Exercise: $M24$ has two irreducible representations of dimension 45: V and its conjugate representation $\overline{V}$.

Let X be a $K3$ manifold. Show that $H^1(X, \mathrm{End}TX) \simeq V \oplus \overline{V}$ naturally.

$N=4$ superconformal algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \delta_{m,-n}\frac{m^3-m}{12}c$$

$$[J^1_m, J^2_n] = J^3_{m+n}, \qquad [J^i_m, J^i_n] = m\frac{c}{3}\delta_{m,-n}$$

$$[L_m, J^i_n] = (m-n)J^i_{m+n}$$

$$[J_m, G^{\pm}_n] = (J \cdot G^{\pm})_{m+n}, \qquad [J_m, \bar{G}^{\pm}_n] = (J \cdot \bar{G}^{\pm})_{m+n}$$

$$[L_m, G^{\pm}_n] = \left(m - \frac{n}{2}\right) G^{\pm}_{m+n}, \qquad [L_m, \bar{G}^{\pm}_n] = \left(m - \frac{n}{2}\right) \bar{G}^{\pm}_{m+n}$$

$$[G^{\pm}_m, G^{\mp}_n] = 2L_{m+n} + \frac{m^2-m}{6}\delta_{m,-n}c \qquad [\bar{G}^{\pm}_m, \bar{G}^{\mp}_n] = 2L_{m+n} + \frac{m^2-m}{6}$$