# *General Gravitational Lenses of Cosmological Systems*

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# Outline I

# Introduction and motivations I

- We are interested in a treatment of weak lenses which allow us to consider the whole curvature of the lens and therefore its whole energy-momentum; as an improvement to the standard formalism of weak lenses [Seitz et al. 1994, Schneider et al. 1992] in which just Newtonian matter distributions are considered.
- Several works in the literature have suggested that a broader approach in the mass content may be useful in the astrophysical description of DM [Gallo Moreschi 2012, Nucamendi et al. 2001, Arbey et al. 2003, Matos et al. 2000].
- In particular, it is desirable to have at hand expressions in terms of the whole curvature of the lens; and so available for any field equations besides Einstein's equations.
- We present a generalization of a previous work [Gallo Moreschi 2011] about general expressions for the optical scalars over a flat background to the cosmological context from first principles, this is recurring to the *geodesic deviation equation*.
- The expressions present two important features: they show the presence of new terms that are usually neglected and at the same time contains in a natural way the possibility of moving lenses.

# Weak gravitational lensing I

## The geodesic deviation equation

- With respect to a null tetrad adapted to the path of the photons $(\ell^a, m^a, \bar{m}^a, n^a)$ the deviation geodesic vector is:

$$\varsigma^a = \varsigma \bar{m}^a + \bar{\varsigma} m^a.$$

- The *geodesic deviation equation*:

$$\boxed{\ell(\ell(\boldsymbol{\mathcal{X}})) = -\mathcal{Q}\boldsymbol{\mathcal{X}},}$$

$$\boldsymbol{\mathcal{X}} = \begin{pmatrix} \varsigma \\ \bar{\varsigma} \end{pmatrix} \qquad \text{and} \qquad \mathcal{Q} = \begin{pmatrix} \Phi_{00} & \Psi_0 \\ \bar{\Psi}_0 & \Phi_{00} \end{pmatrix}.$$

For computing the optical scalars we choose to make the comparison with respect to the flat spacetime.

It is crucial to establish the distance to the source: the unique well defined geometrical distance is the affine distance, namely $\lambda$.

# Weak lensing cosmology I

## In a Robertson-Walker cosmology

- The lens is actually the whole spacetime; i.e. it is not placed at any particular distance. Due to the symmetry there is no notion of deflection angle.
- However, there exist a notion for the optical scalars: $\boxed{\delta\beta = (1-\kappa_c)\delta\theta}$

$$\kappa_c(\lambda) = \frac{4\pi G \varrho_{\mathrm{cr}}}{3!c^2}\left(\Omega_m + \frac{4}{3}\Omega_r\right)\lambda^2 + \frac{8\pi G \varrho_{\mathrm{cr}}}{4!c^3} H_0\Big(5\Omega_m + 8\Omega_r\Big)\lambda_r + \mathscr{O}\left(\lambda^4\right).$$

## An additional lens over the cosmology

- An alteration of the complete homogeneity and isotropy in scales much smaller than the cosmological ones.
- The optical scalars are computed without recurring to the deflection angle

$$\ell(\ell(\boldsymbol{\mathcal{X}})) = -(Q_B + Q_L)\boldsymbol{\mathcal{X}},$$

where

$$Q_B = \begin{pmatrix} \Phi_{00}^B & 0 \\ 0 & \Phi_{00}^B \end{pmatrix}, \qquad Q_L = \begin{pmatrix} \Phi_{00}^L & \Psi_0^L \\ \bar{\Psi}_0^L & \Phi_{00}^L \end{pmatrix}.$$

# Weak lensing cosmology II

**New expressions for the optical scalars in the cosmological context**

- The *optical matrix* $\mathcal{A}$ has the following structure

$$\mathcal{A} = (1 - \kappa_c) \begin{pmatrix} 1 - \kappa^L - \gamma_1^L & -\gamma_2^L \\ -\gamma_2^L & 1 - \kappa^L + \gamma_1^L \end{pmatrix};$$

$$\kappa^L = \int_0^{\lambda_s} \left( \frac{1}{D_A^2} \int_0^{\lambda'} \Phi_{00}^L \, D_A^2 \, d\lambda'' \right) d\lambda',$$

$$\gamma_1^L + i\gamma_2^L = \int_0^{\lambda_s} \left( \frac{1}{D_A^2} \int_0^{\lambda'} \Psi_0^L \, D_A^2 \, d\lambda'' \right) d\lambda'.$$

These equations are more general than the standard ones; they contain the whole curvature and therefore the complete energy-momentum tensor of the lens.

- In the general case one has to notice that $\kappa^L$ and $\gamma^L$ are independent; something that standard mass reconstructions techniques do not take it into account.

## The thin lens approximation

- The optical scalars simplify to

$$\kappa^L = \mathbf{D}_{ls} \int_0^{\lambda_s} \Phi_{00}^L \, d\lambda', \qquad \gamma_1^L + i\gamma_2^L = \mathbf{D}_{ls} \int_0^{\lambda_s} \Psi_0^L \, d\lambda',$$

- The distance factor $\mathbf{D}_{ls}$ only contains information about the cosmology:

$$\mathbf{D}_{ls} = \frac{1}{1+z_l} \frac{D_{A_{ls}} D_{A_l}}{D_{A_s}};$$

where $z_l$ is the cosmological redshift at the position of the lens.

It is important to remark that in the standard literature,
the first factor on the right hand side is missing.

- Geometric model for the lens can be considered; for example spherically [Gallo Moreschi 2011, Gallo Moreschi 2012] and spheroidally symmetric lenses [Boero Moreschi 2016].

## Geometric models for the lens

*When the lens is modeled as static and spherically symmetric* one obtains some useful expressions in terms of the energy-momentum components:

$$\Phi_{00}^{L}(J) = \frac{4\pi G}{c^2}\left[\varrho(\mathbf{r}) + \frac{P_r(\mathbf{r})}{c^2} + \frac{J^2}{c^2\mathbf{r}^2}\left(P_t(\mathbf{r}) - P_r(\mathbf{r})\right)\right],$$

$$\Psi_{0}^{L}(J) = \frac{G}{c^2}\frac{J^2}{\mathbf{r}^2}\left[\frac{3M(\mathbf{r})}{\mathbf{r}^3} - \left(\varrho(\mathbf{r}) + \frac{P_t(\mathbf{r})}{c^2} - \frac{P_r(\mathbf{r})}{c^2}\right)\right];$$

where the impact parameter $J$ and $\lambda$ satisfy:

$$\mathrm{r}^2 = J^2 + \lambda^2.$$

- Even Kerr lenses can be considered in a simple way along this line *(Wednesday)*:

$$\Psi_0^L(X,Z) = \frac{3M}{(r - ia\cos(\theta))^5}\left[Z - i\left(\sqrt{K_o} + X\right)\right]^2.$$

If one is interested in a moving lens then ...

# Moving lenses I

- For a moving lens one might to refer the expression of the optical scalars to the intrinsic rest frame of the lens.

  Instead of calculate the curvature of moving sources one can think in leaving the geometry unaffected and only change the frame of observation by an appropriate boost.

- Both, $\Phi_{00}^{L}$ and $\Psi_{0}^{L}$ behave in the same way under boost.

  Intrinsic optical scalars, $\text{o}^{L} = \{\kappa^{L}, \gamma^{L}\}$ transform in the following way:

$$\text{o}_{v}^{L} = (1 + z_{v})\text{o}_{r}^{L},$$

  Comparing with the usual expressions one has

$$\text{o}_{v}^{L} = \frac{1 + z_{v}}{1 + z_{l}} \frac{D_{A_{ls}} D_{A_{l}}}{D_{A_{s}}} \int_{0}^{\lambda_{s}} C_{r}^{L}\, d\lambda_{r}'$$

  where the part in blue takes into account the motion of the lens and the rest of the expression would corresponds to the usual quotient $\frac{\Sigma}{\Sigma_{cr}}$.

- The derivations of the motion of the lens is straightforward in this approach. (Let see for example [Kopeikin Schaefer 1999, Frittelli 2003, Wucknitz Sperhake 2004])

# Magnifications I

## Magnifications and magnitudes

We define the physical magnification, in terms of the $\lambda$; instead of the usual astrophysical magnification defined in terms of $z$, which requires a cosmological model.

- *Angular magnification* $\mu$ and *intensity magnification* $\tilde{\mu}$:

$$\mu(\lambda) = \frac{1}{(1-\kappa)^2 - (\gamma_1^2 + \gamma_2^2)}, \qquad \tilde{\mu}(\lambda) = \frac{\mathscr{F}(\lambda, z)}{\mathscr{F}_{\text{Mink}}(\lambda, z)};$$

- General definition even for a Robertson-Walker lens. Let us note that $\mu \geq 1$.
- As a consequence of pure geometric arguments; the Etherington's theorem one has

$$\boxed{\mu = \tilde{\mu}}$$

- Cosmological intensity magnification $\mu'_c$ (one adopts a cosmological model):

$$\mu'_c(z) = \frac{\mathscr{F}(z)}{\mathscr{F}_{\text{Milne}}(z)};$$

is only function of redshift as usually employed in practical applications.

It recurs to the flat Milne Universe where one has a relation $\lambda(z)$.

# Magnifications II

### Two different notion of magnitudes

- The above magnifications can be related to the astronomical magnitudes. One can consider two different meanings for the magnitudes:

$$m - M = -\frac{5}{2}\log \mu(\lambda) + 5\log\left(\frac{\lambda(1+z)^2}{\lambda_{\mathrm{10Pc}}}\right)$$

it is valid in any spacetime and only contains information of the optical scalars and kinematic variables.

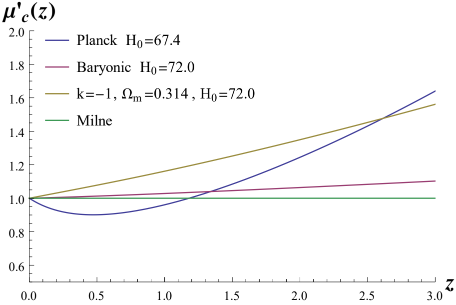
When the redshift is employed as indicator of distances one has:

$$m - M = -\frac{5}{2}\log \mu'_c(z) + 5\log\left(\frac{\lambda_{\mathrm{Milne}}(z)(1+z)^2}{\lambda_{\mathrm{10Pc}}}\right)$$

it only can be used when one has additional structure on the spacetime such as in cosmology where a family of preferred observed is present.

- Let us note that $\mu'_c(z)$ is the quantity intervening in the usual supernovae analysis where a cosmological constant $\Lambda$ is argued.

# Magnifications III




- The notion of 'brighter' and/or 'fainter' depends on the way the observational data are studied.

Let us note that for the ΛCDM concordance model $\mu'_c \leq 1$.
However, if one utilizes the notion $\tilde{\mu} = \mu$ one always would obtain $\mu \geq 1$.

- At the same time, the fluxes do not contain information about a cosmological constant Λ because $\mu$ contains the traceless part of the Ricci and Weyl tensors.

# Summary and final comments

- We have been presented a general formalism for weak lensing immersed in a standard cosmological context where the notion of deflection angle can be avoided and the motion of the lens is contained in a very straightforward way.
- The new equations derived here for the optical scalars allow to deal with more general matter content including sources with non-Newtonian components of the energy–momentum tensor and arbitrary motion.
- The formalism is well suitable for dealing with geometrical models, rather than by just its energy density distributions.
- The use of affine parameter suggest to use a slightly different notion of magnification, namely $\tilde{\mu}(\lambda)$ which do not depend on the choice of a given cosmological model.
- With this notion of mangnification the celebrated supernova luminosity observation, appears as an increase in the luminosity as a function of the affine distance.

Thanks for your attention!

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