Compton effect:
Measuring the energy of the scattered photons as a function of the scattering angle

Objects of the experiment

■ Recording the energy spectra of X-rays scattered on a scattering body under various angles.

■ Determining the energy of the scattered photons as a function of the scattering angle.

■ Comparing the measured energies with the energies calculated from energy and momentum conservation.

Principles

When X-rays pass through matter, part of them is scattered. According to classical physics, the frequency of the radiation should not be changed by the scattering process. However, in 1923 the American physicist *A. H. Compton* observed that the frequency was reduced in some of the X-rays.

In order to explain this phenomenon, the entire scattering process has to be treated in terms of quantum physics, and the X-rays have to be considered, for example, according to the particle aspect. Moreover, it is assumed that the scattering electrons are free, which is a good approximation for the outer atomic electron shells at energies in the range of X-rays. Thus, in a scattering process, a photon with the frequency ν_1, i.e. with the energy $E_1 = h\,\nu_1$, hits a free electron at rest with the rest mass m_0. The photon is scattered by the angle ϑ, and the electron moves at the velocity v with the angle φ relative to the direction of the incoming photon (see Fig. 1). For this collision process, conservation of energy and momentum is postulated as in an elastic collision of two classical particles.

As the photon has no rest mass, energy conservation in the relativistic formulation gives

$$h \cdot \nu_1 + m_0 \cdot c^2 = h \cdot \nu_2 + \frac{m_0 \cdot c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} \qquad \text{(I)}$$

c: velocity of light in vacuum

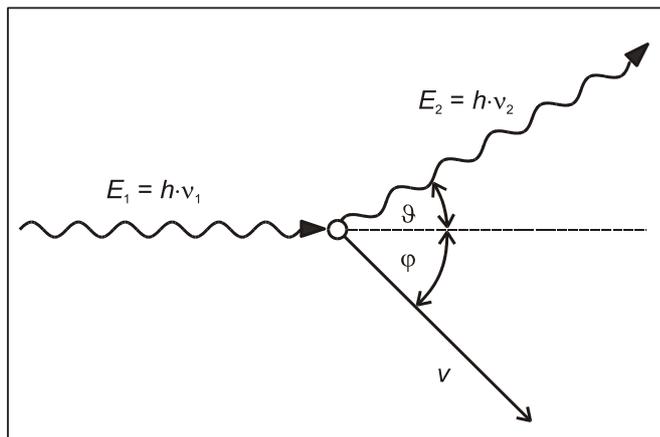and conservation of the components of the momentum gives

$$\frac{h \cdot \nu_1}{c} = \frac{h \cdot \nu_2}{c} \cdot \cos\vartheta + \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} \cdot v \cdot \cos\varphi$$

and

$$0 = \frac{h \cdot \nu_2}{c} \cdot \sin\vartheta + \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} \cdot v \cdot \sin\varphi \qquad \text{(II)}.$$

From Eqs. (I) and (II), the following equation is derived for the energy of the scattered radiation:

$$E_2 = \frac{E_1}{1 + \dfrac{E_1}{m_0 \cdot c^2} \cdot \left(1 - \cos\vartheta\right)} \qquad \text{(III)}.$$

In the experiment, *Compton*'s investigations are repeated on a scattering body made of plexiglass. The results are compared with Eq. (III). The spectrum is recorded with the aid of the X-ray energy detector.

Fig. 1   Schematic illustration of Compton scattering

Apparatus

1 X-ray apparatus with X-ray tube Mo and goniometer	554 811
1 Compton accessory Xray II	554 837
1 X-ray energy detector	559 938
1 Sensor-CASSY	524 010
1 MCA box	524 058
1 CASSY Lab	524 200
1 BNC cable, 1 m	501 02
1 PC with Windows 98/NT or higher version	

Setup

The experimental setup is illustrated in Fig. 2.

–   Put the Zr filter (from the scope of delivery of the X-ray apparatus) onto the beam entrance side of the circular collimator (from the scope of delivery of the Compton accessory Xray II).

–   Mount the circular collimator in the collimator mount of the X-ray apparatus.

–   Guide the connection cable of the table power supply through the empty duct of the X-ray apparatus, and connect it to the Mini-DIN socket of the X-ray energy detector.

–   Fasten the assembly of the X-ray energy detector and the sensor holder in the sensor arm of the goniometer.

–   Use the BNC cable supplied with the X-ray energy detector to connect the signal output of the detector to the BNC socket SIGNAL IN of the X-ray apparatus.

–   Push a sufficient length of the connection cable into the duct so that the sensor arm can perform a complete rotation.
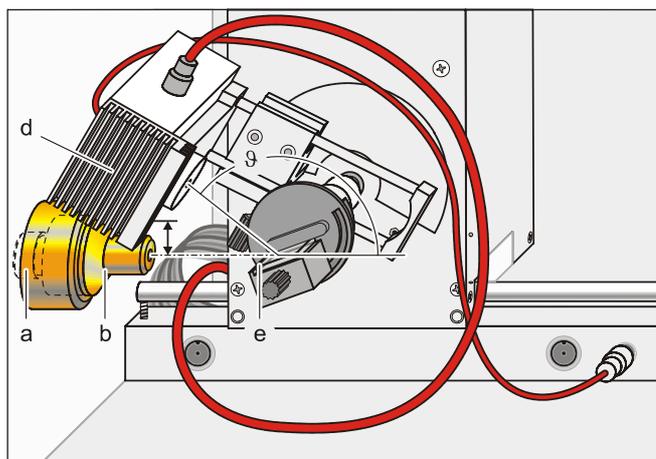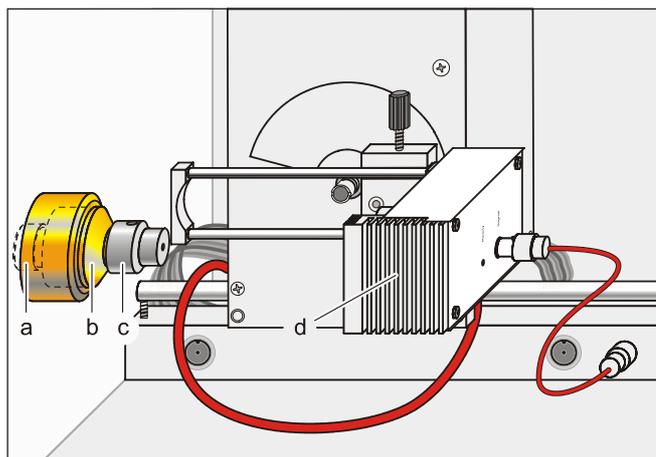

Fig. 2   Experimental setup for measuring the primary beam (above) and for measuring the energy of the scattered photons as a function of the scattering angle (below)
Zr filter (a), circular collimator (b), attenuating aperture (c), X-ray energy detector (d), scattering body (e)

The X-ray apparatus fulfils all regulations on the design of an X-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 807 / 97 Rö).

The built-in protective and shielding fixtures reduce the dose rate outside the X-ray apparatus to less than 1 µSv/h, which is of the order of magnitude of the natural background radiation.

■ Before putting the X-ray apparatus into operation, inspect it for damage and check whether the high voltage is switched off when the sliding doors are opened (see instruction sheet of the X-ray apparatus).

■ Protect the X-ray apparatus against access by unauthorized persons.

Avoid overheating of the X-ray tube.

■ When switching the X-ray apparatus on, check whether the ventilator in the tube chamber starts rotating.

The goniometer is positioned solely by means of electric stepper motors.

■ Do not block the target arm and sensor arm and do not use force to move them.

–   Press the SENSOR key and, using the ADJUST knob, adjust a sensor angle of 150° manually. If necessary, push the goniometer to the right.

–   Adjust the distance between the X-ray energy detector and the axis of rotation so that the detector housing just does not cover the X-ray beam at this sensor angle.

–   Then push the goniometer to the left so that the detector housing just does not touch the circular collimator (approx. 8 cm distance between the circular collimator and the axis of rotation).

–   Connect the Sensor-CASSY to the computer, and plug in the MCA box.

–   Use a BNC cable to connect the output SIGNAL OUT on the terminal panel of the X-ray apparatus to the MCA box.

Carrying out the experiment

–   Connect the table power supply to the mains (after approx. 2 minutes the LED shines green and the X-ray energy detector is ready for operation).

–   Call CASSY Lab, and select the measuring parameters "Multichannel Measurement, 256 Channels, Negative Pulses, Gain -3, Measuring Time 300 s".

Estimating the counting rate in the scattering arrangement:

– Put the plexiglass scattering body on the target stage, and clamp it.

– Press the TARGET pushbutton, and, using the ADJUST knob, adjust the target angle manually to 20°.

– Select the tube high voltage U = 35 kV and the emission current I = 1.00 mA, and switch the high voltage on.

– Start recording the spectrum with ⊙ or with the F9 key.

– Vary the sensor angle slowly between 150° and 30°, and each time read the total counting rate above on the right in the CASSY Lab window.

– Reduce the emission current if the total counting rate clearly exceeds 200 1/s.

Adjusting the counting rate of the primary beam:

– Remove the target holder with the target stage, and take the sensor into the 0° position.

– Put the attenuating aperture onto the circular collimator, and align it carefully (the screws should point upwards and downwards, respectively).

– Reduce the emission current to 0.1 mA, and switch the high voltage on.

– Start recording the spectrum with ⊙ or with the F9 key.

– In steps of 0.1° around 0° look for the sensor angle at which the total counting rate is only slightly greater than the counting rates measured in the scattering arrangement (if necessary, change the emission current slightly).

If no or only a small counting rate is measured:

– Check the alignment of the attenuating aperture and possibly rotate the attenuating aperture by 180°.

Recording the primary spectrum:

The X-rays to be measured produce additional fluorescence X-rays in the housing of the Si-PIN photodiode of the X-ray energy detector, which are also registered. Therefore the Au Lα and the Au Lβ lines are to be expected in the primary spectrum apart from the Mo Kα and the Mo Kβ lines (see Fig. 3). With the aid of these lines, the energy calibration can be carried out.

– Delete registered events, and record the primary spectrum with ⊙ or with the F9 key.

– Next open the "Energy Calibration" dialog window with the shortcut Alt+E, select "Global Energy Calibration", and enter the energies of the Au Kα line (9.71 keV) and the Mo Kα line (17.44 keV [1]).

– Select the menu item "Other Evaluations" → "Calculate Peak Center" in the pop-up menu of the diagram window, mark the region of the Au Kα line, and enter the result in the "Energy Calibration" dialog window.

– Then determine and enter the peak center of the Mo Kα line.

Recording the spectra in the scattering arrangement:

– Remove the absorption screen.

– Mount the target holder with the target stage on the goniometer.

– Put the plexiglass scattering body on the target stage, and clamp it.

– Select the emission current I = 1.00 mA (or the current determined previously for estimating the counting rate), and switch the high voltage on.

– Adjust a target angle of 20° and a sensor angle of 30°.

– Record a new spectrum with ⊙ or with the F9 key.

– Then record further spectra at constant target angles for the sensor angles 60°, 90°, 120° and 150°.

– Store the entire measurement with an appropriate name.

Measuring example

Fig. 3 shows the primary spectrum, i.e. the emission spectrum of the X-ray tube with Mo anode after monochromatization with a Zr filter.

In Fig. 4 a superposition of the spectra recorded under various scattering angles ϑ in an energy interval around the Mo Kα line is shown. It can be seen that the energy of the scattered radiation decreases with increasing scattering angle. The intensity of the scattered radiation has its minimum at ϑ = 90°.

Fig. 3   Emission spectrum of the X-ray tube with Mo anode after monochromatization with a Zr filter (ϑ = -0.1°)
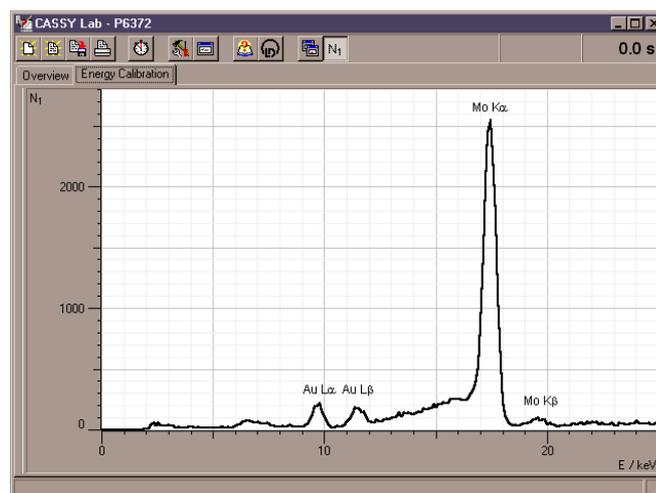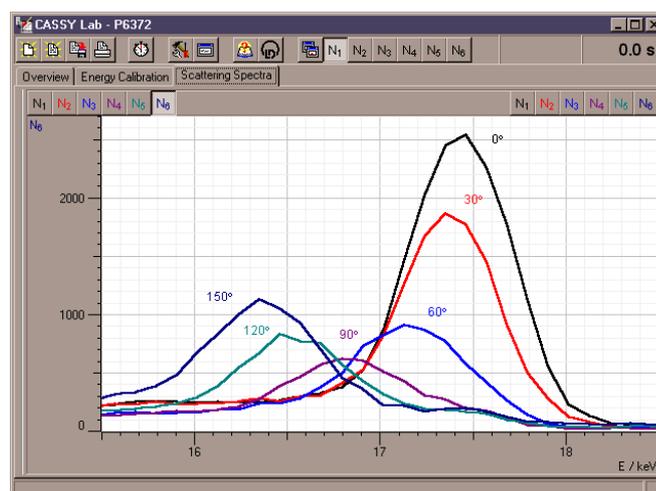

Fig. 4   Energy interval of the primary spectrum (0°) and the energy spectra measured under the scattering angles 30°, 60°, 90°, 120° and 150°.
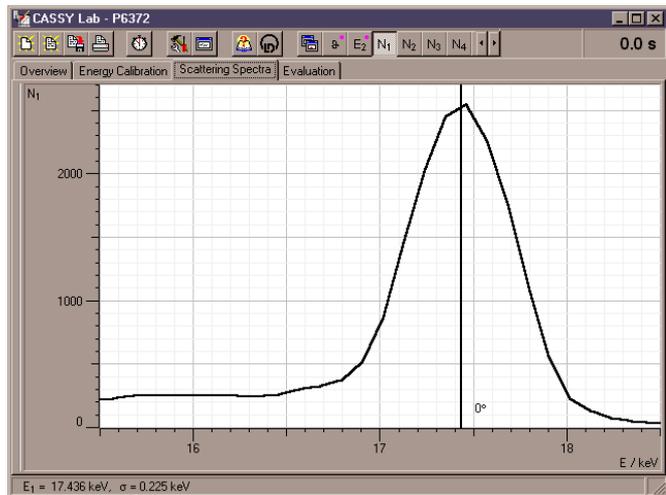
Evaluation

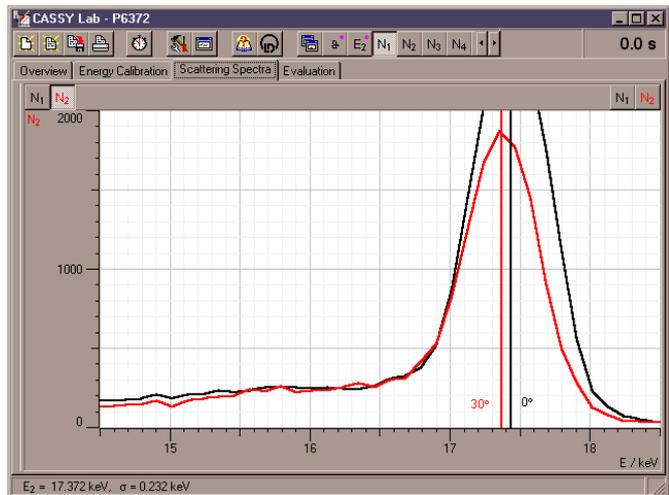Fig. 5a Section of the spectrum N_1 (0°, primary spectrum) with the unshifted Mo Kα line

Fig. 5b Shifted Mo Kα line from the spectrum N_2 (30°) and unshifted line from N_1.
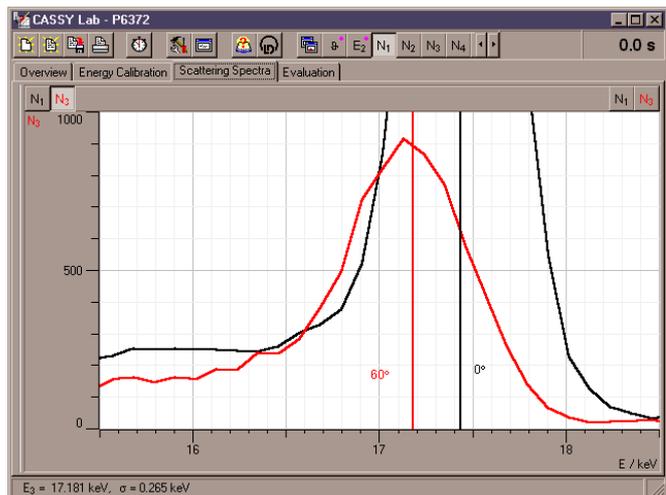

Fig. 5c Shifted Mo Kα line from the spectrum N_3 (60°) and unshifted line from N_1.
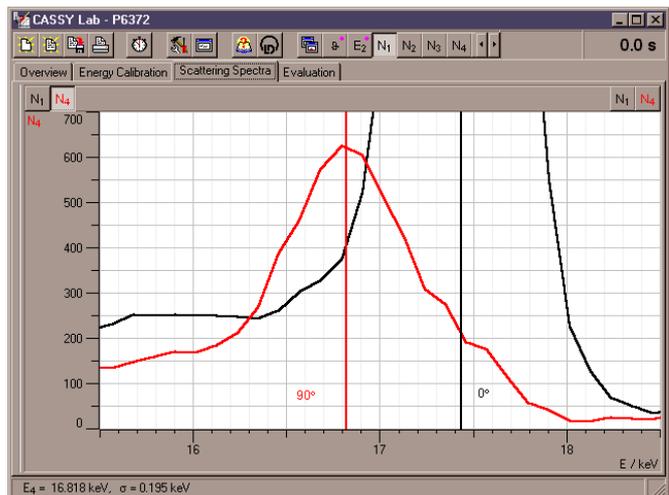

Fig. 5d Shifted Mo Kα line from the spectrum N_4 (90°) and unshifted line from N_1.
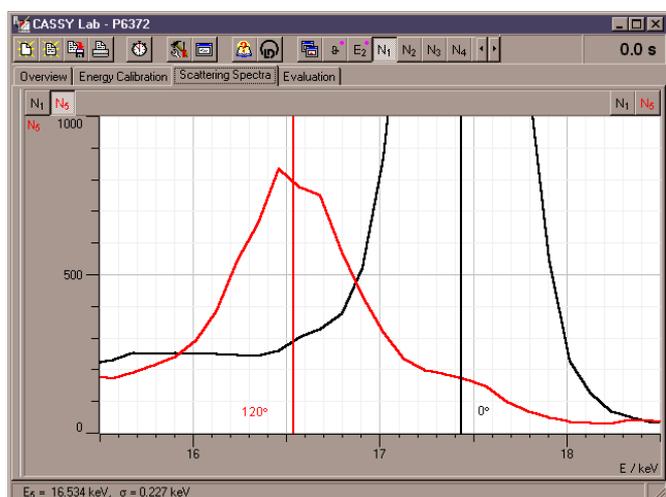

Fig. 5e Shifted Mo Kα line from the spectrum N_5 (120°) and unshifted line from N_1.
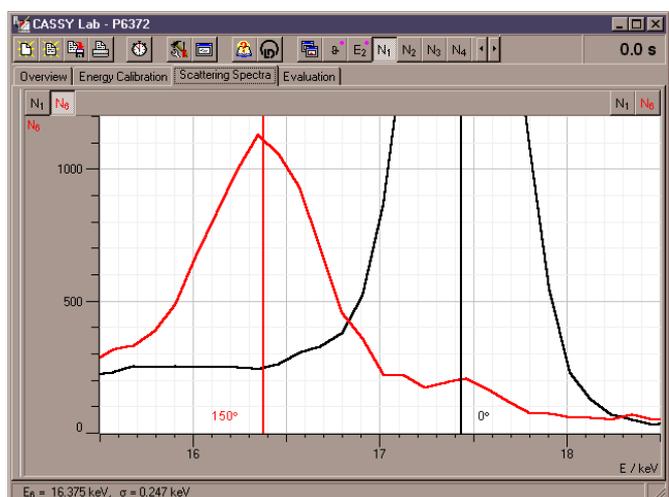

Fig. 5f Shifted Mo Kα line from the spectrum N_6 (150°) and unshifted line from N_1.

Preparation for further evaluation in CASSY Lab:

– Create the new quantity "Scattering Angle" (as parameter, symbol: &J, unit: °, from: 0, to: 180, decimal places: 0).

– Create the new quantity "Energy" (as parameter, symbol: E_2, unit: keV, from: 0, to: 20, decimal places: 2).

– Create the new display "Evaluation" with the scattering angle as x-axis and the energy as y-axis.

Determining the energy as a function of the scattering angle:

– Select an energy spectrum and a suitable interval.

– Call the menu item "Other Evaluations" → "Calculate Peak Center" in the pop-up menu of the diagram window, and mark the region of the energy-shifted peak (starting from $\vartheta = 90°$ the energy resolution of the detector is sufficient to separate the unshifted and the shifted peaks, see Fig. 5d to 5f).

– Enter the peak center obtained and the associated scattering angle in the table of the display "Evaluation" (see Fig. 6).

Comparison of the measured energies with the energies calculated from energy and momentum conservation:

– Select the display "Evaluation", and open the "Free Fit" dialog window with the shortcut Alt+F.

– Enter f(x,A,B,C,D) = 17.44/(1+17.44*(1-cos(x))/A) and the initial value for A: 511 (constant).

– Click on "Continue with marking a range", and mark the data points in the diagram.

The result is a theoretical curve calculated according to Eq. (III) with the parameters $E_1 = 17.44$ keV and $m\,c^2 = 511$ keV, which is in good agreement with the measured values (see Fig. 6).
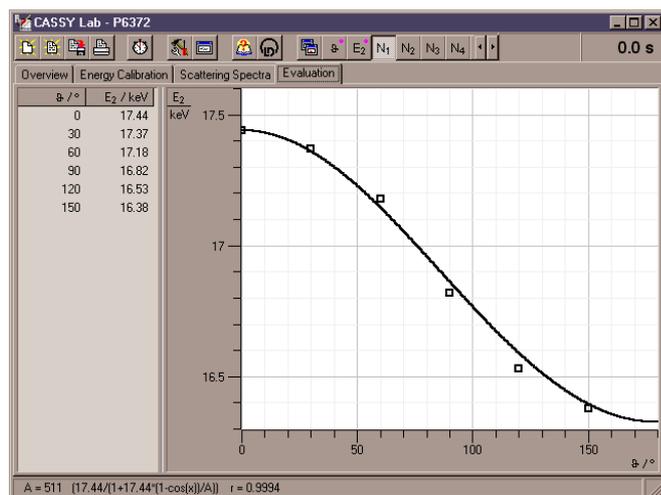
Results

When X-rays pass through matter, part of them is scattered and experiences an energy shift (Compton effect).

The energy shift can be calculated by describing the scattering process as a collision between an X-ray photon and a free electron at rest and by postulating the conservation of energy and momentum in this process.

Supplementary information

Alternatively, the comparison between measurement and theory can be carried out as a free fit with the free fit parameter A (the "rest mass" of the collision partner of the X-ray photon).

As a result a value for the parameter A is obtained, which agrees with the "rest mass" of an electron ($m\,c^2 = 511$ keV) to a good approximation.

Literature

[1] weighted mean value from
C. M. Lederer and V. S. Shirley, Table of Isotopes, 7th Edition, 1978, John Wiley & Sons, Inc., New York, USA.

Fig. 6   Energy E_2 determined from the measured energy spectra as a function of the scattering angle ϑ and curve calculated according to Eq. (III) .