# Gas de Bose-Einstein (Reichl-Huang)

$$Z_\mu^{(\mathrm{BE})}(T,V) = \sum_{n_o=0}^{\infty} \cdots \sum_{n_\infty=0}^{\infty} e^{-\beta \sum_{j=0}^{\infty} n_j(\epsilon_j-\mu)} = \prod_{j=0}^{\infty} \left[ \sum_{n_j=0}^{\infty} e^{-\beta(\epsilon_j-\mu)n_j} \right]$$

$\mu < \epsilon_j \;\; \forall j \quad \left( t^n = e^{-\beta(\epsilon_j-\mu)\, n} < 1 \right)$

$$\hookrightarrow \quad Z_\mu^{(\mathrm{BE})}(T,V) = \prod_{j=0}^{\infty} \left[ \frac{1}{1-e^{-\beta(\epsilon_j-\mu)}} \right]$$

$\epsilon_j$ depende solo de $\boldsymbol{p}_\ell \quad \rightarrow \quad j = (\ell, m_s) \qquad m_s = -s, -s+1, \ldots, s$

proyección del espín $s$

$Z_\mu^{(\mathrm{BE})}$ : $g_s = 2s+1$ factores idénticos ...

$$Z_\mu^{(\mathrm{BE})}(T,V) = \prod_{\ell=0}^{\infty} \left[ \frac{1}{1-e^{-\beta(\epsilon_\ell-\mu)}} \right]^{g_s}$$

$$(s=0 \quad \leftrightarrow \quad g_s = 1) \qquad \hookrightarrow \quad \Omega_{\mathrm{BE}}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1-e^{-\beta(\epsilon_\ell-\mu)}\right]$$

# Gas de Bose-Einstein

$$\Omega_{\rm BE}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell - \mu)}\right]$$

$$\langle N \rangle = -\left(\frac{\partial \Omega_{\rm BE}}{\partial \mu}\right)_{T,V} = \sum_{\ell=0}^{\infty} \frac{1\; g_s}{e^{\beta(\epsilon_\ell-\mu)} - 1} = \sum_{\ell=0}^{\infty} \langle n_\ell \rangle$$

$\uparrow$

número medio de partículas en $\boldsymbol{p}_\ell$

*fugacidad* $z \equiv e^{\beta\mu}$ $\rightarrow$ $\langle n_\ell \rangle = \dfrac{z\; g_s}{e^{\beta\epsilon_\ell} - z} \geq 0$ $\Rightarrow$ $e^{\beta\epsilon_\ell} > z \;\; \forall\; \ell$

$\hookrightarrow$ mínima $\epsilon_\ell$ : $0$ $\rightarrow$ $\mathbf{0 < z < 1}$ $\Rightarrow$ $\boldsymbol{\mu < 0}$

estado fundamental : $\langle n_o \rangle = \dfrac{z\; g_s}{1 - z}$ ¿ $z \rightarrow 1$ ?

# Gas de Bose-Einstein

$$\Omega_{\rm BE}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell-\mu)}\right] \qquad \langle N\rangle = \sum_{\ell=0}^{\infty} \frac{1\; g_s}{e^{\beta(\epsilon_\ell-\mu)}-1}$$

partículas libres : $\epsilon = \frac{\boldsymbol{p}^2}{2m} = \frac{\hbar^2\boldsymbol{k}^2}{2m}$ condiciones periódicas : $k_{x,y,z}L = 2\pi\,\ell_{x,y,z}$

$$L\,\Delta k_{x,y,z} = 2\pi\,\Delta\ell_{x,y,z} \quad\Rightarrow\quad \Delta k_{x,y,z} = \frac{2\pi}{L}\Delta\ell_{x,y,z}$$

límite termodinámico : $\langle N\rangle$ muy grande $\leftrightarrow$ $V$ muy grande $\Rightarrow$ $\boldsymbol{p} = \hbar\boldsymbol{k}$ continuas

$$\sum_{\ell} f(\boldsymbol{\ell})\,\Delta\ell_x\,\Delta\ell_y\,\Delta\ell_z \;\to\; \frac{V}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{k}\; f(\boldsymbol{k}) = \frac{V}{h^3}\int \mathrm{d}^3\boldsymbol{p}\; f(\boldsymbol{p})$$

dependencia a través de $\epsilon = p^2/(2m)$ :

$$\sum_{\ell} f(\boldsymbol{\ell})\,\Delta\ell_x\,\Delta\ell_y\,\Delta\ell_z \;\to\; \frac{4\pi V}{h^3}\int \mathrm{d}p\; p^2\; f(p) = \frac{4\sqrt{2}\pi m^{3/2}V}{h^3}\int \mathrm{d}\epsilon\;\sqrt{\epsilon}\; f(\epsilon)$$

# Gas de Bose-Einstein

$$\Omega_{\mathrm{BE}}(T,V,\mu) = kT\, g_s \sum_{\ell=0}^{\infty} \ln\left[1 - e^{-\beta(\epsilon_\ell-\mu)}\right] \qquad \langle N\rangle = \sum_{\ell=0}^{\infty} \frac{1\; g_s}{e^{\beta(\epsilon_\ell-\mu)}-1}$$

$$\frac{\Omega_{\mathrm{BE}}(T,V,\mu)}{V} = \frac{kT\, g_s}{V}\ln(1-z) - \frac{kT\, g_s}{\lambda^3} g_{5/2}(z) \qquad z \equiv e^{\beta\mu}$$

$$\frac{1}{v} \equiv \frac{\langle N\rangle}{V} = \frac{\langle n_o\rangle}{V} + \frac{1\; g_s}{\lambda^3} g_{3/2}(z) \qquad \langle n_o\rangle = \frac{z\; g_s}{1-z}$$

$$g_{5/2}(z) = -\frac{4}{\sqrt{\pi}}\int_0^\infty \mathrm{d}x\; x^2 \ln(1-ze^{-x^2}) = \sum_{j=1}^{\infty}\frac{z^j}{j^{5/2}}$$

$$g_{3/2}(z) = \frac{4}{\sqrt{\pi}}\int_0^\infty \mathrm{d}x\; \frac{x^2}{z^{-1}e^{x^2}-1} = z\frac{\partial}{\partial z} g_{5/2}(z) = \sum_{j=1}^{\infty}\frac{z^j}{j^{3/2}}$$

sustitución $x^2 = \beta p^2/(2m)$

# Gas de Bose-Einstein

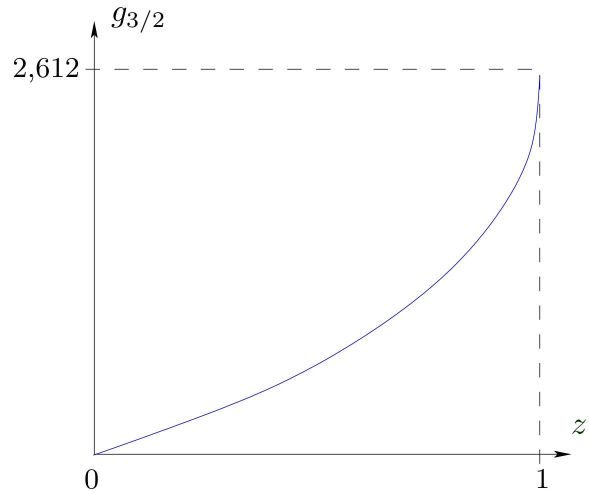
$$\frac{1}{v} \equiv \frac{\langle N \rangle}{V} = \frac{\langle n_o \rangle}{V} + \frac{1}{\lambda^3}\, g_s\, g_{3/2}(z) \qquad \langle n_o \rangle = \frac{z\, g_s}{1-z} = \frac{1\, g_s}{e^{-\beta\mu}-1}$$

$T$ suficientemente altas o $v$ grandes (bajas densidades)

término $\ell = 0$ aporta un *diferencial*

$\hookrightarrow$ innecesario separarlo

$g_{3/2}(z) \geq 0$ $\left(g'_{3/2}(z) \geq 0\right)$ acotada




$T$ ó $v$ suficientemente bajos :

$g_{3/2}$ llega al máximo valor $(= 2,612)$

para $z$ máximo

$\hookrightarrow$ ¡ el término $g_{3/2}(z)/\lambda^3$ no alcanza ! ***condensación de Bose-Einstein***

el aporte de $\boldsymbol{p} = 0$ deja de ser diferencial $\langle n_o \rangle / V \neq 0$

# Gas de Bose-Einstein

$$\frac{1}{v} = \begin{cases} \frac{1}{\lambda^3} \frac{g_s}{} \, g_{3/2}(z) & \text{si} \quad \frac{\lambda^3}{g_s v} \leq g_{3/2}(1) = 2,612 \\ \frac{1}{V} \frac{z \, g_s}{1-z} + \frac{1}{\lambda^3} \frac{g_s}{} \, g_{3/2}(1) & \text{si} \quad \frac{\lambda^3}{g_s v} > g_{3/2}(1) \end{cases}$$

$$\frac{1}{v} \equiv \frac{\langle N \rangle}{V} = \frac{\langle n_o \rangle}{V} + \frac{1 \, g_s}{\lambda^3} \, g_{3/2}(z)$$

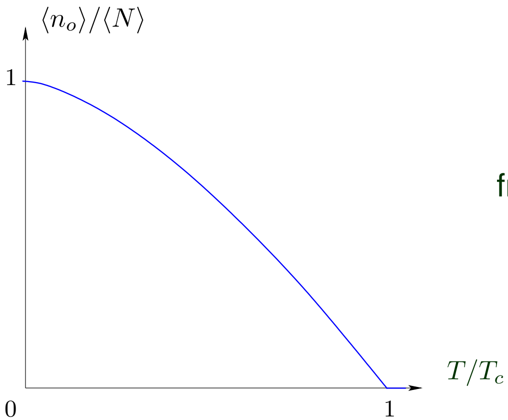
dado $v$ , hay una ***temperatura crítica***

$$T_c = \frac{2\pi\hbar^2}{mk} \left( \frac{1}{2,612 \, g_s \, v} \right)^{2/3}$$

dada $T$ , hay un ***volumen específico crítico*** :

$$v_c = \frac{1}{2,612 \, g_s} \left( \frac{2\pi\hbar^2}{mkT} \right)^{3/2}$$

# Gas de Bose-Einstein




$$\frac{1}{v} \equiv \frac{\langle N\rangle}{V} = \frac{\langle n_o\rangle}{V} + \frac{1}{\lambda^3} g_s\, g_{3/2}(z)$$

fracción de partículas en $\boldsymbol{p} = 0$

$$\frac{\langle n_o\rangle}{\langle N\rangle} = 1 - \frac{2{,}612\, g_s}{\lambda^3}\frac{V}{\langle N\rangle} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

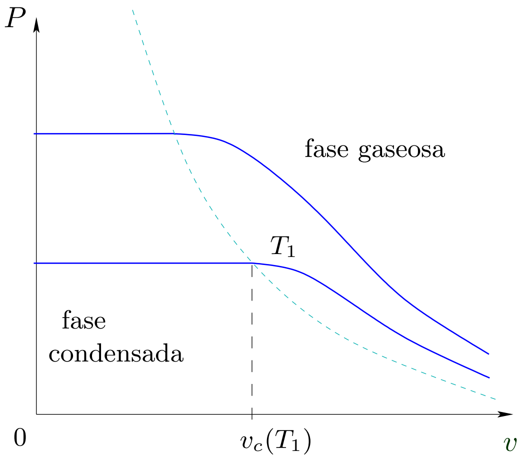
(si $T > T_c$ esta fracción se anula)

ec. Euler : $U = TS - PV + \mu\langle N\rangle \quad \rightarrow \quad \Omega = -PV = kT\, g_s\, \ln(1-z) - \frac{kTV\, g_s}{\lambda^3}\, g_{5/2}(z)$

$$\hookrightarrow \quad \begin{cases} P^{>} = \dfrac{kT\, g_s}{\lambda^3}\, g_{5/2}(z) & (T > T_c \ \text{ ó } \ v > v_c) \\[2ex] P^{<} = \dfrac{kT\, g_s}{\lambda^3}\, g_{5/2}(1) & (T < T_c \ \text{ ó } \ v < v_c) \end{cases}$$

$\searrow$ *independiente* de $v$

# Gas de Bose-Einstein



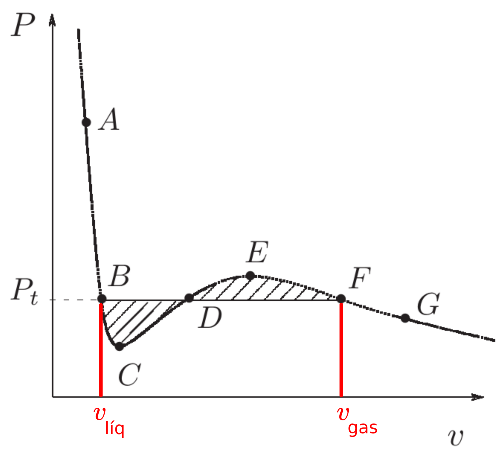

$$P^< = \frac{kT\, g_s}{\lambda^3}\, g_{5/2}(1) \underset{T\to 0}{\longrightarrow} 0$$

$\hookrightarrow$ estado $\boldsymbol{p} = 0$ (fase condensada)

**no** aporta a la presión

región horizontal : coexistencia

Van der Waals ...



# Gas de Bose-Einstein

presión de vapor (curva cortada) :

$g_s = 1$

$$P_o(T) = \frac{kT}{\lambda^3}\, g_{5/2}(1) \qquad \left( = \frac{2\pi\hbar^2 g_{5/2}(1)}{m\left[g_{3/2}(1)\right]^{5/3}} \frac{1}{v_c^{5/3}} \right)$$

ec. Clausius-Clapeyron :

$$\hookrightarrow \qquad \frac{\mathrm{d}P_o}{\mathrm{d}T} = \frac{5}{2}\frac{k\, g_{5/2}(1)}{\lambda^3} = \frac{1}{T\, v_c}\left[\frac{5}{2}\, kT \frac{g_{5/2}(1)}{g_{3/2}(1)}\right] = \frac{\ell}{T\,\Delta v}$$

$$\nearrow$$

$$\Delta v = v_c \qquad (v_{\mathrm{cond}} = 0)$$

se cumple si el calor latente de la transformación (por partícula) es

$$\ell = \frac{5}{2}\, kT\, \frac{g_{5/2}(1)}{g_{3/2}(1)}$$

(próximamente en su pantalla)

discontinuidades en derivadas *primeras* de energía libre de Gibbs

$\rightarrow$ **transición de fase de *primer orden***

# Gas de Bose-Einstein

$$G = \mu \langle N \rangle \quad \rightarrow \quad \begin{cases} G^{>} = \langle N \rangle kT \; \ln z \\ \\ G^{<} = 0 \end{cases}$$

$$\Omega = -PV = -\frac{kT\,1\,V}{\lambda^3}\, g_{5/2}(z)$$

$$S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu} \quad \rightarrow \quad \begin{cases} S^{>} = \dfrac{5}{2}\, \dfrac{k\,V}{\lambda^3}\, g_{5/2}(z) - \langle N \rangle k \; \ln z \\ \\ S^{<} = \underbrace{\dfrac{5}{2}\, \dfrac{k\,V}{\lambda^3}\, g_{5/2}(1)} \end{cases}$$

solo aporta la fase gaseosa $(\boldsymbol{s}_{\text{cond}} = \mathbf{0})$

¿ ?

OJO : $S = -\left(\dfrac{\partial G}{\partial T}\right)_{V,\langle \boldsymbol{N} \rangle}$

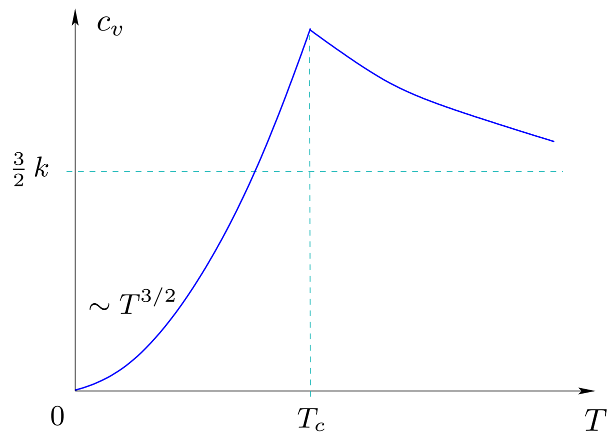
ejercicio : calor latente de la transformación $\ell$

# Gas de Bose-Einstein

$$U = \begin{pmatrix} \Omega + TS + \mu\langle N\rangle \\ G + TS - PV \end{pmatrix} = \frac{3}{2}PV \quad \Rightarrow \quad \begin{cases} U^{>} = \dfrac{3}{2}\dfrac{kTV}{\lambda^3}\, g_{5/2}(z) \\ U^{<} = \dfrac{3}{2}\dfrac{kTV}{\lambda^3}\, g_{5/2}(1) \end{cases}$$

$$c_v = \frac{T}{\langle N\rangle}\left(\frac{\partial S}{\partial T}\right)_{V,\langle N\rangle} \quad \rightarrow$$

$$\begin{cases} c_v^{>} = \dfrac{15}{4}\dfrac{k\,v}{\lambda^3}\, g_{5/2}(z) - \dfrac{9}{4}\, k\dfrac{g_{3/2}(z)}{g_{1/2}(z)} \\ c_v^{<} = \dfrac{15}{4}\dfrac{kv}{\lambda^3}\, g_{5/2}(1) \end{cases}$$




“transición lambda”

*gas de fotones* : $\langle N\rangle$ no agrega otro vínculo ... $\mu = 0$

# Gas de Bose-Einstein

KUNGL.
VETENSKAPSAKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

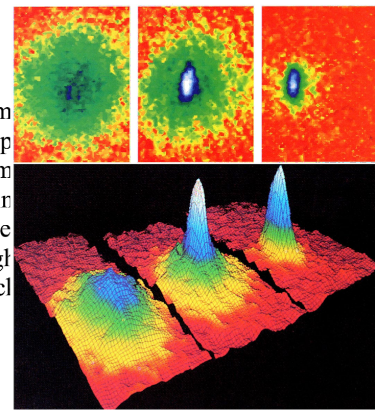
Information Department, P.O. Box 50005, SE-104 05 Stockholm, Sweden
Phone: +46 8 673 95 00, Fax: +46 8 15 56 70, E-mail: info@kva.se, Web site: www.kva.se

**Advanced information on the Nobel Prize in Physics 2001**

## Bose-Einstein Condensation in Alkali Gases

### 1. Bose-Einstein Condensation

At the beginning of the twentieth century, the quantum nature of therm
electromagnetic radiation was a subject of intense interest. This was sp
Planck's discovery that the spectral distribution of light emerging from
could be explained only if the radiators emitting the energy occurred ir
states. This induced Albert Einstein to conclude in 1905 that it was the
that was created and converted in bursts of energy; these bursts are ligh
were later to be called photons. The vision of such discrete energy pacl



1960s : *láseres* $\rightarrow$ para enfriar (1975)

... desplazamiento Doppler ... MOT (trampa magneto-óptica)

1995 : primer condensado Rb a $170\,\mathrm{K}$ Cornell-Wiedmann (JILA) Ketterle (MIT)

1997 : Nobel para Cohen-Tannoudji - Philips por métodos de enfriamiento $10^{-9}\,\mathrm{K}$

# Gas de Bose-Einstein

▷ BEC : **física básica**

▷ trampas ópticas $\longrightarrow$ "redes ópticas" ( *optical lattices* )

(sólido que puede *configurarse* )

▷ superconductividad : pares de $e^-$ fuertemente correlacionados

coherencia $\longrightarrow$ resistividad eléctrica nula

transición conductor-superconductor $\sim$ BEC

▷ computación cuántica : muchos *q-bits* en el mismo estado inicial

▷ láseres atómicos (coherencia) (por favor)

▷ *exciton-polariton lasers* - *spasers* transición de Berezinskii-Kosterlitz-Thouless