\ g_s}{e^{\beta(\epsilon_\ell-\mu)} - 1}$$

$$\frac{\Omega_{\mathrm{BE}}(T,V,\mu)}{V} = \frac{kT\, g_s}{V}\ln(1-z) - \frac{kT\, g_s}{\lambda^3}\, g_{5/2}(z) \qquad z \equiv e^{\beta\mu}$$

$$\frac{1}{v} \equiv \frac{\langle N\rangle}{V} = \frac{\langle n_o\rangle}{V} + \frac{1\ g_s}{\lambda^3}\, g_{3/2}(z) \qquad \langle n_o\rangle = \frac{z\ g_s}{1-z}$$

$$g_{5/2}(z) = -\frac{4}{\sqrt{\pi}}\int_0^\infty \mathrm{d}x\ x^2\ \ln(1 - z e^{-x^2}) = \sum_{j=1}^{\infty}\frac{z^j}{j^{5/2}}$$

$$g_{3/2}(z) = \frac{4}{\sqrt{\pi}}\int_0^\infty \mathrm{d}x\ \frac{x^2}{z^{-1}e^{x^2}-1} = z\frac{\partial}{\partial z} g_{5/2}(z) = \sum_{j=1}^{\infty}\frac{z^j}{j^{3/2}}$$

sustitución $x^2 = \beta p^2/(2m)$

Gas de Bose-Einstein

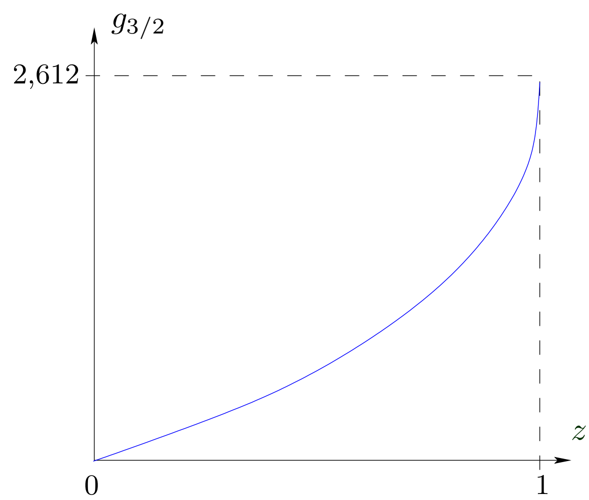
$$\frac{1}{v} \equiv \frac{\langle N \rangle}{V} = \frac{\langle n_o \rangle}{V} + \frac{1}{\lambda^3} \frac{g_s}{} g_{3/2}(z) \qquad \langle n_o \rangle = \frac{z\ g_s}{1-z} = \frac{1\ g_s}{e^{-\beta\mu} - 1}$$

T suficientemente altas o v grandes (bajas densidades)

término $\ell = 0$ aporta un *diferencial*

$\hookrightarrow$ innecesario separarlo

$g_{3/2}(z) \geq 0 \quad \left(g'_{3/2}(z) \geq 0\right)$ acotada



T ó v suficientemente bajos :

$g_{3/2}$ llega al máximo valor $(= 2,612)$

para z máximo

$\hookrightarrow$ ¡ el término $g_{3/2}(z)/\lambda^3$ no alcanza ! ***condensación de Bose-Einstein***

el aporte de $\boldsymbol{p} = 0$ deja de ser diferencial $\langle n_o \rangle / V \neq 0$

Gas de Bose-Einstein

$$\frac{1}{v} = \begin{cases} \frac{1}{\lambda^3} g_s \; g_{3/2}(z) & \text{si} \quad \frac{\lambda^3}{g_s v} \leq g_{3/2}(1) = 2,612 \\ \frac{1}{V}\frac{z\, g_s}{1-z} + \frac{1}{\lambda^3} g_s \; g_{3/2}(1) & \text{si} \quad \frac{\lambda^3}{g_s v} > g_{3/2}(1) \end{cases}$$

$$\frac{1}{v} \equiv \frac{\langle N \rangle}{V} = \frac{\langle n_o \rangle}{V} + \frac{1}{\lambda^3} g_s \; g_{3/2}(z)$$

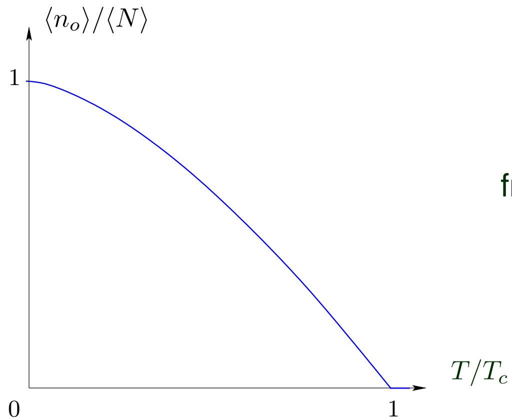
dado v , hay una ***temperatura crítica***

$$T_c = \frac{2\pi\hbar^2}{mk}\left(\frac{1}{2,612\, g_s\, v}\right)^{2/3}$$

dada T , hay un ***volumen específico crítico*** :

$$v_c = \frac{1}{2,612\, g_s}\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}$$

Gas de Bose-Einstein



$$\frac{1}{v} \equiv \frac{\langle N \rangle}{V} = \frac{\langle n_o \rangle}{V} + \frac{1}{\lambda^3} g_s \, g_{3/2}(z)$$

fracción de partículas en $\boldsymbol{p} = 0$

$$\frac{\langle n_o \rangle}{\langle N \rangle} = 1 - \frac{2,612 \, g_s}{\lambda^3} \frac{V}{\langle N \rangle} = 1 - \left(\frac{T}{T_c} \right)^{3/2}$$

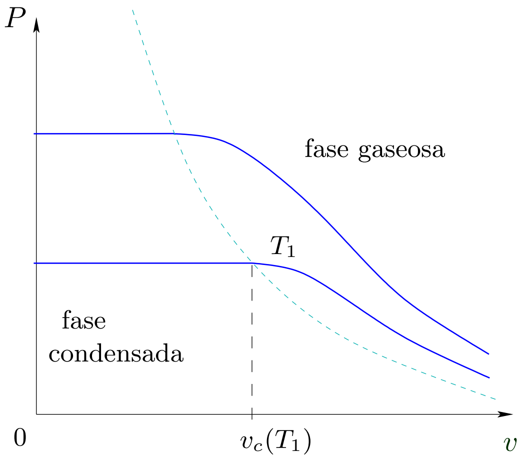
(si $T > T_c$ esta fracción se anula)

ec. Euler : $U = TS - PV + \mu \langle N \rangle \quad \rightarrow \quad \Omega = -PV = kT \, g_s \, \ln(1-z) - \frac{kTV \, g_s}{\lambda^3} g_{5/2}(z)$

$$\hookrightarrow \quad \begin{cases} P^{>} = \dfrac{kT \, g_s}{\lambda^3} g_{5/2}(z) & (T > T_c \text{ ó } v > v_c) \\ P^{<} = \dfrac{kT \, g_s}{\lambda^3} g_{5/2}(1) & (T < T_c \text{ ó } v < v_c) \end{cases}$$

$\searrow$ *independiente* de v

Gas de Bose-Einstein



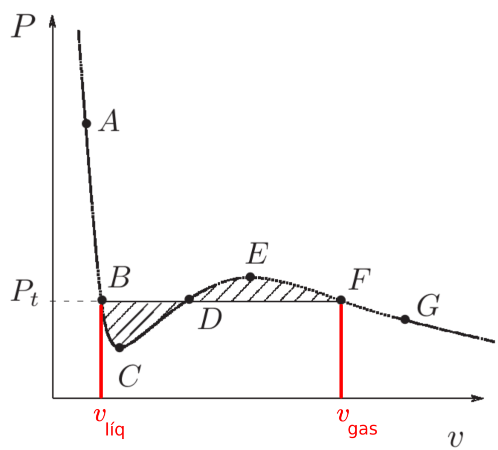
$$P^< = \frac{kT\, g_s}{\lambda^3}\, g_{5/2}(1) \underset{T\to 0}{\longrightarrow} 0$$

$\hookrightarrow$ estado $\boldsymbol{p} = 0$ (fase condensada)

no aporta a la presión

región horizontal : coexistencia

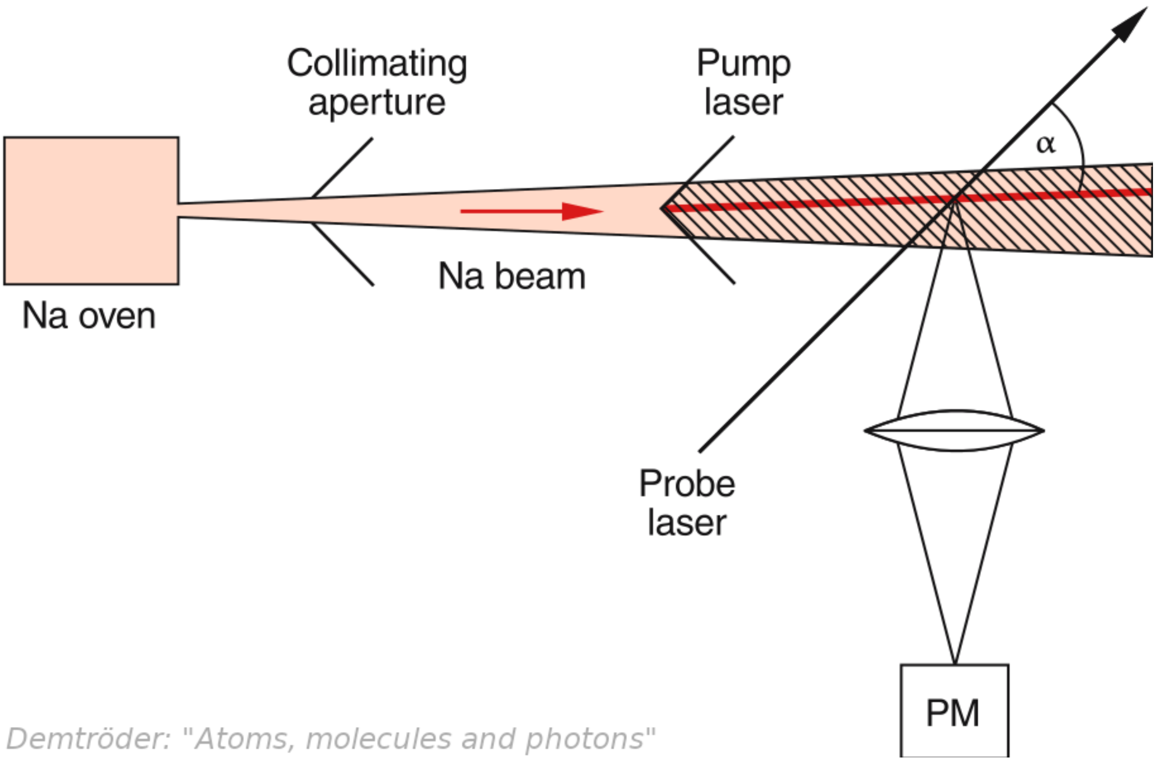
... van der Waals ...



$\rightarrow$ **transición de fase de *primer orden***

Muy bajas temperaturas

“enfriamiento óptico”



Demtröder: "Atoms, molecules and photons"

frenado por retroceso de fotones *(deceleration by photon recoil)*

Muy bajas temperaturas

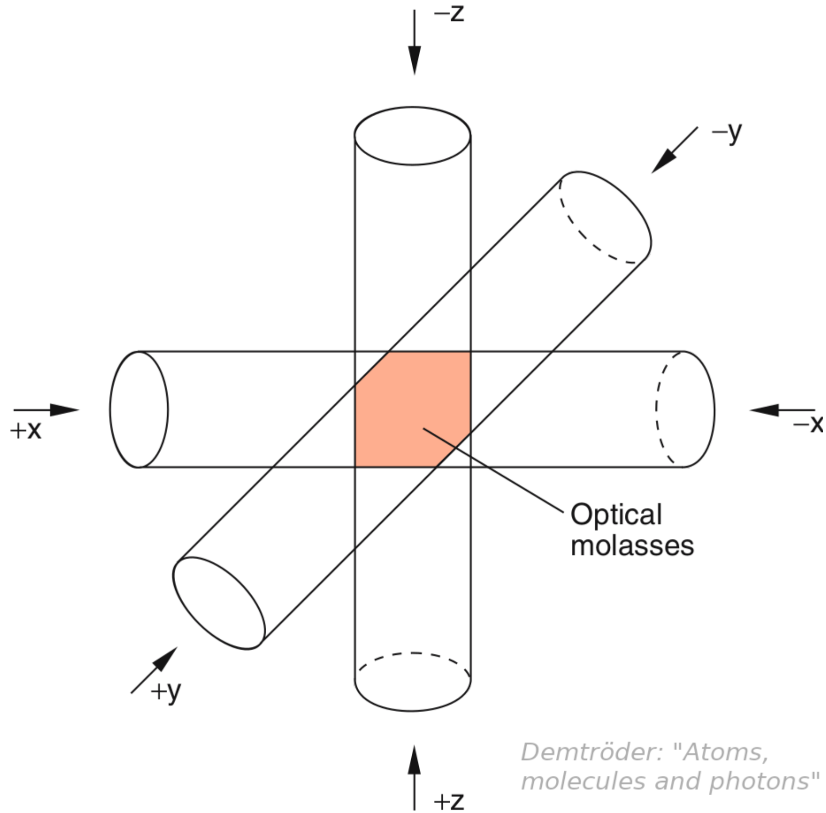
"melazas ópticas"

mediante 6 láseres combinados

absorción / excitación / emisión

desplazamiento Doppler

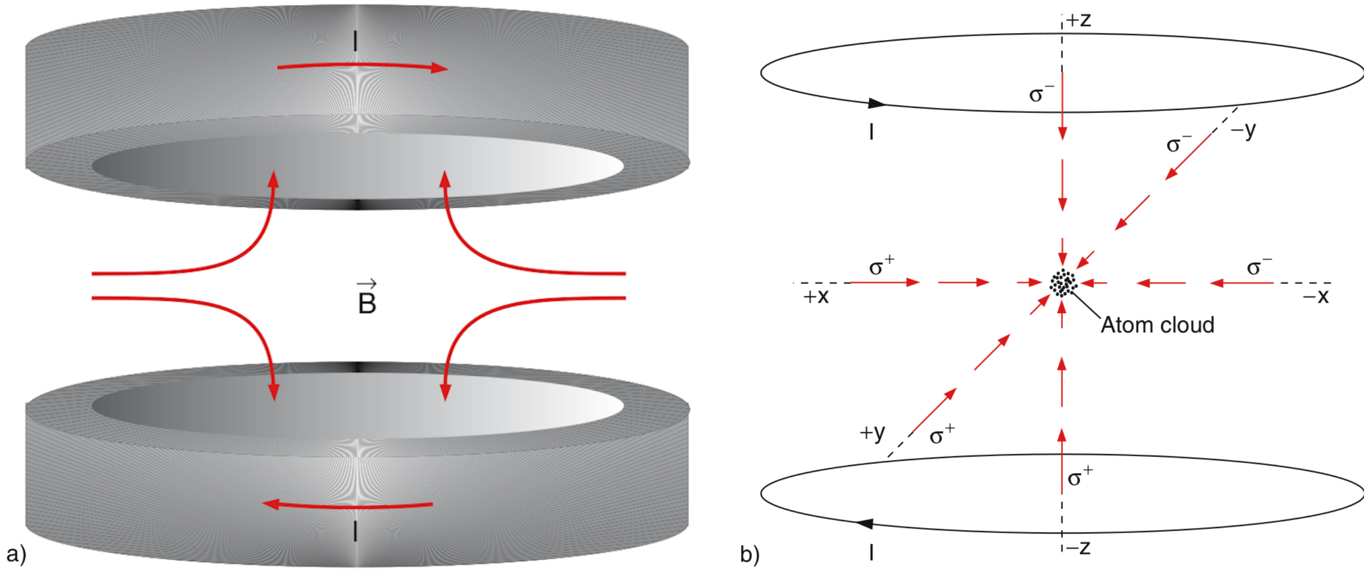
(limitación tecnológica)



Demtröder: "Atoms, molecules and photons"

Muy bajas temperaturas

“trampas magneto-ópticas”



Demtröder: "Atoms, molecules and photons"

desdoblamiento Zeeman $\propto \boldsymbol{B}$

Condensación de Bose-Einstein

1924-1925 : Bose y Einstein predicen condensación

He superfluido : ¿ bosones ?

$\hookrightarrow$ transición de fase a $2{,}17\,\mathrm{K}$ "transición lambda"

1995 : primera comprobación experimental

Science **269** (1995) 198

REPORTS

Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor

M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman,* E. A. Cornell

A Bose-Einstein condensate was produced in a vapor of rubidium-87 atoms that was confined by magnetic fields and evaporatively cooled. The condensate fraction first appeared near a temperature of 170 nanokelvin and a number density of 2.5×10^{12} per cubic centimeter and could be preserved for more than 15 seconds. Three primary signatures of Bose-Einstein condensation were seen. (i) On top of a broad thermal velocity distribution, a narrow peak appeared that was centered at zero velocity. (ii) The fraction of the atoms that were in this low-velocity peak increased abruptly as the sample temperature was lowered. (iii) The peak exhibited a nonthermal, anisotropic velocity distribution expected of the minimum-energy quantum state of the magnetic trap in contrast to the isotropic, thermal velocity distribution observed in the broad uncondensed fraction.

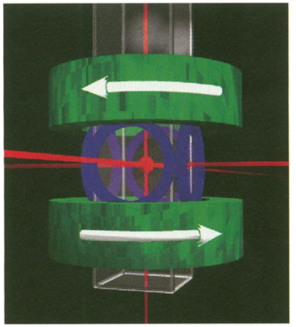


Fig. 1. Schematic of the apparatus. Six laser beams intersect in a glass cell, creating a magne-

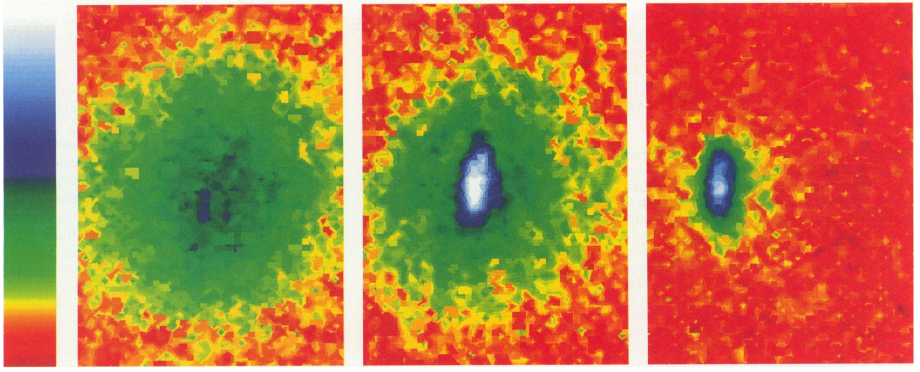


Fig. 2. False-color images display the velocity distribution of the cloud (**A**) just before the appearance of the condensate, (**B**) just after the appearance of the with thermal equilibrium. The condensate fraction (mostly blue and white) is elliptical, indicative that it is a highly nonthermal distribution. The elliptical pattern